Chapter One

ELECTRIC CHARGES AND FIELDS

SUMMARY

1. Electric and magnetic forces determine the properties of atoms, molecules and bulk matter.
2. From simple experiments on frictional electricity, one can infer that there are two types of charges in nature; and that like charges repel and unlike charges attract. By convention, the charge on a glass rod rubbed with silk is positive; that on a plastic rod rubbed with fur is then negative.
3. Conductors allow movement of electric charge through them, insulators do not. In metals, the mobile charges are electrons; in electrolytes both positive and negative ions are mobile.
4. Electric charge has three basic properties: quantisation, additivity and conservation.

   Quantisation of electric charge means that total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) i.e., $q = n\,e$, where $n = 0, \pm1, \pm2, \pm3, ...$ Proton and electron have charges $+e$, $-e$, respectively. For macroscopic charges for which n is a very large number, quantisation of charge can be ignored.

   Additivity of electric charges means that the total charge of a system is the algebraic sum (i.e., the sum taking into account proper signs) of all individual charges in the system.

   Conservation of electric charges means that the total charge of an isolated system remains unchanged with time. This means that when bodies are charged through friction, there is a transfer of electric charge from one body to another, but no creation or destruction of charge.
5. *Coulomb's Law*: The mutual electrostatic force between two point charges q_1 and q_2 is proportional to the product $q_1 q_2$ and inversely proportional to the square of the distance r_{21} separating them. Mathematically,

   $$\mathbf{F}_{21} = \text{force on } q_2 \text{ due to } q_1 = \frac{k\ (q_1 q_2)}{r_{21}^2}\,\hat{\mathbf{r}}_{21}$$

   where $\hat{\mathbf{r}}_{21}$ is a unit vector in the direction from q_1 to q_2 and $k = \dfrac{1}{4\pi\varepsilon_0}$ is the constant of proportionality.

   In SI units, the unit of charge is coulomb. The experimental value of the constant ε_0 is

   $$\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$$

   The approximate value of k is

   $$k = 9 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}}$$
6. The ratio of electric force and gravitational force between a proton and an electron is

   $$\frac{k\,e^2}{G\ m_e m_p} \cong 2.4 \times 10^{39}$$
7. *Superposition Principle*: The principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s). For an assembly of charges $q_1, q_2, q_3, ...$, the force on any charge, say q_1, is

the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by the Coulomb's law for two charges stated earlier.

8. The electric field **E** at a point due to a charge configuration is the force on a small positive test charge q placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude $|q|/4\pi\varepsilon_0 r^2$; it is radially outwards from q, if q is positive, and radially inwards if q is negative. Like Coulomb force, electric field also satisfies superposition principle.
9. An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point. The relative closeness of field lines indicates the relative strength of electric field at different points; they crowd near each other in regions of strong electric field and are far apart where the electric field is weak. In regions of constant electric field, the field lines are uniformly spaced parallel straight lines.
10. Some of the important properties of field lines are: (i) Field lines are continuous curves without any breaks. (ii) Two field lines cannot cross each other. (iii) Electrostatic field lines start at positive charges and end at negative charges —they cannot form closed loops.
11. An electric dipole is a pair of equal and opposite charges q and $-q$ separated by some distance $2a$. Its dipole moment vector **p** has magnitude $2qa$ and is in the direction of the dipole axis from $-q$ to q.
12. Field of an electric dipole in its equatorial plane (i.e., the plane perpendicular to its axis and passing through its centre) at a distance r from the centre:

$$\mathbf{E} = \frac{-\mathbf{p}}{4\,\pi\varepsilon_o}\,\frac{1}{(a^2+r^2)^{3/2}}$$

$$\cong \frac{-\mathbf{p}}{4\,\pi\varepsilon_o\,r^3}, \qquad \textit{for } r >> a$$

Dipole electric field on the axis at a distance r from the centre:

$$\mathbf{E} = \frac{2\,\mathbf{p}r}{4\,\pi\varepsilon_0(r^2-a^2)^2}$$

$$\cong \frac{2\mathbf{p}}{4\,\pi\varepsilon_0 r^3} \qquad \textit{for} \quad r >> a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charge.

13. In a uniform electric field **E**, a dipole experiences a torque $\boldsymbol{\tau}$ given by

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

but experiences no net force.

14. The flux $\Delta\phi$ of electric field **E** through a small area element $\Delta\mathbf{S}$ is given by

$$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S}$$

The vector area element $\Delta\mathbf{S}$ is

$$\Delta\mathbf{S} = \Delta S\,\hat{\mathbf{n}}$$

where ΔS is the magnitude of the area element and $\hat{\mathbf{n}}$ is normal to the area element, which can be considered planar for sufficiently small ΔS.

For an area element of a closed surface, $\hat{\mathbf{n}}$ is taken to be the direction of *outward* normal, by convention.

15. *Gauss's law*: The flux of electric field through any closed surface *S* is $1/\varepsilon_0$ times the total charge enclosed by *S*. The law is especially useful in determining electric field **E**, when the source distribution has simple symmetry:

(i) *Thin infinitely long straight wire of uniform linear charge density* λ

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{n}}$$

where r is the perpendicular distance of the point from the wire and $\hat{\mathbf{n}}$ is the radial unit vector in the plane normal to the wire passing through the point.

(ii) Infinite thin plane sheet of uniform surface charge density σ

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the plane, outward on either side.

(iii) *Thin spherical shell of uniform surface charge density* σ

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \qquad (r \geq R)$$

$$\mathbf{E} = 0 \qquad (r < R)$$

where r is the distance of the point from the centre of the shell and R the radius of the shell. q is the total charge of the shell: $q = 4\pi R^2\sigma$.

The electric field outside the shell is as though the total charge is concentrated at the centre. The same result is true for a solid sphere of uniform volume charge density. The field is zero at all points inside the shell

Physical quantity	Symbol	Dimensions	Unit	Remarks
Vector area element	$\Delta\mathbf{S}$	[L^2]	m^2	$\Delta\mathbf{S} = \Delta S\,\hat{\mathbf{n}}$
Electric field	$\mathbf{E}$	[MLT^{-3}A^{-1}]	V m^{-1}	
Electric flux	ϕ	[ML^3 T^{-3}A^{-1}]	V m	$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S}$
Dipole moment	$\mathbf{p}$	[LTA]	C m	Vector directed from negative to positive charge
Charge density				
linear	λ	[L^{-1} TA]	C m^{-1}	Charge/length
surface	σ	[L^{-2} TA]	C m^{-2}	Charge/area
volume	ρ	[L^{-3} TA]	C m^{-3}	Charge/volume

POINTS TO PONDER

1. You might wonder why the protons, all carrying positive charges, are compactly residing inside the nucleus. Why do they not fly away? You will learn that there is a third kind of a fundamental force, called the strong force which holds them together. The range of distance where this force is effective is, however, very small ~10^{-14} m. This is precisely the size of the nucleus. Also the electrons are not allowed to sit on top of the protons, i.e. inside the nucleus, due to the laws of quantum mechanics. This gives the atoms their structure as they exist in nature.
2. Coulomb force and gravitational force follow the same inverse-square law. But gravitational force has only one sign (always attractive), while Coulomb force can be of both signs (attractive and repulsive), allowing possibility of cancellation of electric forces. This is how gravity, despite being a much weaker force, can be a dominating and more pervasive force in nature.
3. The constant of proportionality k in Coulomb's law is a matter of choice if the unit of charge is to be defined using Coulomb's law. In SI units, however, what is defined is the unit of current (A) via its magnetic effect (Ampere's law) and the unit of charge (coulomb) is simply defined by (1C = 1 A s). In this case, the value of k is no longer arbitrary; it is approximately 9×10^9 N m^2 C^{-2}.
4. The rather large value of k, i.e., the large size of the unit of charge (1C) from the point of view of electric effects arises because (as mentioned in point 3 already) the unit of charge is defined in terms of magnetic forces (forces on current–carrying wires) which are generally much weaker than the electric forces. Thus while 1 ampere is a unit of reasonable size for magnetic effects, 1 C = 1 A s, is too big a unit for electric effects.
5. The additive property of charge is not an 'obvious' property. It is related to the fact that electric charge has no direction associated with it; charge is a scalar.
6. Charge is not only a scalar (or invariant) under rotation; it is also invariant for frames of reference in relative motion. This is not always true for every scalar. For example, kinetic energy is a scalar under rotation, but is not invariant for frames of reference in relative motion.
7. Conservation of total charge of an isolated system is a property independent of the scalar nature of charge noted in point 6. Conservation refers to invariance in time in a given frame of reference. A quantity may be scalar but not conserved (like kinetic energy in an inelastic collision). On the other hand, one can have conserved vector quantity (e.g., angular momentum of an isolated system).
8. Quantisation of electric charge is a basic (unexplained) law of nature; interestingly, there is no analogous law on quantisation of mass.
9. Superposition principle should not be regarded as 'obvious', or equated with the law of addition of vectors. It says two things: force on one charge due to another charge is unaffected by the presence of other charges, and there are no additional three-body, four-body, etc., forces which arise only when there are more than two charges.
10. The electric field due to a discrete charge configuration is not defined at the locations of the discrete charges. For continuous volume charge distribution, it is defined at any point in the distribution. For a surface charge distribution, electric field is discontinuous across the surface.

11. The electric field due to a charge configuration with total charge zero is not zero; but for distances large compared to the size of the configuration, its field falls off faster than $1/r^2$, typical of field due to a single charge. An electric dipole is the simplest example of this fact.

EXERCISES

1.1 What is the force between two small charged spheres having charges of 2×10^{-7}C and 3×10^{-7}C placed 30 cm apart in air?

1.2 The electrostatic force on a small sphere of charge 0.4 µC due to another small sphere of charge −0.8 µC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

1.3 Check that the ratio $ke^2/G\, m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

1.4 (a) Explain the meaning of the statement 'electric charge of a body is quantised'.
(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

1.5 When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

1.6 Four point charges q_A = 2 µC, q_B = –5 µC, q_C = 2 µC, and q_D = –5 µC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 µC placed at the centre of the square?

1.7 (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?
(b) Explain why two field lines never cross each other at any point?

1.8 Two point charges q_A = 3 µC and q_B = –3 µC are located 20 cm apart in vacuum.
(a) What is the electric field at the midpoint O of the line AB joining the two charges?
(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

1.9 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, –15 cm) and B: (0,0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?

1.10 An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.

1.11 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.
(a) Estimate the number of electrons transferred (from which to which?)
(b) Is there a transfer of mass from wool to polythene?

1.12 (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

1.13 Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

1.14 Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

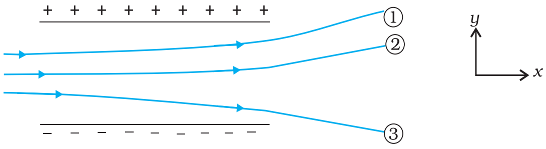


FIGURE 1.33

1.15 Consider a uniform electric field $\mathbf{E} = 3 \times 10^3\ \hat{\mathbf{i}}$ N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the *yz* plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the *x*-axis?

1.16 What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

1.17 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 Nm^2/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

1.18 A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (*Hint:* Think of the square as one face of a cube with edge 10 cm.)

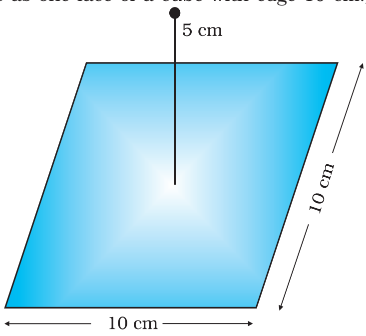


FIGURE 1.34

1.19 A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

1.20 A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

1.21 A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

1.22 A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m^2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

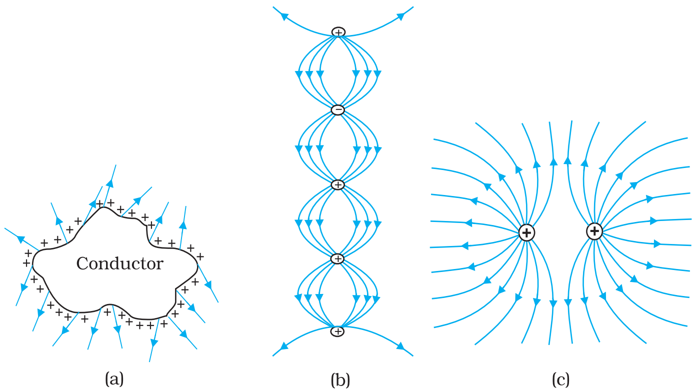
1.23 An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

1.24 Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is **E**: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

ADDITIONAL EXERCISES

1.25 An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. (g = 9.81 m s^{-2}; e = 1.60×10^{-19} C).

1.26 Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic field lines?



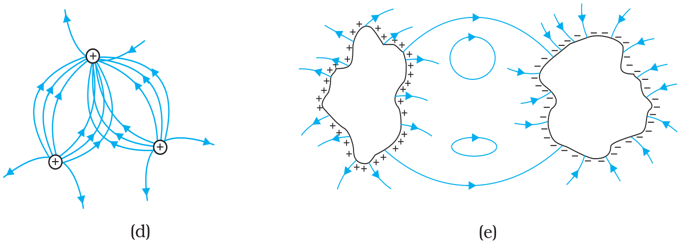


FIGURE 1.35

1.27 In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative *z*-direction ?

1.28 (a) A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge *Q*. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge *q* is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is *Q* + *q* [Fig. 1.36(b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

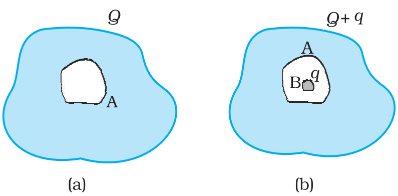


FIGURE 1.36

1.29 A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $(\sigma/2\varepsilon_0)\ \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

1.30 Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [*Hint:* Use Coulomb's law directly and evaluate the necessary integral.]

1.31 It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) *e*, and the 'down' quark (denoted by d) of charge (–1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

1.32 (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where **E** = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

1.33 A particle of mass m and charge $(-q)$ enters the region between the two charged plates initially moving along x-axis with speed v_x (like particle 1 in Fig. 1.33). The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m\, v_x^2)$.

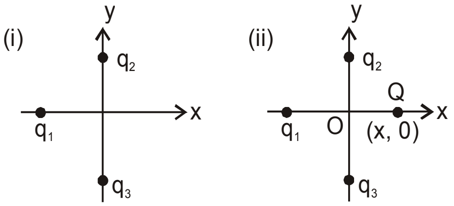
Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

1.34 Suppose that the particle in Exercise in 1.33 is an electron projected with velocity $v_x = 2.0 \times 10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.)

Chapter 1:- Electric Charge

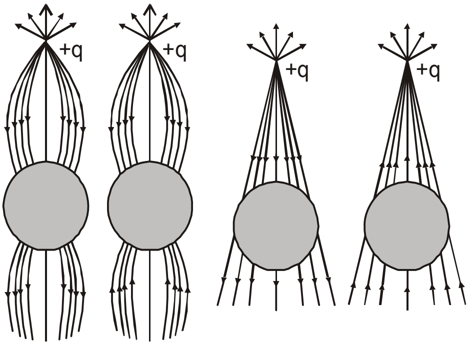
Multiple Choice Questions (MCQs)

1. In figure two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the + x-direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at (x, 0), the force on q_1



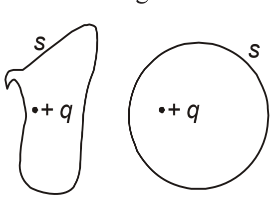
(a) shall increase along the positive x-axis
(b) shall decrease along the positive x-axis
(c) shall point along the negative x-axis
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

2. A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by

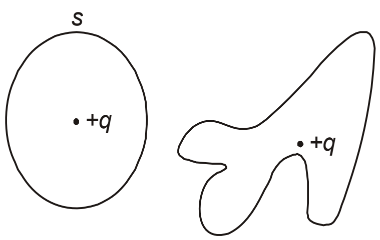


(a) (b) (c) (d)

3. The electric flux through the surface



(i) (ii)

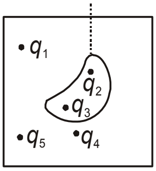


(iii) (iv)

(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

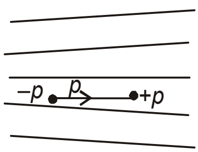
4. Five charges q_1, q_2, q_3,q_4, and q_5 are fixed at their positions as shown in Figure, **S** is a Gaussian surface. The Gauss' law is given by

$$\int_s E.dS = \frac{q}{\varepsilon_0}.$$ Which of the following statements is correct?



(a) E on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only
(c) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5.
(d) Both E on the LHS and q on the RHS will have contributions from q_2 and q_4 only

5. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct?



(a) The dipole will not experience any force
(b) The dipole will experience a force towards right
(c) The dipole will experience a force towards left
(d) The dipole will experience a force upwards

6. A point charge + q is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is
(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge.

7. A hemisphere is uniformely charged positively. The electric field at a point on a diameter away from the centre is directed
(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

Multiple Choice Questions (MCQs) (More than one option correct)

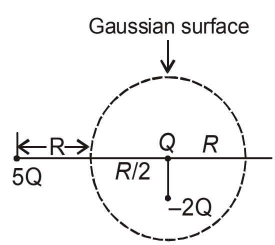
1. If $\int_s \text{E.dS} = 0$ over a surface, then
(a) the electric field inside the surface and on it is zero
(b) the electric field inside the surface is necessarily uniform
(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it
(d) all charges must necessarily be outside the surface

2. The electric field at a point is
(a) always continuous
(b) continuous if there is no charge at that point
(c) discontinuous only if there is a negative charge at that point
(d) discontinuous if there is a charge at that point

3. If there were only one type of charge in the universe, then
(a) $\oint_s \text{E.dS} \neq 0$ on any surface
(b) $\oint_s \text{E.dS} = 0$ if the charge is outside the surface
(c) $\oint_s \text{E.dS}$ could not be defined
(d) $\oint_s \text{E.dS} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface

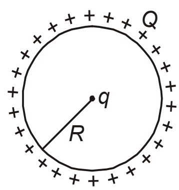
4. Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region,
(a) the electric field is necessarily zero
(b) the electric field is due to the dipole moment of the charge distribution only
(c) the dominant electric field is $\propto \frac{1}{r_3}$, for large r, where r is the distance from a origin in this regions
(d) the work done to move a charged particle along a closed path, away from the region, will be zero

5. Refer to the arrangement of charges in figure and a Gaussian surface of radius R with Q at the centre. Then,



(a) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$
(b) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$
(c) flux through the surface of sphere due to 5Q is zero
(d) field on the surface of sphere due to –2Q is same everywhere

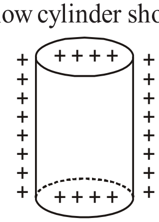
6. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring figure. Then,



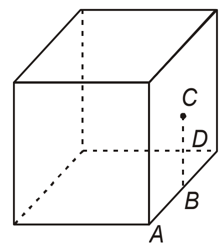
(a) if $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre
(b) if $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring
(c) if $q < 0$, it will perform SHM for small displacement along the axis
(d) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$

Very Short Answer Questions

1. An arbitrary surface encloses a dipole. What is the electric flux through this surface?
2. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on
(i) the inner surface
(ii) the outer surface?
3. The dimensions of an atom are of the order of an Angstrom. Thus, there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero?
4. If the total charge enclosed by a surface is zero, does it imply that the elecric field everywhere on the surface is zero? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero.
5. Sketch the electric field lines for a uniformly charged hollow cylinder shown in figure.



6. What will be the total flux through the faces of the cube as given in the figure with side of length a if a charge q is placed at?



(a) A a corner of the cube
(b) B mid-point of an edge of the cube
(c) C centre of a face of the cube
(d) D mid-point of B and C

Short Answer Questions

1. A paisa coin is made up of Al-Mg alloy and weight 0.75g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.
2. Consider a coin of Question 20. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by

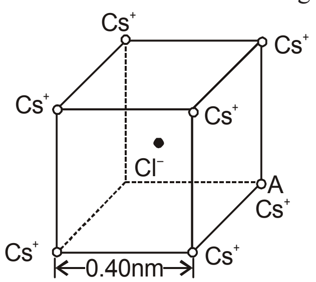
(i) 1 cm ($\sim \frac{1}{2} \times$ diagonal of the one paisa coin)

(ii) 100 m (~length of a long building)

(iii) 10^6 m (radius of the earth). Find the force on each such point charge in each of the three cases. What do you conclude from these results?

3. Figure represents a crystal unit of cesium chloride, CsCl The cesium atoms, represented by open circles are situated at the corners of a cube of side 0.40nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.
(i) What is the net electric field on the Cl atom due to eight Cs atoms?
(ii) Suppose that the Cs atom at the corner A is missing. What is the net force now on the

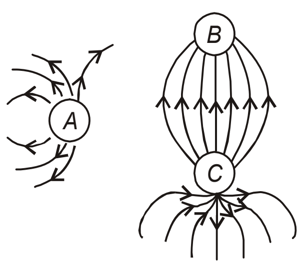
Cl atom due to seven remaining Cs atoms?



°Cs + • Cl^-

4. Two charges q and –3q are placed fixed on x-axis separated by distance d. Where should a third charge 2q be placed such that it will not experience any force ?

5. Figure shows the electric field lines around three point charges A, B and C



(i) Which charges are positive?

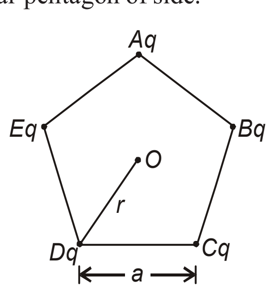
(ii) Which charge has the largest magnitude? Why?

(iii) In which region or regions of the picture could the electric field be zero? Justify your answer.

(a) Near A (b) Near B

(c) Near C (d) Nowhere

6. Five charges, q each are placed at the corners of a regular pentagon of side.



(a) (i) What will be the electric field at 0, the centre of the pentagon?

(ii) What will be the electric field at 0 if the charge from one of the corners (say A) is removed?

(iii) What will be the electric field at 0 if the charge q at A is replaced by –q?

(b) How would your answer to (a) be affected if pentagon is replaced by n-sided regular polygon with charge q at each of its corners?

Long Answer Questions

1. In 1959 Lyttleton and Bondi suggested that the expansion of the universe could be explained if matter carried a net charge. Suppose that the universe is made up of hydrogen atoms with a number density N, which is maintained a constant. Let the charge on the proton be $e_p = -(1 + y)e$ where e is the electronic charge.

(a) Find the critical value of y such that expansion may start.

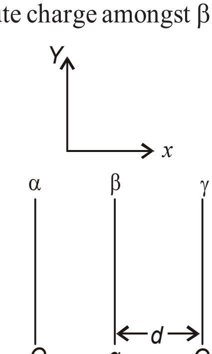
(b) Show that the velocity of expansion is proportional to the distance from the centre.

2. Consider a sphere of radius R with charge density distributed as $p(r) = kr$ for $r \leq R = 0$ for $r > R$.

(a) Find the electric field as all points r.

(b) Suppose the total charge on the sphere is 2e where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

3. Two fixed, identical conducting plates (α and β), each of surface area S are charged to –Q and q respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge q at a distance d (figure). The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.



(a) Find the electric field acting on the plate γ before collision.
(b) Find the charges on β and γ after the collision.
(c) Find the velocity of the plate γ after the collision and at a distance d from the plate β.

4. There is another useful system of units, besides the SI/MKS. A system, called the CGS (Centimeter-Gram-Second) system. In this system, Coulomb's law is given by

$$F = \frac{Qq}{r^2}\hat{r}.$$

where the distance r is measured in cm ($= 10^{-2}\mu$), F in dynes ($= 10^{-5}$N) and the charges in electrostatic units (es units), where 1 es unit of charge $= \frac{1}{[3]} \times 10^{-9}$ C. The number [3] actually arises from the speed of light in vaccum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c, then is $c = 3 \times 10^8$ m/s.

(i) Show that the Coulomb's law in CGS units yields 1 esu of charge = 1 $(\text{dyne})^{1/2}$ cm. Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

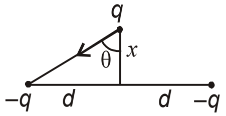
(ii) Write 1 esu of charge = xC, where x is a dimensionless number. Show that this gives $\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2}\frac{Nm^2}{C^2}$. With $x = \frac{1}{[3]} \times 10^{-9}$,

we have $\frac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9 \frac{Nm^2}{C^2}$,

$$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \frac{Nm^2}{C^2} \text{(exactly)}.$$

5. Two charges –q each are fixed separated by distance 2d. A third charge q of mass m placed at the mid-point is displaced slightly by $x(x << d)$ perpendicular to the line joining the two fixed charged as shown in figure. Show that q will perform simple harmonic oscillation of time period.

$$T = \left[\frac{8\pi^3\varepsilon_0 md^3}{q^2}\right]^{1/2}$$

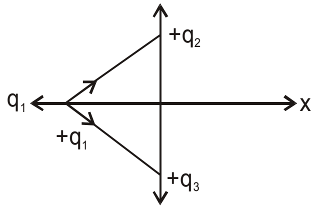


6. Total charge –Q is uniformly spread along length of a ring of radius R. A small test charge + q of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.
(a) Show that the particle executes a simple harmonic oscillation.
(b) Obtain its time period.

NCERT EXEMPLAR SOLUTIONS

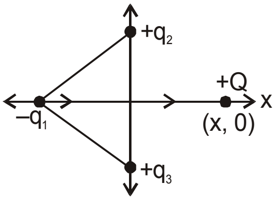
Multiple Choice Questions (MCQs)

1. **(a)** The force on q_1 depend on the force acting between q_1 and q_2 and q_1 and q_3 so that the net force acting on q_1 by q_2 and q_1 by q_3 is along the + x-direction, so the force acting between q_1, q_2 and q_1, q_3 is attractive force as shown in figure :



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Then the force acting between q_1 and charge Q (positive) is also know as attractive force and then the net force on q_1 by q_2, q_3 and Q are along the same direction as shown in the figure.

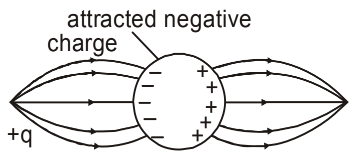


The figure shows that the force on q_1 shall increase along the positive x-axis due to the positive charge Q.

2. **(a)** If a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive Charge and electric field passes through a charged body. This leaves an excess of positive charge on the (right) surface of sphere due to the induction process.

Both type of charges are bound in the (isolated conducting) sphere and cannot escape. They, therefore, reside on the surface.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as shown in figure.



An electric field lines start from positive charge and ends at negative charge.

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity shown in figure (a).

3. **(d)** **By Gauss's law :** The total of the electric flux out of a closed surface is equal to the charge enclosed devided by the permittivity i.e., $\phi = \frac{Q}{\varepsilon_0}$.

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface. So all the given figures have same electric flux as all of them also has same single positive charge.

4. **(b)** Gauss's law states that total electric flux of an enclosed surface is given by, $\oint_s E.dS = \frac{q}{\varepsilon_0}$, includes the sum of all charges enclosed by the surface.

The charges may be located anywhere inside the surface, and out side the surface. Then, the electric field on the left side of equation is due to all the charges, both inside and outside S.

So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

5. **(c)** The electric field lines, are directed away from positively charged source and directed toward negatively charged source. In electric field force are directly proportional to the electric field strength hence, higher the electric field strength greater the force and vice-versa.

The space between the electric field lines is increasing, from left to right so strength of electric field decreases with the increase in the space between electric field lines. Then the force on charges also decreases from left to right.

Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

6. **(a)** When a positive point charge $+q$ is placed near an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane. This is called induction process and the electric field on a isolated conducting plane at point is directly projected in a plane perpendicular to the field and away from the plane.

7. **(a)** Consider a point on diameter away from the centre of hemisphere uniformly positively charged, then the electric field is perpendicular to the diameter and the component of electric intensity parallel to the diameter cancel out.

Multiple Choice Questions (More Than One Option)

1. **(c, d)** In general Gauss's law is applied to any closed surface.

In Gauss's law, $\oint_s E.dS$ means the algebraic sum of number of flux lines entering the surface and number of flux lines leaving the surface.

When $\oint_s E.dS = 0$, the number of flux lines entering the surface must be equal to the number of flux lines leaving it. If there is no change inside.

$\oint_s E.dS = \frac{q}{\varepsilon_0}$, where q is charge enclosed by the surface, and $\in_0$ is permitivity.

If $\oint_s E.dS = 0, q = 0$ then the resultant charge enclosed by the surface must be zero.

So, all other charges must necessarily be outside the surface. This is because charges outside because of the fact that charges outside the surface do not contribute to the electric flux.

2. **(b, d)**
Due to a charge Q the electric field at a point in space may be defined as the electric force per unit charge on a changed object at that point. Thus, electric field due to the charge Q will be continuous, if there is no charge at that point. It will be discontinuous if there is a charge at that point.

3. **(b, d)** By Gauss' law, $\oint_s E.dS = \frac{q}{\varepsilon_0}$, where q is the net charge enclosed within the surface.

If $\oint_s E.dS = 0$, then q = 0, i.e., net charge enclosed by the surface must be zero. So all the charge is outside the surface. Here, electric flux doesn't depend on the type or nature of charge.

4. **(c, d)**
The electric field due to dipole is proportional to $(1/r^3)$. When there are various types of charges in a region, but the net charge is zero, the region is, supposed to contain a number of electric dipoles.
So, at points outside the region the dominant electric field $\propto \frac{1}{r^3}$ for large r.
Then, electric field is conservative, work done to move a charged particle along a closed path, away from the region will be zero

5. **(a, c)**

By Gauss' law $\left(\oint_s E.dS = \frac{q}{\epsilon_0}\right)$ where q is the charge inclosed by the surfaces, so, net charge inside the surface is $(-2Q + Q = -Q)$
∴ Total flux through the surface of the sphere

$= \frac{-Q}{\varepsilon_0}$

Now, assuming charge 5Q lies outside the Gaussian surface, thus it will not contribute to electric flux through the given Gaussian surface.

6. **(a, b, c)**
The main concepts are used
(i) Interaction between charges.
(ii) Stable and unstable equilibrium.
The positive charge Q is uniformly distributed at the-outer surface of the ring. Thus, electric field inside the sphere is zero.
Then the positive charge of the ring will interact equally a charge placed at centre of ring will be in unstable equilibrium.
So the effect of electric field on charge q due to the positive charge Q is zero.
Now, there are two forces are active between charges (Q and q).
(i) attractive force. (ii) repulsive force.
There are two cases obtained :
Case I : When charge q > 0 i.e., q is a positive charge, there creates a repulsive force between charge q and Q.
The repulsive forces of charge Q from all around the charge q will push it towards.ihe centre if it is displaced from the centre of the ring.
Case II : When charge q < 0 i.e., q is a negative charge then there is an attractive force between charge Q and q.
If q is displaced slightly small from the centre then the positive charges nearer to this charge will attract it towards itself and charge q will never return to the centre.

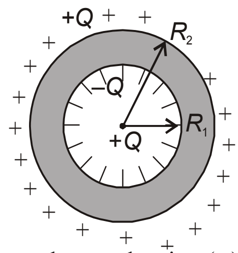
Very Short Answer Questions

1. By Gauss's law

$\oint_s E.dS = \frac{q}{\varepsilon_0}$ where, q is the net charge inside that enclosed surface.
Now, the net charae on Gaussion surface due to a dipole is $-q + q = 0$.
So, electric flux through a surface enclosing a dipole $\phi = \frac{-q+q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$

2. As we know that the surface density is $\left(\sigma = \frac{Q}{A}\right)$

As per given data the charge placed at the centre of the spherical cavity is positively charged. So, the charge (–Q) created at the inner surface of the sphere, due to induction by charge +Q at centre and due to this charge created at outer surface of the sphere is + Q.



So, surface charge density (σ) on inside the surface = $\frac{-Q}{4\pi R_1^2}$

Now, surface charge density (σ) on outside the surface = $\frac{+Q}{4\pi R_2^2}$

3. Electrostatic fields are caused by the presence of charges.
But there can be no excess charge on the inside surface of an isolated conductor. So, the net charge inside a conductor is zero despite the fact that the dimensions of an atom are of the order of an Angstrom. So force between electron and proton is very large. Electric field outside the atom will be $E \propto \frac{2aq}{r^3}$

where (2a) – average distance between P^+ and e^- in atom
r – very large distance between point and atomic dipole when ($r >> a$), so electric field tends to zero.

4. Gauss' law states that

$$\oint_s E.dS = \frac{q}{\varepsilon_0}$$

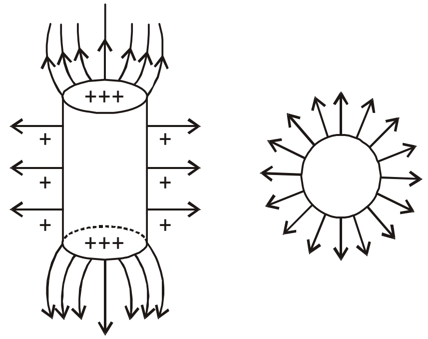
The electric field in the LHS is due to all the charges both inside and outside the surface. The term q on the right side of the equation given by Gauss' law represent only the total charge inside and outside of the surface.
So, despite being total charge inside enclosed by a sorface zero, it doesn't imply that the electric field everywhere on the surface is zero, the field may be normal to the surface.

Also, if the electric field everywhere on a Gaussian surface is zero, it doesn't imply that net charge inside it is zero.It is possible, only when outside charge will be zero.

i.e., Putting $E = 0$ in $\oint E.dS = \frac{q}{\varepsilon_0}$

So, $q = 0$.

5. As per given figure, there is no any negative charge only positive charge is spreading uniformly.
Thus, the electric field lines of force emerge away perpendicularly from surface. So it will start from positive charges and move towards infinity as no negative charge. So, no end point.



6. **(a)** To make symmetric identical cube.
So there are eight corners in a cube. Hence, total charge distribution for a cube is $\frac{q}{8 \times 1} = \frac{q}{8}$

By Gauss's law the electric flux at $A = \frac{q}{8\varepsilon_0}$

(b) Charge q at B symmetric, we can consider the identical cube of side $\left(\frac{q}{2}\right)$. So the charge distribution for one cube $Q = \frac{q}{8 \times \frac{1}{2}} = \frac{q}{4}$.

So, total flux through the faces of the given cube $= q/4\,\varepsilon_0$.

(c) When the charge q is placed at C, the centre of a face of the cube, it is being shared equally by 2 cubes. So charge distribution for one cube is $= \frac{q}{8 \times \frac{1}{4}} = \frac{q}{2}$. So, total flux through the faces of the given cube $= q/2\,\varepsilon_0$.

(d) Similarly, when charge q placed at Q, the mid-point of B and C. It is being shared equally by 2 cubes, So charge distribution for one cube is $= \dfrac{q}{8 \times \dfrac{q}{4}} = \dfrac{q}{2}$. So, total flux through the faces of the given cube = $q/2\ \varepsilon_0$.

Short Answer Questions

1. As, given that

Mass of a paisa coin = 0.75g

diagonal = 17 mm,

As we know that,

Atomic mass of aluminium = 26.9815g

Avogadro's number = 6.023×10^{23}

So, the number of mole in (0.75 g) = $\dfrac{0.75}{26.9815}$

then the number of aluminium atoms in a paisa coin,

$$N = \frac{6.023 \times 10^{23}}{26.9815} \times 0.75 = 1.6742 \times 10^{22}$$

Megnitude of charge on a proton or electron
$= 1.6 \times 10^{-19}$ C

As charge number of Al is 13, each atom of Al contains 13 protons and 13 electrons.

∴ Magnitude of positive and negative charges in one paisa coin = N ze

$= 1.6742 \times 10^{22} \times 13 \times 1.6 \times 10^{-19}$ C

$= 3.48 \times 10^{4}$ C

This is a very large amount of charge. So, we can conclude that even a 0.75 g Al and Mg contains enormous amount of positive and negative charge (these are) equal in magnitude.

2. Here charges are equal and opposite, so by columb's law, force of attraction between charges is :

$$F = \frac{1}{4\pi \epsilon_0}\left(\frac{q_1 q_2}{r^2}\right) \quad \text{where, } (q_1 = q_2 = q)$$

As given that :

$q = \pm 34.8\text{KC} = \pm 3.48 \times 10^{4}$C

$r_1 = 1\text{cm} = 10^{-2}$m, $r_2 = 100$ m, $r_3 = 10^{6}$ m

and $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ \text{N} - \text{m}^2/\text{C}^2$

(i) $F_1 = \dfrac{|q|^2}{4\pi\varepsilon_0 r_1^2} = \dfrac{9\times10^9 (3.48\times10^4)^2}{(10^{-2})^2}$

$= 1.09 \times 10^{23}$ N

(ii) $F_2 = \dfrac{|q|^2}{4\pi\varepsilon_0 r_2^2} = \dfrac{9\times10^9 (3.48\times10^4)^2}{(100)^2}$

$= 1.09 \times 10^{15}$ N

(iii) $F_3 = \dfrac{|q|^2}{4\pi\varepsilon_0 r_3^2} = \dfrac{9\times10^9 (3.48\times10^4)^2}{(10^6)^2}$

$= 1.09 \times 10^{7}$ N

So, when ± charges in ordinary neutral matter are separated as point charges, they exert an enormous force. Hence very difficult to disturb electrical neutrality of matter.

3. (i) Here, the distance of chlorine atom at the centre of the cube is equal distance from all the eight corners of cube where cesium atoms are placed. So, due to symmetry the forces due to all Cs ions, on Cl atom will cancel out.

Hence, $E = \dfrac{F}{q}$ where $F = 0$

So, $E = 0$

(ii) The, net force on Cl atom at A would be.

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2},$$

where, r = distance between Cl ion and Cs ion

Applying Pythagorous theorem then,

$$r = \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9}\ \text{m}$$

$r = 0.346 \times 10^{-9}$ m,

We know that,

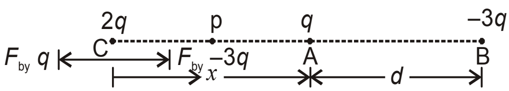
$$e = (1.6 \times 10^{-19}), \frac{1}{4\pi \epsilon_0} = 4 \times 10^{9}\left(\frac{N - n^2}{e^2}\right)$$

So, the force $F = \dfrac{q^2}{4\pi\varepsilon_0 r^2} = \dfrac{e^2}{4\pi\varepsilon_0 r^2}$

$(\because q_1 = q_2 = e)$

$$= \frac{9 \times 10^9 (1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2} = 1.92 \times 10^{-9}\ \text{N}$$

4. Consider the figure given below :



Let consider 2q at C to the left of q at distance X form q. Force on 2q at C (left of q) is in opposite direction.

So net force will be zero is magnitude is equal

So, $F_{CA} + F_{CB} = 0$

or, $(F_{CA}) = (-F_{CB})$

$$\frac{q_C q_A}{(4\pi \in_0)(r^2_{(CA)})} = \frac{-q_C q_B}{(4\pi \in_0)(r^2_{(CB)})}$$

As given that, $(q_A = q)$, $(q_B = -3q)$, $(q_C = +2q)$

When force of repulsion on it due to q is balanced by force of attraction on it due to (–3q), at B, where AB = d

Thus, force of attraction by (–3q) = Force of repulsion by q

$$\frac{2q \times q}{4\pi\varepsilon_0 x^2} = \frac{2q \times 3q}{4\pi\varepsilon_0 (x+d)^2}$$

$$\frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$$(x + d^2) = 3x^2$$

$$x^2 + d^2 + 2xd = 3x^2 = 2x^2 - d^2$$

So, $2x^2 - 2dx - d^2 = 0$

$$x = -\frac{d}{2} \pm \frac{\sqrt{3}\,d}{2}$$

Negative sign be between q and –3q so not considerable.

So, $x = \frac{d}{2} + \frac{\sqrt{3}\,d}{2} = \frac{d}{2}(1+\sqrt{3})$ to the left of q

5. **(a)** (i) As given figure, the electric lines of force emerge out positive charge from A and C. So, charges A and C must be positive.

(ii) The number of electric lines of forces enamating is maximum for charge C because the density of electric lines of force from a charge increase the intensity of electric field, or magnitude of charge increases. So, C must have the largest magnitude.

(iii) The point between two like charges where electrostatic force is zero is called neutral point. So, the neutral point lies between A and C only.

Now the position of neutral point depends on the strength of the forces of charges. Here, more number of electric lines of forces shows larger magnitude (strength) of charge C than A. So. neutral point lies near A.

6. **(a)** (i) As the given figure the point O is equal distance from all the charges at the end point of pentagon. So, due to symmetric to all the five equal charge +q then, the forces due to all the charges are cancelled out. As a result electric field at O is zero.

(ii) When charge q is removed a negative chare –q will develop at A then electric

field $E = \frac{-q \times 1}{4\pi\varepsilon_0 r^2}$ along OA.

(iii) If charge q at A is replaced by –q, then two negative charges –2q will develop there. So, the electric field is

$$E = \frac{-2q}{4\pi\varepsilon_0 r^2} \text{ along OA}$$

(b) If Pentagon is replaced by n sided regular polygon with charge q at each of its corners, the electric field at O would continue to be zero as symmetricity of the charges is due to the regularity of the polygon. It doesn't depend on the number of sides or the number of charges.

Long Answer Questions

1. **(a)** Consider universe is a sphere with radius R. H atoms and its constituent hydrogen atoms are distributed uniformly in the sphere.

As hydrogen atom contains one proton and one electron, charge on each hydrogen atom.

$$e_H = e_P + e = -(1+Y)\,e + e = -Ye = (Ye)$$

∵ charge on proton $e_P = -(1+Y)e$

If E is electric field intensity at distance R, on the surface of the sphere, then according to Gauss' theorem,

$$\oint E.ds = \frac{q}{\varepsilon_0}$$

i.e., $E(4\pi R^2) = \frac{4}{3}\frac{\pi R^3 N|Ye|}{\varepsilon_0}$

$$E = \frac{4\pi}{3}\frac{N|Ye|R^3}{\varepsilon_0 \times 4\pi R^2} = \frac{1}{3}\frac{N|Ye|R}{\epsilon_0} \quad \text{... (i)}$$

Now suppose, mass of each hydrogen atom $\simeq$ m_p = Mass of a proton, G_R (gravitational field) at distance R on the sphere.

Then, $-4nR^2 G_R = 4\pi G m_p \left(\frac{4}{3}\pi R^3\right)N$

$$G_R = \frac{-4}{3}\pi G m_p NR \quad \text{... (ii)}$$

So, Gravitational force on this atom is

$$F_G = m_p \times GR = \frac{-4\pi}{3} G m_p^2 NR \quad \text{... (iii)}$$

Coulomb force on hydrogen atom at R is

$F_C = (Ye).E$

From equation (i) putting the value of (E),

$$F_C = \left[\frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0}\right]$$

So, to start expansion $F_C > F_G$ and critical value of Y to start expansion would be when

$$F_C = F_G$$

$$\frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} = \frac{4\pi}{3} G m_p^2 NR$$

$$Y^2 = (4\pi\varepsilon_0) G \left(\frac{m_p}{e}\right)^2$$

$$X^2 = \frac{1}{9\times10^9} \times (6.68\times10^{-11}) \left(\frac{(1.66\times10^{10^{-27}})^2}{(1.6\times10^{-19})^2}\right)$$

$Y^2 = 79.8 \times 10^{-38}$

$Y = \sqrt{79.8\times10^{-38}} = 8.9 \times 10^{-19} \simeq 10^{-18}$

So critical value of Y is of the order of (10^{-18}) that universe start to expand.

For expansion repulsive force F_C must be greater than, attractive gravitational force.

(b) Net force experience by the hydrogen atom.

So, $F = F_C - F_G$

$$= \frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 NR$$

When acceleration of hydrogen atom is represent by (d^2R/dt^2) then

$$m_p \frac{d^2R}{dt^2} = F = \frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 NR$$

$$= \left(\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N\right) R$$

So, $$\frac{d^2R}{dt^2} = \frac{1}{m_p}\left[\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N\right] R$$

$$\left(\text{If } \alpha^2 = \frac{1}{m_p}\left[\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3} G m_p^2 N\right]\right)$$

then, $$\frac{d^2R}{dt^2} = \alpha^2 R \quad \text{... (iv)}$$

The general solution of Eq (iv) is $R - Ae^{\alpha t} + Be^{-\alpha t}$

We are looking for expansion of universe,

So, $B = 0$ and $R = Ae^{\alpha t}$

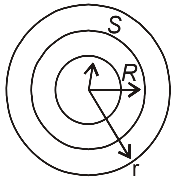
then velocity of expansion,

$$V = \frac{dR}{dt}$$

$$Ae^{\alpha t}(\alpha) = \alpha Ae^{\alpha t} = \alpha R$$

Thus, $v \propto R$ i.e., velocity of expansion is proportional to the distance from the centre.

2. **(a)** Consider a sphere S of radius R and two hypothetic sphere of radius $r < R$ and $r > R$.



According to Gauss's law, $\oint E.dS = \frac{q}{\varepsilon_0}$

Case I : Consider, for Point r < R, electric field intensity will be

$$\oint E.dS = \frac{1}{\varepsilon_0}\int \rho dV \qquad (q = \rho dV)$$

Volume of Gaussion surface (V)

As we know that,

For dV, $V = \frac{4}{3}\pi r^3 \Rightarrow dV = 3 \times \frac{4}{3}\pi r^2 dr = 4\pi r^2 dr$

$[\rho(r) = kr \text{ for } (r < R)]$

So, $$\oint E.dS = \frac{1}{\varepsilon_0} 4\pi K \int_0^r r^3 dr$$

$$(E)\, 4\pi r^2 = \frac{4\pi K}{\varepsilon_0}\frac{r^4}{4}$$

$$E = \frac{1}{4\varepsilon_0} Kr^2$$

So, direction of E is radially outwards.

Case II : By Gauss's law

$$\oint E.dS = \frac{q}{\epsilon_0}$$

For points r > R, electric field intensity will be

$$\oint E.dS = \frac{1}{\varepsilon_0}\int \rho dV \qquad (\because\ q = \rho dV)$$

For dV, $$V = \frac{4}{3}\pi r^3$$

$$dV = 3 \times \frac{4}{3}\pi r^2 dr$$

$$= 4\pi r^2 dr$$

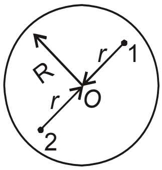
$$\rho(r) = Kr \ \text{ for } r > R$$

$$E(4\pi r^2) = \frac{4\pi K}{\varepsilon_0}\int_0^R r^3 dr = \frac{4\pi K(R^4 - 0)}{\varepsilon_0 \quad 4}$$

$$E = \frac{K}{4\varepsilon_0}\frac{R^4}{r^2}$$

Charge density is again positive. So, the direction of E is radially outward.

(b) The two protons must be on the opposite sides of the centre along a diameter following, the rule of symmetry. This can be shown by the figure so, total negative charge on the sphere is 2e, it is distributed in sphere of radius R.



Symmetrically,

so, the charge on sphere is

$$q = \int_0^R \rho dV = \int_0^R (Kr)4\pi r^2 dr$$

$$q = 4\pi K \frac{R^4}{4} = 2e$$

$$\therefore \qquad K = \frac{2e}{\pi R^4}$$

If protons 1 and 2 are embedded at distance r from the centre O of the sphere as shown thus attractive force on proton 1 due to charge distribution inside the charge sphere at r < R from part (a) is

$$F_1 = eE = \frac{-eKr^2}{4\varepsilon_0}, \qquad \left(\because\ E = \frac{Kr^2}{4\epsilon_0}\right)$$

Repulsive force on proton 1 due to proton 2 is

$$F_2 = \frac{e^2}{4\pi\varepsilon_0 (2r)^2}$$

Net force on proton 1,

$$F = F_1 + F_2$$

$$F = \frac{-eKr^2}{4\varepsilon_0} + \frac{e^2}{16\pi\varepsilon_0 r^2}$$

By putting the value of (K)

So, $$F = \left[\frac{-er^2}{4\varepsilon_0}\frac{2e}{\pi R^4} + \frac{e^2}{16\pi\varepsilon_0 r^2}\right]$$

Therefore, net force on proton 1 will be zero, so

$$F = 0 \quad \text{or} \quad F_1 + F_2 = 0$$

$$\frac{-er^2\, 2e}{4\epsilon_0 \pi R^4} + \frac{e^2}{16\pi\epsilon_0 r^2} = 0$$

$$\frac{er^2\, 2e}{4\varepsilon_0 \pi R^4} = \frac{e^2}{16\pi\varepsilon_0 r^2}$$

$$r^4 = \frac{R^4}{8} \quad \text{or} \quad r = \frac{R}{(8)^{1/4}}$$

This is the distance of each of the two protons from the centre of the sphere.

3. **(a)** Net electric field at plate γ before collision is equal to the sum of electric field at plate γ due to plate α and β.

The electric field at plate γ due to plate α

when $\alpha = \dfrac{\sigma}{2\in_0}$ is $E_1 = \dfrac{-Q}{S(2\varepsilon_0)}$, (towards left).

The electric field at plate γ due to plate β

when $\beta = \dfrac{q}{2S\in_0}$ is $E_2 = \dfrac{q}{S(2\varepsilon_0)}$,

(towards right).

Hence, the net electric field at plate γ before collision.

$E = E_1 + E_2 = \dfrac{q-Q}{S(2\varepsilon_0)}$, if $Q > q$ (towards left).

(b) During collision, plates β and γ are together. Their potentials become same.

Suppose charge on plate β is q_1 and charge on plate γ is q_2. At any point O, in between the two plates, the electric field must be zero.

Electric field at O due to plate $\alpha = \dfrac{-Q}{S(2\varepsilon_0)}$,

(towards left)

Electric field at O due to plate $\beta = \dfrac{q_1}{S(2\varepsilon_0)}$,

(towards right)

Electric field at O due to plate $\gamma = \dfrac{-q_2}{S(2\varepsilon_0)}$,

(towards left)

So, net electric field at O is zero, therefore

$$\frac{-Q}{S(2\varepsilon_0)} + \frac{-q_2}{2S\varepsilon_0} + \frac{q_1}{2S\varepsilon_0} = 0$$

$$\frac{Q+q_2}{S(2\varepsilon_0)} = \frac{q_1}{S(2\varepsilon_0)}$$

$\therefore \quad Q + q_2 = q_1$

$Q = q_1 - q_2$ ...(i)

If there is no loss of charge on collision.

$Q + q = q_1 + q_2$ ...(ii)

On solving Eqs. (i) and (ii), we have,

$q_1 = (Q + q/2)$, (charge on plate β)

$q_2 = (q/2)$ (charge on plate γ)

(c) After collision, at a distance d from plate β, Consider the velocity of plate γ be v. After the collision, electric field at plate γ is

$$E_2 = \frac{-Q}{2\varepsilon_0 S} + \frac{(Q+q/2)}{2\varepsilon_0 S} = \frac{q/2}{2\varepsilon_0 S}$$

(towards right)

Electric field on plate γ just before collision due to plate α and β

$$E_1 = \frac{-Q_1}{2\varepsilon_0 S} + \frac{q}{2\varepsilon_0 S} = -\frac{(Q-q)}{2\varepsilon_0 S}$$

Just before collision, electric field at plate γ is

$$E_1 = -\left\{\frac{Q-q}{2\varepsilon_0 S}\right\}$$

If F_1 is force on plate γ before collision, then

$$F_1 = -E_1 Q = \frac{(Q-q)Q}{2\varepsilon_0 S}$$

So, total work done by the electric field is round trip movement of plate γ

$$W = (F_1 + F_2)d = \frac{\left[(Q-q)Q + (q/2)^2\right]d}{2\varepsilon_0 S}$$

$$W = \frac{(Q-q/2)^2 d}{2\varepsilon_0 S}$$

If m is mass of plate γ, the KE gained by plate,

According to work-energy theorem,

$$W = \frac{1}{2}mv^2$$

So, $\quad \dfrac{1}{2}mv^2 = \dfrac{(Q-q/2)^2 d}{2\varepsilon_0 S}$

$$V = (Q-q/2)\left(\frac{d}{m\varepsilon_0 S}\right)^{1/2}$$

4. (i) As given that, the relation,

$$F = \frac{Qq}{r^2}\hat{r} = 1 \text{ dyne} = \frac{[1 \text{ esu of charge}]^2}{[1 \text{ cm}]^2}$$

So, 1 esu of charge $= (1 \text{ dyne})^{1/2} \times 1$ cm

$= F^{1/2}.L = [MLT^{-2}]^{1/2}L$

1 esu of charge $= M^{1/2}L^{3/2}T^{-1}$

So, esu of charge is represented in terms of fractional powers $\frac{1}{2}$ of M and $\frac{3}{2}$ of L

(ii) Consider 1 esu of charge = x C, where x is a dimensionless number Coulomb force on two charges, each of magnitude 1 esu separated by 1 cm is dyne = 10^{-5}N. This situation is equivalent to two charges of magnitude x C separated by 10^{-2} m.

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r^2}\right)$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{x^2}{(10^{-2})^2} = 1 \text{ dyne} = 10^{-5} \text{ N}$$

$[\because \ q_1 = q_2 = xC]$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2}\frac{Nm^2}{C^2} \quad ...(i)$$

Putting $x = \frac{1}{|3|\times 10^9}$ in eq. (i)

$$\text{So,} \quad \frac{1}{4\pi\varepsilon_0} = 10^{-9}\times|3|^2\times 10^{18}\frac{Nm^2}{C^2}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times 10^9\frac{Nm^2}{C^2}$$

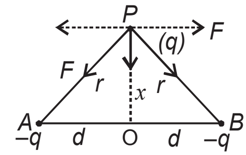
5. In simple harmonic, force on charge q in motion must be proportional to its distance from the centre O and is directed towards O.

As given that, two charge –q at A and B

$AB = [AO + OB] = 2d \quad (\because \ AO = OB = d)$

x is small distance perpendicular to O, i.e.

x < d mass of charge q.



So, force of attraction at P towards A and B are each

$$F = \frac{q(q)}{4\pi\varepsilon_0 r^2}, \text{ where } AP = BP = r$$

Horizontal components of these forces F_n are cancel out. Vertical components along PO add. If ∠APO = O, the net force on q along PO is

$F' = 2F \cos Q$

$$= \frac{2q^2}{4\pi\varepsilon_0 r^2}\left(\frac{x}{r}\right) = \frac{2q^2x}{4\pi\varepsilon_0(d^2+x^2)^{3/2}}$$

$$\text{When,} \quad x << d, F' = \frac{2q^2x}{4\pi\varepsilon_0 d^3} = Kx$$

$$\text{where} \quad K = \frac{2q^2}{4\pi\varepsilon_0 d^3}$$

So, $F \propto x$

Thus, force on charge q is proportional to its displacement from the centre O and it is directed towards O.

Hence, motion of charge q would be simple harmonic, where

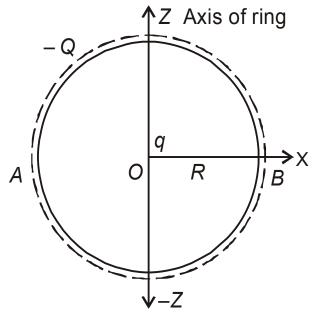
$$\omega = \sqrt{\frac{K}{m}}$$

$$\text{and } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m4\pi\varepsilon_0 d^3}{2q^2}}$$

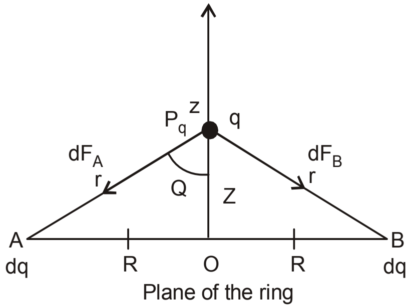
$$\left[\because \ K = \frac{2q^2}{4\pi\varepsilon_0 d^3}\right] \quad \left[T = \frac{8\pi^3\varepsilon_0 md^3}{q^2}\right]^{1/2}$$

6. In a simple harmonic oscillation retarding force on q is proportional to the amount of negative displacement from an equilibrium position.

The figure according to question



A gentle push on charge q along the axis of the ring gives rise to the situation shown in the figure.



Taking line elements of charge at A and B, having unit length, then charge on each elements.

$$dF = 2\left(-\frac{Q}{2\pi R}\right)q \times \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cos\theta$$

So, total force on the charge q, due to entire ring

$$F = -\frac{Qq}{\pi R}(\pi R)\cdot\frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cdot\frac{2}{r}$$

$$F = -\frac{Qqz}{4\pi\varepsilon_0 (Z^2 + R^2)^{3/2}}$$

$$F = -\frac{Qqz}{4\pi\varepsilon_0 R^3} = -Kz,$$

if $Z << R$, $Z^2 << R^2$ or Z^2 can be neglected

where, $\left(\frac{Qq}{4\pi\varepsilon_0 R^3} = \text{constant} = K\right)$

So, $(F \propto -z)$

Thus force on q is proportional to negative of its displacement. Therefore, motion of q is simple harmonic.

$$\omega = \sqrt{\frac{K}{m}}$$

and $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$ $\quad T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 R^3 m}{Qq}}$

$$\left(\because K = \frac{Qq}{4\pi\varepsilon_0 R^3}\right)$$

Chapter Two

ELECTROSTATIC POTENTIAL AND CAPACITANCE

EXERCISES

2.1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

2.2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

2.3 Two charges 2 μC and –2 μC are placed at points A and B 6 cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

2.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere?

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

2.6 Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

2.9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

ADDITIONAL EXERCISES

2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

2.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

2.15 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$(\mathbf{E}_2 - \mathbf{E}_1)\,\hat{\mathbf{n}} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{\mathbf{n}}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\sigma\,\hat{\mathbf{n}}/\varepsilon_0$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

2.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

2.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

2.21 Two charges $-q$ and $+q$ are located at points (0, 0, $-a$) and (0, 0, a), respectively.

(a) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0) ?

(b) Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point (5,0,0) to (–7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

2.22 Figure 2.34 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).

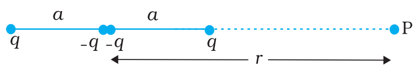


FIGURE 2.34

2.23 An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

2.25 Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.

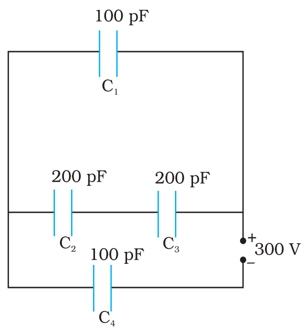


FIGURE 2.35

2.26 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

2.27 A 4 µF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 µF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

2.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

2.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show

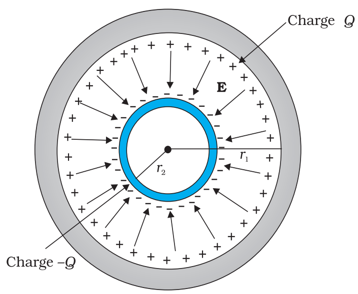


FIGURE 2.36

that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

2.30 A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.
(a) Determine the capacitance of the capacitor.
(b) What is the potential of the inner sphere?
(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

2.31 Answer carefully:
(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1\, Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres?
(b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true ?
(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will travel along the field line passing through that point?
(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?
(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?
(f) What meaning would you give to the capacitance of a single conductor?
(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

2.34 Describe schematically the equipotential surfaces corresponding to
(a) a constant electric field in the z-direction,
(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(c) a single positive charge at the origin, and
(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

2.35 In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

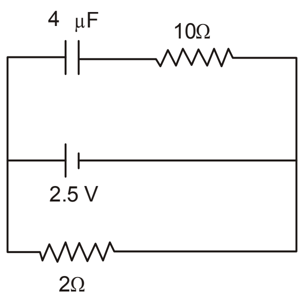
2.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

2.37 Answer the following:
(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)
(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m^2. Will he get an electrc shock if he touches the metal sheet next morning?
(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?
(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?
(Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Chapter 2:- Electrostatic Potential

Multiple Choice Questions (MCQs)

1. A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be



(a) 0 μC (b) 4 μC
(c) 16 μC (d) 8 μC

2. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

3. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

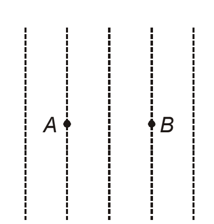


Fig. (i)

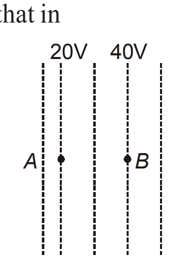


Fig. (ii)

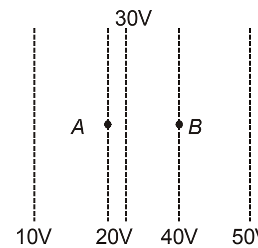


Fig. (iii)

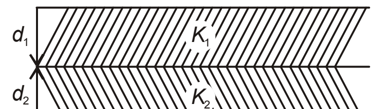
4. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard S_1 at any point inside the sphere, electric intensity is zero. S_2 at any point inside the sphere, the electrostatic potential is 100V. Which of the following is a correct statement?

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independant

5. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness d (= $d_1 + d_2$) and effective dielectric constant K. The K is



(a) $\dfrac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (b) $\dfrac{K_1d_1 + K_2d_2}{K_1 + K_2}$

(c) $\dfrac{K_1K_2(d_1 + d_2)}{(K_1d_1 + K_2d_2)}$ (d) $\dfrac{2K_1K_2}{K_1 + K_2}$

Multiple Choice Questions (MCQs) (More than one option correct)

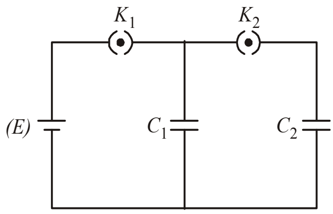
1. Consider a uniform electric field in the z-direction. The potential is a constant
(a) in all space
(b) for any x for a given z
(c) for any y for a given z
(d) on the x-y plane for a given z

2. Equipotential surfaces
(a) are closer in regions of large electric fields compared to regions of lower electric fields
(b) will be more crowded near sharp edges of a conductor
(c) will be more crowded near regions of large charge densities
(d) will always be equally spaced

3. The work done to move a charge along an equipotential from A to B.
(a) cannot be defined a $-\int_A^B E.dl$
(b) must be defined as $-\int_A^B E.dl$
(c) is zero
(d) Can have a non zero value

4. In a region of constant potential
(a) the electric field is uniform
(b) the electric field is zero
(c) there can be no charge inside the region
(d) the electric field shall necessarily change if a charge is placed outside the region.

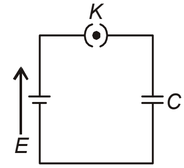
5. In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important).
[Take Q_1' and Q'_2 as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.]



Then,
(a) charge on C_1 gets redistributed such that $V_1 = V_2$
(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$
(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$
(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

6. If a conductor has a potential $V \neq 0$ and there are no charge anywhere else outside, then
(a) there must be charges on the surface or inside itself
(b) there cannot be any charge in the body of the conductor
(c) there must be charges only on the surface
(d) there must be charges inside the surface

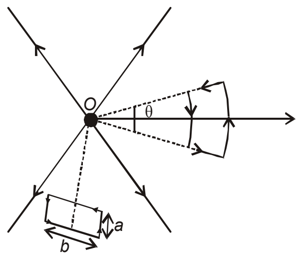
7. A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations.



A. Key K is kept closed and plates of capacitors are moved apart using insulating handle.
B. Key K is opened and plates of capacitors are moved apart using insulating handle.
Choose the correct option(s).
(a) In A Q remains same but C changes
(b) In B V remains same but C changes
(c) In A V remains same and hence Q changes
(d) In B Q remains same and hence V changes.

Very Short Answer Questions

1. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.
2. Do free electrons travel to region of higher potential or lower potential ?
3. Can there be a potential difference between two adjacent conductors carrying the same charge?
4. Can the potential function have a maximum or minimum in free space?
5. A test charge q is made to move in the electric field of a point charge Q along two different closed paths. Figure first path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?

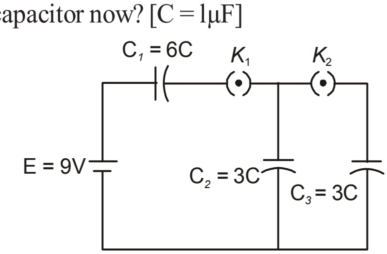


Short Answer Questions

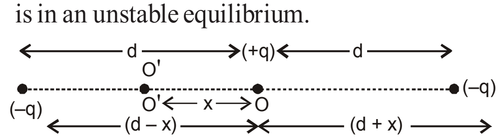
1. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.
2. A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed . State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.
3. Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must intermediate in potential between that of the charged body and that of infinity.
4. Calculate potential energy of a point charge –q placed along the axis due to a charge +Q uniformly distributed along a ring of radius R. Sketch PE, as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if –q is displaced slightly from the centre of the ring (along the axis)?
5. Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Long Answer Questions

1. Find the equation of the equipotentials for an infinite cylinder of radius r_0 carrying charge of linear density λ.
2. Two point charges of magnitude + q and –q are placed at (–d/2, 0, 0) and (d/2, 2, 0), respectively. Find the equation of the equipotential surface where the potential is zero.
3. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $a = 2V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.
4. A capacitor is made of two circular plates of radius R each, separated by a distance d << R. The capacitor is connected to a constant voltage. A thin conducting disc of radius r << R and thickness t << r is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.
5. (a) In a quark model of elementary particles, a neutron is made of one up quarks [charge (2/3) e] and two down quarks [charges –(1/3) e]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.
   (b) Repeat above exercise for a proton which is made of two up and one down quark.
6. Two metal spheres, one of radius R and the other of radius 2R, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them ?
7. In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors?
   Then K_1 was opened and K_2 was closed (order is important), what will be the charge on each capacitor now? [C = 1μF]



8. Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.
9. Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, –d) respectively. Find locus of points where the potential is zero.
10. Two charges –q each are separated by distance 2d. A third charge + q is kept at mid-point 0. Find potential energy of + q as a function of small distance x from 0 due to –q charges. Sketch PE Vs/x and convince yourself that the charge at 0 is in an unstable equilibrium.



NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(d)** As capacitor offers infinite resistance in dc-circuit.
So, current flows through 2Ω resistance from left to right, given by

$$I = \frac{V}{R+r} = \frac{2.5V}{2+0.5} = \frac{2.5}{2.5} = 1\,A$$

So, the potential difference across 2Ω resistance $V = IR = 1 \times 2 = 2$ volt.
Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.
As current does not flow through capacitor branch so no potential drop will be accross 10Ω resistance.The charge on capacitor

$$q = CV = (4\,\mu F) \times 2V = 8\,\mu C$$

2. **(c)** The direction of electric field is always perpendicular to the direction of electric field and equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.
The positively charged particle experiences the electrostatic force in the direction of electric field i.e., from high electrostatic potential to low electrostatic potential. Thus, the work done by the electric field on the positive charge, so electrostatic potential energy of the positive charge decreases because speed of charged particle moves in the direction of field due to force $q\vec{E}$.

3. **(c)** The work done (in displacing a charge particle) by a electric force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases are equal (20V) and same charge is moving from A to B, so work done is (ΔVq) same in all three cases.

4. **(c)** As we know that the relation between electric field intensity E and electric potential V is

$$E = -\frac{dV}{dr}$$

Electric field intensity $E = 0$ then $\frac{dV}{dr} = 0$

This imply that $V =$ constant
Thus, E = 0 inside the charged conducting sphere then the constant electrostatic potential 100V at every where inside the sphere and it verifies the shielding effect also.

5. **(a)** Here we have to findout the shape of equipotential surface, these surface are perpendicular to the field lines, so there must be electric field which can not be without charge.
So, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = K_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points which are at same potential, form spherical surface.
The lines of field from point charge are radial. So the equipotential surface perpendicular to field lines from a sphere.

6. **(c)** The capacitance of parallel plate capacitor filled with dielectric of thickness d_1 and dielectric constant K_1 is

$$C_1 = \frac{K_1 \varepsilon_o A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric of thickness d_2 and dielectric constant K_2 is

$$C_2 = \frac{K_2 \varepsilon_o A}{d_2}$$

Since both capacitors are in series combination, then the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{K_1\varepsilon_0 A}{d_1}\frac{K_2\varepsilon_0 A}{d_2}}{\frac{K_1\varepsilon_0 A}{d_1} + \frac{K_2\varepsilon_0 A}{d_2}}$$

$$C = \frac{K_1 K_2 \varepsilon_0 A}{K_1 d_2 + K_2 d_1} \quad \text{...(i)}$$

So multiply the numerator and denominator of equation (i) with $(d_1 + d_2)$

$$C = \frac{K_1 K_2 \varepsilon_0 A}{(K_1 d_2 + K_2 d_1)} \times \frac{(d_1+d_2)}{(d_1+d_2)}$$

$$= \frac{K_1 K_2 (d_1+d_2)}{(K_1 d_2 + K_2 d_1)} \times \frac{\varepsilon_0 A}{(d_1+d_2)} \quad \text{...(ii)}$$

So, the equivalent capacitances is

$$C=\frac{K\varepsilon_0 A}{(d_1+d_2)} \quad \text{... (iii)}$$

Comparing, (ii) and (iii), the dielectric constant of new capacitor

$$K=\frac{K_1K_2(d_1+d_2)}{K_1d_2+K_2d_1}$$

Multiple Choice Questions (More Than One Option correct)

1. **(b, c, d)**
The direction of electric field is perpendicular to the equipotential surfaces. Here the electric field is in +z direction. And the electric field is always remain in the direction in which the potential decreases. Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.
So, electric field in z-direction suggests that equipotential surfaces will be the plane perpendicular to z-axis along x-y plane. Therefore the potential is a constant for any x for a given z and for any y for a given z and on the x-y plane for a given z.

2. **(a, b, c)**
In equipotential surface, at any two point, potential difference is zero or equal potential.
As we know that the relation between the electric field intensity E and electric potential v is :

$$E=-\frac{dv}{dr}$$

The electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.
The electric field intensity is larger near to sharp edges of charged conductor due to larger charge densities $\sigma=\left(\frac{V}{A}\right)$

As A is very small.

And the electric field $E=\frac{Kq}{r^2}$

So, potential or field decreases as size of body increase or (vice-versa). Hence the equipotential surface will more crowded if charge density (σ) increases.

3. **(c)** Work done in displacing a charge particle is given by $W_{12}=q(V_2-V_1)$ and the line integral of electrical field from point 1 to 2 gives potential difference $(V_2-V_1)=-\int_1^2 E.dl$.
The charge q is moving in the electric field then work done $w=-q\int_1^2 (E)dl$

For the potential on equipotential surface, $(V_2-V_1)=0$. Hence, work done on moving a charge is zero ($w=0$).

4. **(b, c)** As we know,
the relation between electric field intensity E and electric potential V is $E=\frac{-dV}{dr}$ as $E=0$ and for $V=$ constant.

$$\frac{dV}{dr}=0$$

Thus electric field intensity $E=0$.

5. **(a, d)** When K_1 is closed and K_2 is open, the capacitor C_1 gets charged by battery. Now when K_1 is opened and K_2 is closed.
The charge stored by capacitor C_1 gets redistributed between C_1 and C_2 till their potentials become same i.e., $V_2=V_1$. By law of conservation of charge, the charge stored in capacitor C_1 when K_1 is closed and K_2 is open is equal to sum of charges on capacitors C_1 and C_2 when K_1 is opened and K_2 is closed
Let charge Q on C_1, which is charged by battery, then after re-distribution of charge Q by law of conservation of charge $(Q_1'+Q_2'=Q)$.

6. **(a, b)** The main concept used are :
1. The charge resides on the outer surface of a closed charged conductor
2. Net charge inside the conductor is zero.
   The excess charge can reside only on the surface of conductor and inside net positive and negative charge is zero, so any charge can reside inside the hollow shell or body. So, inside the solid material of conducting body there is no charge, it comes to outer surface.

7. **(c, d)**
Case A When K is kept closed and plates of capacitors are moved apart using insulating handle the space between two plates increases then the capacitance decrease by relation

$\left(C = \frac{K\varepsilon_0 A}{d}\right)$ and hence, the charge stored decreases as Q = CV (potential continue to be the same as capacitor is still connected with cell).

Case B When K is opened and plates of capacitors are moved apart using insulating handle, charge stored by disconnected charged capacitor remains conserved and capacitance decrease by moving apart plates of capacitor. So by relation Q = CV, here K is open, so charge Q remains same in turn (V) will increase on decreasing the capacitance.

Very Short Answer Questions

1. The electric potentials on spheres due to their charge in terms of their charge densities,

So, $\sigma = \frac{q}{A}$,

$V = \frac{Kq}{R}$

then, $V_1 = \frac{Kq_1}{R_1}$, $V_2 = \frac{Kq_2}{R_2}$

Since, the two spheres are at the same potential, therefore,

$V_1 = V_2$,

So, $\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{Kq_1 R_1}{4\pi R_1^2} = \frac{Kq_2 R_2}{4\pi R_2^2}$

or, $\sigma_1 R_1 = \sigma_2 R_2$

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$ (Given: $R_1 > R_2$)

This imply that $\sigma_1 < \sigma_2$

So charge density of the smaller sphere (R_2) is more than that of the larger sphere (R_1).

2. The free electrons has negative charge that experiences the direction of electrostatic force is opposite to the direction of electric field. The electric field always directed from higher potential to lower travel.
Hence, direction of free electrons is from lower potential to higher poetntial.

3. Yes, if the sizes are different.

The capacity of conductor $C = \frac{Q}{V}$, Q is the charge of conductor and V is the potential of conductor. For given charge potential $V \propto 1/C$, so two adjacent conductors carrying the same charge of different cross section area may have different potentials.

4. Potential function does not have a maximum or minimum in free space because the absence of atmosphere or free space around conductor prevents the phenomenon of electric discharge or potential leakage.

5. As electric field is conservative, work done in closed loop is always zero and it does not depend on the nature of closed loop conservative forces are those forces, work done by which depends only on initial position and final position of object charge, but not on the path through which it goes from initial position to final position. Hence work done in both the cases will be zero.

Short Answer Questions

1. Let us consider that inside the enclosed equipotential surface the potential is not same. Let the potential just inside the surface is different to that of the surface causing in a potential gradient $\left(\frac{dv}{dr}\right) = E$.

If $E \neq 0$, so, electric field will exist from inside surface which is equal to $E = -\frac{dV}{dr}$

The field lines pointing inwards or outwards from the surface. These lines cannot be again on the equipotential surface. It is possible only when the other end of the field lines are originated from the charges inside.

Which contradicts the original assumption. Hence, the entire volume inside must be euipotential surface has no charge.

2. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is

$$C = \frac{K\varepsilon_0 A}{d},$$

Where 'K' is positive and more then one

So, by removing dielectric medium from capacitor keeping A and d constant. So, capacitance of capacitor will decrease, K = 1.

When battery and dielectric slab removed from capacitor, then charge stored will remain the same as due to conservation of charge.

The energy stored in charge capacitor $V = \frac{q^2}{2C}$;

where, q is constant, energy stored $V \propto 1/C$ and

C decreases with the removal of dielectric medium, therefore energy stored in capacitor will increases. Since q is constant and $V = q/C$ and C decreases which in turn potential (V) will increase and therefore E increases as $E = V/d$. The distance between plote of capacitor is same and potential is increase. So electric field between the plates of capacitor will increase.

3. The main concept used is the electric field $E = \frac{dV}{dr}$ that suggest the electric potential decreases along the direction of electric field.

Consider a charged conductor and an insulated uncharged conductor.

Now take any path from the charged conductor to the uncharged conductor along the direction of electric field. So, the electric potential decrease along this path from a charge conductor body to infinity.

Now, another path from the uncharged conductor to infinity will again continually lower the potential further. This ensures that the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

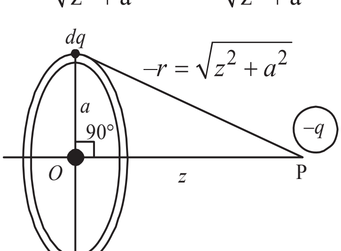
4. Let us consider a ring of radius (R = a) having charge Q, distributed uniformly over a ring.

Now take a point P to be at a distance z from the centre of the ring or perpendicular to plane of ring as shown in figure. The charge element dq is at a distance z from point P. So, V can be written as

$$V = k_e\int\frac{dq}{r} = k_e\int\frac{dq}{\sqrt{z^2+a^2}} \quad (\because\ r = \sqrt{z^2+a^2})$$

where, $k = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{z^2+a^2}}\int dq = \frac{k_eQ}{\sqrt{z^2+a^2}}$$



Considering –q charge at P, the potential energy is

$$U = W = q \times \text{potential difference}$$

$$= \frac{k_eQ(-q)}{\sqrt{z^2+a^2}}$$

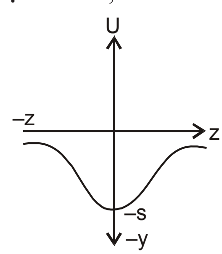
or,
$$U = \frac{1}{4\pi\varepsilon_0}\frac{-Qq}{\sqrt{z^2+a^2}}$$

$$= \frac{1}{4\pi\varepsilon_0 a}\frac{-Qq}{\sqrt{1+\left(\frac{z}{a}\right)^2}}$$

Let, $\frac{Qq}{4\pi\epsilon_0\ a} = S$, a new constant

Then,
$$U = \frac{-S}{\left[1+\left[\frac{z}{a}\right]^2\right]^{1/2}}$$

$\because\ z >>> a;\ z^2 >>> a^2$ at $z = 0$ $(U = -S)$



Hence, the variation of potential energy with z is shown in the figure. The charge –q displaced would perform oscillations.

5. Let us consider a ring of radius (R = a) having charge Q, distributed uniformly.

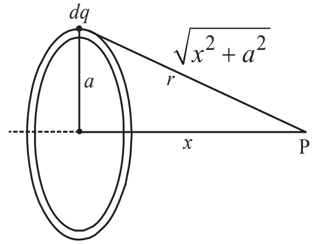
Now take point P to be at a distance x from the centre of the ring or perpendicular to plane of ring as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e\int\frac{dq}{r} = k_e\int\frac{dq}{\sqrt{x^2+a^2}}$$

$$(\because\ r = \sqrt{(x^2+a^2)})$$

Where, $k_e = \frac{1}{4\pi\varepsilon_0}$, thus each element dq is at the same distance from point P, so we get the net potential is :

$$V = \frac{k_e}{\sqrt{x^2+a^2}}\int dq = \frac{k_eQ}{\sqrt{x^2+a^2}}$$

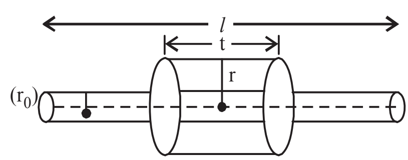


Hence, the net electric potential

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

Long Answer Questions

1. Consider a cylindrical Gaussian surface of radius r_0 and length l.



$$\int_0^{2\pi l} E.dS = \frac{1}{\varepsilon_0}\lambda l$$

Then, applying Gauss's theorem

So, $$\left[E_r S\cos\theta\right]_0^{2\pi l} = \frac{\lambda l}{\varepsilon_0}$$

or, $$E_r 2\pi r l\cos 0^\circ = \frac{\lambda l}{\varepsilon_0},$$

$$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l \qquad (\because \cos\theta = 1)$$

So, $$E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

As given r_0 is the radius of infinite cylinder,

So, $$V(r) - V(r_0) = -\int_{r_0}^{r} E.dl = \frac{\lambda}{2\pi\varepsilon_0}\log_e\frac{r_0}{r}$$

Since, $$\int_{r_0}^{r}\frac{\lambda}{2\pi\varepsilon_0 r}dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{1}{r}dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\log_e\frac{r}{r_0}$$

For a given V,

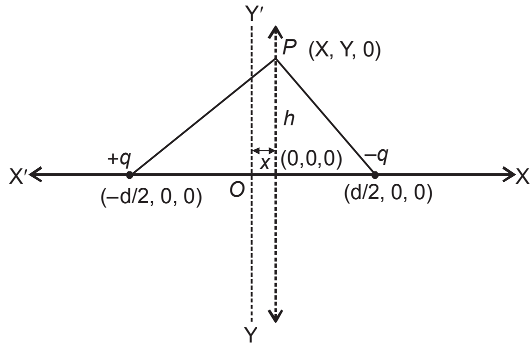
$$\log_e\left[\frac{r}{r_0}\right] = \frac{-2\pi\varepsilon_0}{\lambda}\left[V(r) - V(r_0)\right]$$

$$r = r_0 e^{-2\pi\varepsilon_0 V r_0 l/\lambda} e + 2\pi\varepsilon_0 V(r)/\lambda$$

$$r = r_0 e^{-2\pi\varepsilon_0\left[V(r) - V(r_0)/\lambda\right]}$$

This is the required equations of equipotentials.

2. Consider the required plane lies at a distance x from the origin as shown in figure.



The potential at the point P

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{\left[(x+d/2)^2+h^2\right]^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[(x-d/2)^2+h^2\right]^{1/2}} = 0$$

$$\frac{1}{\left[(x+d/2)^2+h^2\right]^{1/2}} = \frac{1}{\left[(x-d/2)^2+h^2\right]^{1/2}}$$

or $$(x-d/2)^2+h^2 = (x+d/2)^2+h^2$$

$$x^2 - dx + d^2/4 = x^2 + dx + d^2/4$$

or, $$2dx = 0$$

$$x = 0$$

This equation of the required plane is $x = 0$ i.e., y – z plane.

3. Let C is the capacitance of the capacitor (C) without the dielectric, then the charge on the capacitor is $Q_1 = CU$ where U is the final potential of the capacitor.

If the capacitor filled with the dielectric ε has a capacitance εC. Hence, the charge on the capacitor is

$$Q_2 = \varepsilon CU = (\alpha U)CU = \alpha CU^2$$

$$(\therefore \varepsilon = \alpha U)$$

The initial charge on the capacitor is given by

$$Q_0 = CU_0$$

From the conservation of charges, $Q_0 = Q_1 + Q_2$

or, $$CU_0 = CU + \alpha CU^2$$

$$\alpha U^2 + U - U_0 = 0,$$

$$\therefore \quad U = \frac{-1 \pm\sqrt{1+4\alpha U_0}}{2\alpha}$$

Putting $U_0 = 78$ V and $\alpha = 2$ per volt, we get

$$= \frac{-1 \pm \sqrt{625}}{4}$$

$$= \frac{-1 \pm 25}{4}$$

$$= \frac{24}{4} = 6 \text{ Volt}$$

4. Let initially the disc is in touch with the bottom plate, so the entire plate is a equipotential.

The Magnitude of electric field E on the disc (when potential difference V) is applied between plates of capacitor is :

$$E = \frac{V}{d} \qquad \left[\therefore E = \frac{-dV}{dr}\right]$$

Let charge q' is transferred to the disc during the process.

Therefore by Gauss' theorem,

$$\oint E.dS = \frac{q'}{\epsilon_0}$$

$$\therefore \quad q' = +\varepsilon_0 \frac{V}{d} \pi r^2$$

Electrostatic repulsive force acting on disc in upward direction.

$$\text{So,} \quad F = q'E = \frac{V}{d}\pi r^2 \varepsilon_0 . \frac{V}{d} = \frac{V^2}{d^2}\pi r^2 \epsilon_0$$

The repulsive force acting on the disc will be balanced by weight (mg).

$$\varepsilon_0 \frac{V^2}{d^2}\pi r^2 = mg$$

If the disc is to be lifted, then the minimum voltage

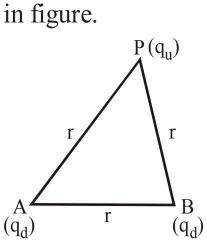
$$V = \sqrt{\frac{mgd^2}{\pi\varepsilon_0 r^2}}$$

This is the required expression.

5. **(a)** As we know that the potential energy i.e.

$$U = \frac{Kq_1q_2}{r}, \left(K = \frac{1}{4\pi\varepsilon_0}\right)$$

This system is made up of three charges shown in figure.



The potential energy of the system is equal to the algebraic sum of PE of each pair. So,

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_dq_d}{r} - \frac{q_uq_d}{r} - \frac{q_uq_d}{r}\right\}$$

(Taking sign of charge)

$$\therefore \quad q_d = \frac{1}{3}e$$

$$\therefore \quad q_u = \frac{2}{3}e$$

$$= \frac{9\times10^9}{10^{-15}}(1.6\times10^{-19})^2[\{(1/3)^2 - (2/3)(1/3) - (2/3)(1/3)\}]$$

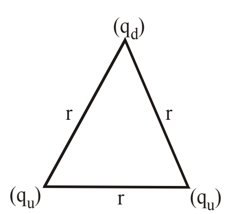
$$= 2.304 \times 0^{-13}\left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68 \times 10^{-14} J$$

$= 4.8 \times 10^5$ eV $= 0.48$ meV

$= 5.11 \times 10^{-4}\,(m_nc^2)$

(b) P.E. of proton consists of 2 upward and 1 downward quark.

As given that, $r = 10^{-15}$ m, $q_d = +\frac{1}{3}e$, $q_u = \frac{2}{3}e$



$$\text{So, } U_p = \left[\frac{1}{4\pi\varepsilon_0}\left(\frac{q_u \times q_u}{r}\right)\right] + \frac{q_u(-q_u)}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$$

$$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - 2q_d] = 0$$

6. Let the charges on two metal spheres are Q_1 and Q_2 before coming in contact, then the charges are,

$$Q_1 = \sigma.4\pi R^2$$

($\therefore \sigma$ = surface charge density same for both sphere)

$$Q_2 = \sigma.4\pi(2R^2) = 4(\sigma.4\pi R^2) = 4Q_1$$

Let the charges on two metal spheres, after coming in contact becomes Q'_1 and Q'_2.

Now applying law of conservation of charges is

$$Q'_1 + Q'_2 = Q_1 + Q_2$$

$$= 5Q_1 = 5(\sigma.4\pi R^2)$$

After coming in contact, they acquire equal potentials. So, we get

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q'_2}{R}$$

Putting the value of Q'_1 and Q'_2 in above equation.

$\therefore \quad Q'_1 = \frac{5}{3}(\sigma.4\pi R^2)$

and $\quad Q'_2 = \frac{10}{3}(\sigma.4\pi R^2)$

$\therefore \quad \sigma_1 = \frac{5}{3\sigma}$ and

$\therefore \quad \sigma_2 = \frac{5}{6}\sigma$

7. In the given circuit when, K_1 is closed and K_2 is open, the capacitors C_1 and C_2 will charge and potential difference develops accross V_1 and V_2 respectively. So, we have

$$V_1 + V_2 = E$$

or $\quad V_1 + V_2 = 9V \quad$ ..... (i)

Also, in series combination ,

$$V = \frac{q}{C}$$

or, $\quad V \propto 1/C$

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$V_1 : V_2 = \frac{1}{6} : \frac{1}{3}$$

$$3V_2 = 6V_1, (V_2 = 2V_1)$$

Putting with the value of V_2 in equation (i) then

$$V_1 + 2V_1 = 9$$

$$3V_1 = 9, V_1 = 3V$$

then, $\quad V_2 = 6V$

so, $\quad Q_1 = C_1V_1 = 6C \times 3 = 18\ \mu C$

$\quad Q_2 = C_2V_2 = 3 \times 6$

$\quad = 18\ \mu C \quad (\therefore a = 1\ \mu f)$

and $\quad Q_3 = 0$

Then, K_1 was opened and K_2 was closed, the parallel combination of C_2 and C_3 is in series with C_1.

$$Q_2 = Q'_2 + Q_3$$

and considering common potential of parallel combination as V. then we get :

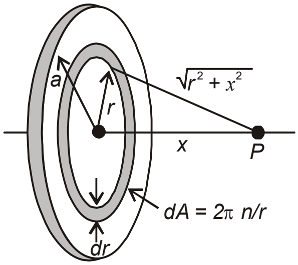
$$C_2V + C_3V = Q_2$$

$$V = \frac{Q_2}{C_2 + C_3} = \frac{18}{3C + 3C} = 3 \text{ Volt}$$

On solving, $\quad Q'_2 = C_2V = 3C \times 3 = 9\ \mu C$

and $\quad Q_3 = C_3V = 3C \times 3 = 9\ \mu C$

8. Consider a point P lies at a distance x from the centre of the disk or perpendicular to the plane of disc. Let the disc is divided into a number of charged rings as shown in figure.



The electric potential of each ring, of radius r and thickness dr, have charge dq is

$$\sigma\, dA = \sigma 2\pi r dr$$

and potential is dv due to ring at point P, will be

$$dV = \frac{K_e dq}{r'}$$

dq is the charge on the ring = σ.area of ring

$$dq = -(\pi(r + dr)^2 - \pi r^2]$$

$$= -\pi[(r + dr)^2 - r^2]$$

$$dq = \sigma\pi[r^2 + dr^2 + 2rdr - r^2]$$

Because dr is very small therefore dr^2 is negligible, so

$$dq = (2\pi r\sigma dr)$$

and, $\quad dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} \quad (\therefore r' = \sqrt{r^2 + x^2})$

$$= \frac{k_e \sigma 2\pi r dr}{\sqrt{r^2 + x^2}} \quad (\therefore dq = 2\pi r\sigma dr)$$

where $k_e = \frac{1}{4\pi\varepsilon_0}$ the total electric potential at P, is

$$V = \pi k_e \sigma \int_0^a \frac{2rdr}{\sqrt{r^2 + x^2}}$$

$$= \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2}\, 2r\, dr$$

$$V = 2\pi k_e \sigma \left[(x^2 + a^2)^{1/2} - x \right]$$

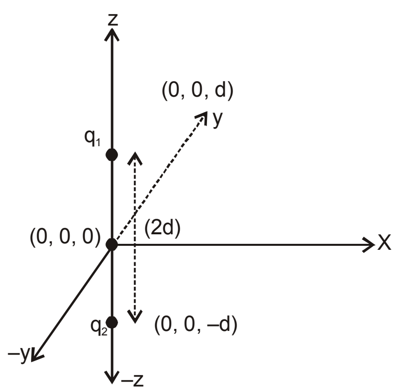
So, we have by substring

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{a^2}\left[\sqrt{x^2+a^2}-x\right]$$

So, $V = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}\left[\sqrt{x^2+R^2}-x\right]$

9.



As shown in figure above any arbitrary point on the required plane is (x, y, z). The two charges lies on z-axis at a separation of 2d.

Let potential at the point P due to two charges is zero then

$$V_1 + V_2 = 0$$

$$\frac{kq_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{kq_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

$$\therefore \frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} = \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$

$$\frac{q_1}{q_2} = \frac{-\sqrt{x^2+y^2+(z-d)^2}}{\sqrt{x^2+y^2+(z+d)^2}}$$

On squaring both sides and using $\frac{x}{a} = \frac{y}{b}$

$$\left[\frac{x+a}{x-a} = \frac{y+b}{y-b}\right]$$

Then simplifying, we get

$$x^2+y^2+z^2+\left[\frac{(q_1/q_2)^2+1}{(q_1/q_2)^2-1}\right](2zd)+d^2 = 0$$

This is the equation of a sphere with centre at (a, b, c) are required. Point is on z-axies so

$$a = 0, b = 0 \text{ and } z = 2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]$$

So, $\left(0, 0, -2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right)$

Where as, the centre and radius of sphere $(x-a)^2+(y-b)^2+(z-c)^2 = r^2$ is (a, b, c) and r respectively.

10. As shown in figure (Question) above third charge +q is slightly displaced from mean position towards first charge. So, the total potential energy of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d-x)} + \frac{-q^2}{(d+x)}\right\}, \left(\because U = \frac{kq_1q_2}{r_1}\right)$$

$$U = \frac{-q^2}{4\pi\varepsilon_0}\frac{2d}{(d^2-x^2)}$$

$$\frac{dU}{dx} = \frac{-q^2 2d}{4\pi\varepsilon_0}\cdot\left[\frac{2x}{(d^2-x^2)^2}\right]$$

The equilibrium of (+q) is at P at the distance x from mid point of line joining two charges. Then, the system will be in equilibrium, if

$$F = -\frac{dU}{dx} = 0$$

On solving, x = 0. So for, +q charge to be in stable/unstable equilibrium, finding second derivative of PE.

$$\frac{d^2U}{dx^2}\left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left[\frac{2}{(d^2-x^2)^2} - \frac{8x^2}{(d^2-x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\frac{1}{(d^2-x^2)^3}\left[2(d^2-x^2)^2 - 8x^2\right]$$

At, x = 0

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{d^6}\right)(2d^2), \text{ which is} < 0.$$

This shows that system will be unstable equilibrium.

Chapter Three

CURRENT ELECTRICITY

SUMMARY

1. *Current* through a given area of a conductor is the net charge passing per unit time through the area.
2. To maintain a steady current, we must have a closed circuit in which an external agency moves electric charge from lower to higher potential energy. The work done per unit charge by the source in taking the charge from lower to higher potential energy (i.e., from one terminal of the source to the other) is called the electromotive force, or *emf*, of the source. Note that the emf is not a force; it is the voltage difference between the two terminals of a source in open circuit.
3. *Ohm's law*: The electric current I flowing through a substance is proportional to the voltage V across its ends, i.e., $V \propto I$ or $V = RI$, where R is called the *resistance* of the substance. The unit of resistance is ohm: $1\Omega = 1 \text{ V A}^{-1}$.
4. The *resistance* R of a conductor depends on its length l and constant cross-sectional area A through the relation,

   $$R = \frac{\rho l}{A}$$

   where ρ, called *resistivity* is a property of the material and depends on temperature and pressure.
5. *Electrical resistivity* of substances varies over a very wide range. Metals have low resistivity, in the range of 10^{-8} Ω m to 10^{-6} Ω m. Insulators like glass and rubber have 10^{22} to 10^{24} times greater resistivity. Semiconductors like Si and Ge lie roughly in the middle range of resistivity on a logarithmic scale.
6. In most substances, the carriers of current are electrons; in some cases, for example, ionic crystals and electrolytic liquids, positive and negative ions carry the electric current.
7. *Current density* **j** gives the amount of charge flowing per second per unit area normal to the flow,

   $$\mathbf{j} = nq\,\mathbf{v}_d$$

   where n is the number density (number per unit volume) of charge carriers each of charge q, and $\mathbf{v}_d$ is the *drift velocity* of the charge carriers. For electrons $q = -e$. If **j** is normal to a cross-sectional area **A** and is constant over the area, the magnitude of the current I through the area is $nev_d A$.

8. Using $E = V/l$, $I = nev_d A$, and Ohm's law, one obtains

$$\frac{eE}{m} = \rho \frac{ne^2}{m} v_d$$

The proportionality between the *force eE* on the electrons in a metal due to the external field E and the drift velocity v_d (not acceleration) can be understood, if we assume that the electrons suffer collisions with ions in the metal, which deflect them randomly. If such collisions occur on an average at a time interval τ,

$v_d = a\tau = eE\tau/m$

where a is the acceleration of the electron. This gives

$$\rho = \frac{m}{ne^2 \tau}$$

9. In the temperature range in which resistivity increases linearly with temperature, the *temperature coefficient of resistivity* α is defined as the fractional increase in resistivity per unit increase in temperature.

10. Ohm's law is obeyed by many substances, but it is not a fundamental law of nature. It fails if
    (a) V depends on I non-linearly.
    (b) the relation between V and I depends on the sign of V for the same absolute value of V.
    (c) The relation between V and I is non-unique.

    An example of (a) is when ρ increases with I (even if temperature is kept fixed). A rectifier combines features (a) and (b). GaAs shows the feature (c).

11. When a source of emf ε is connected to an external resistance R, the voltage V_{ext} across R is given by

$$V_{ext} = IR = \frac{\varepsilon}{R + r} R$$

where r is the *internal resistance* of the source.

12. (a) Total resistance R of n resistors connected in *series* is given by

$$R = R_1 + R_2 + ... + R_n$$

    (b) Total resistance R of n resistors connected in *parallel* is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$

13. *Kirchhoff's Rules* –
    (a) *Junction Rule:* At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
    (b) *Loop Rule:* The algebraic sum of changes in potential around any closed loop must be zero.

14. The *Wheatstone bridge* is an arrangement of four resistances – R_1, R_2, R_3, R_4 as shown in the text. The null-point condition is given by

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

using which the value of one resistance can be determined, knowing the other three resistances.

15. The *potentiometer* is a device to compare potential differences. Since the method involves a condition of *no* current flow, the device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources.

Physical Quantity	Symbol	Dimensions	Unit	Remark
Electric current	I	[A]	A	SI base unit
Charge	Q, q	[T A]	C	
Voltage, Electric potential difference	V	$[M L^2 T^{-3} A^{-1}]$	V	Work/charge
Electromotive force	ε	$[M L^2 T^{-3} A^{-1}]$	V	Work/charge
Resistance	R	$[M L^2 T^{-3} A^{-2}]$	Ω	$R = V/I$
Resistivity	ρ	$[M L^3 T^{-3} A^{-2}]$	Ω m	$R = \rho l/A$
Electrical conductivity	σ	$[M^{-1} L^{-3} T^3 A^2]$	S	$\sigma = 1/\rho$
Electric field	$\mathbf{E}$	$[M L T^{-3} A^{-1}]$	$V m^{-1}$	$\frac{\text{Electric force}}{\text{charge}}$
Drift speed	v_d	$[L T^{-1}]$	$m s^{-1}$	$v_d = \frac{e E \tau}{m}$
Relaxation time	τ	[T]	s	
Current density	$\mathbf{j}$	$[L^{-2} A]$	$A m^{-2}$	current/area
Mobility	μ	$[M L^3 T^{-4} A^{-1}]$	$m^2 V^{-1} s^{-1}$	v_d / E

POINTS TO PONDER

1. Current is a scalar although we represent current with an arrow. Currents do not obey the law of vector addition. That current is a scalar also follows from it's definition. The current I through an area of cross-section is given by the scalar product of two vectors:

   $I = \mathbf{j} \cdot \Delta\mathbf{S}$

   where $\mathbf{j}$ and $\Delta\mathbf{S}$ are vectors.
2. Refer to V-I curves of a resistor and a diode as drawn in the text. A resistor obeys Ohm's law while a diode does not. The assertion that $V = IR$ is a statement of Ohm's law is not true. This equation defines resistance and it may be applied to all conducting devices whether they obey Ohm's law or not. The Ohm's law asserts that the plot of I versus V is linear i.e., R is independent of V.

   Equation $\mathbf{E} = \rho\,\mathbf{j}$ leads to another statement of *Ohm's* law, i.e., a conducting material obeys Ohm's law when the resistivity of the material does not depend on the magnitude and direction of applied electric field.
3. Homogeneous conductors like silver or semiconductors like pure germanium or germanium containing impurities obey Ohm's law within some range of electric field values. If the field becomes too strong, there are departures from Ohm's law in all cases.
4. Motion of conduction electrons in electric field $\mathbf{E}$ is the sum of (i) motion due to random collisions and (ii) that due to $\mathbf{E}$. The motion

due to random collisions averages to zero and does not contribute to v_d (Chapter 11, Textbook of Class XI). v_d , thus is only due to applied electric field on the electron.

5. The relation $\mathbf{j} = \rho\,\mathbf{v}$ should be applied to each type of charge carriers separately. In a conducting wire, the total current and charge density arises from both positive and negative charges:

   $\mathbf{j} = \rho_+\,\mathbf{v}_+ + \rho_-\,\mathbf{v}_-$

   $\rho = \rho_+ + \rho_-$

   Now in a neutral wire carrying electric current,

   $\rho_+ = -\,\rho_-$

   Further, $v_+ \sim 0$ which gives

   $\rho = 0$

   $\mathbf{j} = \rho_-\,\mathbf{v}$

   Thus, the relation $\mathbf{j} = \rho\,\mathbf{v}$ does not apply to the total current charge density.

6. Kirchhoff's junction rule is based on conservation of charge and the outgoing currents add up and are equal to incoming current at a junction. Bending or reorienting the wire does not change the validity of Kirchhoff's junction rule.

EXERCISES

3.1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?

3.2 A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

3.3 (a) Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

3.4 (a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

3.5 At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is 1.70×10^{-4} °C^{-1}.

3.6 A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

3.7 A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.

3.8 A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is 1.70×10^{-4} °C^{-1}.

3.9 Determine the current in each branch of the network shown in Fig. 3.30:

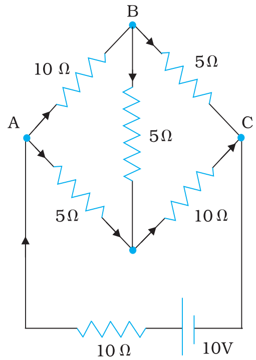


FIGURE 3.30

3.10 (a) In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end *A*, when the resistor *Y* is of 12.5 Ω. Determine the resistance of *X*. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?

(b) Determine the balance point of the bridge above if *X* and *Y* are interchanged.

(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

3.11 A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

3.12 In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

3. 13 The number density of free electrons in a copper conductor estimated in Example 3.1 is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

ADDITIONAL EXERCISES

3. 14 The earth's surface has a negative surface charge density of 10^{-9} C m^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe). (Radius of earth = 6.37×10^6 m.)

3.15 (a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

3.16 Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. (ρ_{Al} = 2.63×10^{-8} Ω m, ρ_{Cu} = 1.72×10^{-8} Ω m, Relative density of Al = 2.7, of Cu = 8.9.)

3.17 What conclusion can you draw from the following observations on a resistor made of alloy manganin?

Current A	Voltage V	Current A	Voltage V
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

3.18 Answer the following questions:

(a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?

(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.

(c) A low voltage supply from which one needs high currents must have very low internal resistance. Why?

(d) A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?

3.19 Choose the correct alternative:

(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

(c) The resistivity of the alloy manganin is nearly independent of/ increases rapidly with increase of temperature.

(d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of ($10^{22}/10^{23}$).

3.20 (a) Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

(b) Given the resistances of 1 Ω, 2 Ω, 3 Ω, how will be combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω, (iii) 6 Ω, (iv) (6/11) Ω?

(c) Determine the equivalent resistance of networks shown in Fig. 3.31.

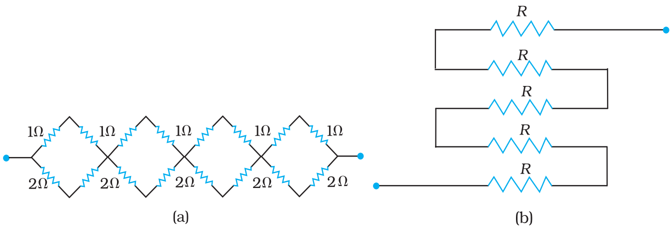


FIGURE 3.31

3.21 Determine the current drawn from a 12V supply with internal resistance 0.5Ω by the infinite network shown in Fig. 3.32. Each resistor has 1Ω resistance.

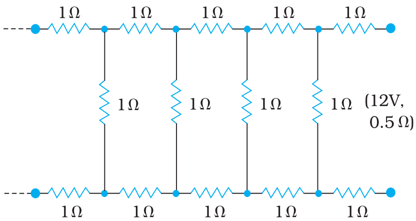


FIGURE 3.32

3.22 Figure 3.33 shows a potentimeter with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents upto a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found similarly, turns out to be at 82.3 cm length of the wire.

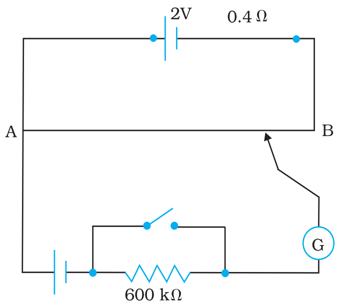


FIGURE 3.33

(a) What is the value ε?
(b) What purpose does the high resistance of 600 kΩ have?

(c) Is the balance point affected by this high resistance?
(d) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0V instead of 2.0V?
(e) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?

3.23 Figure 3.34 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

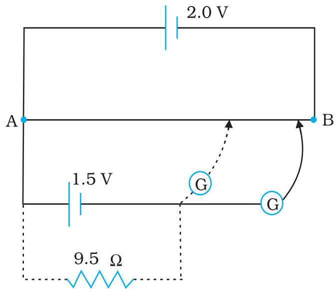


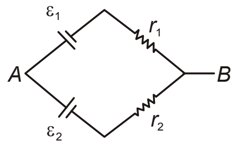
FIGURE 3.34

Chapter 3:- Current Electricity

Multiple Choice Questions (MCQs)

1. Consider a current carrying wire (current I) in the shape of a circle.
   (a) source of emf
   (b) electric field produced by charges accumulated on the surface of wire
   (c) the charges just behind a given segment of wire which push them just the right way by repulsion
   (d) the charges ahead

2. Two batteries of emf ε_1 and $\varepsilon_2 (\varepsilon_2 > \varepsilon_1)$ and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.
   (a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, i.e., $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
   (b) The equivalent emf ε_{eq} is smaller than ε_1
   (c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
   (d) ε_{eq} is independent of internal resistances r_1 and r_2



3. A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?
   (a) He should measure I_1 more accurately
   (b) He should change 5 to 1000Ω and repeat the experiment
   (c) He should change S to 3Ω and repeat the experiment
   (d) He should given up hope of a more accurate measurement with a meter bridge

4. Two cells of emfs approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.
   (a) The battery that runs the potentiometer should have voltage of 8V
   (b) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V
   (c) The first portion of 50 cm of wire itself should have a potential drop of 10 V
   (d) Potentiometer is usually used for comparing resistances and not voltages

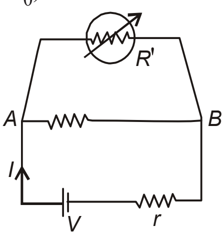
5. A metal rod of length 10 cm and a rectangular cross-section of $1\text{cm} \times \frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be
   (a) maximum when the battery is connected across $1 \text{ cm} \times \frac{1}{2}$ cm faces
   (b) maximum when the battery is connected across 10 cm × 1 cm faces
   (c) maximum when the battery is connected across $10 \text{ cm} \times \frac{1}{2}$ cm faces
   (d) same irrespective of the three faces

6. Which of the following characteristics of electrons determines the current in a conductor?
   (a) Drift velocity alone
   (b) Thermal velocity alone
   (c) Both drift velocity and thermal velocity
   (d) Neither drift nor thermal velocity

Multiple Choice Questions (MCQs) (More than one option correct)

1. Kirchhoff's junction rule is a reflection of
   (a) conservation of current density vector
   (b) conservation of charge
   (c) the fact that the momentum with which a charged particle approaches a junction is unchanged (as a vector) as the charged particle leaves the junction
   (d) the fact that there is no accumulation of charges at a junction

2. Consider a simple circuit shown in figure stands for a variable resistance R'. R' can vary from R_0 to infinity, r is internal resistance of the battery ($r << R << R_0$).



(a) Potential drop across AB is nearly constant as R' is varied
(b) Current through R' is nearly a constant as R' is varied
(c) Current I depends sensitively on R'
(d) $I \geq \frac{V}{r+R}$ always

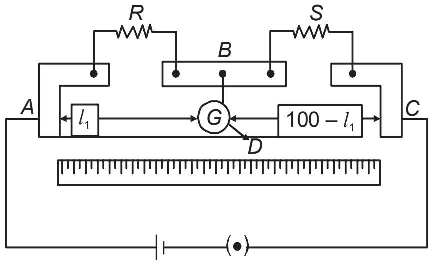
3. Temperature dependence of resistivity ρ(T) of semiconductors, insulators and metals is significantly based on the following factors
(a) number of charge carriers can change with temperature T
(b) time interval between two successive collisions can depend on T
(c) length of material can be a function of T
(d) mass of carriers is a function of T

4. The measurement of an unknown resistance R is to be carried out using. Wheatstones bridge as given in the figure below. Two students perform an experiment in two ways. The first students takes $R_2 = 10\Omega$ and $R_1 = 5\Omega$. The other student takes $R_2 = 1000\Omega$ and $R_1 = 500\Omega$. In the standard arm, both take $R_3 = 5\Omega$.

Both find $R = \frac{R_2}{R_1}, R_3 = 10\Omega$ within errors.

(a) The errors of measurement of the two students are the same
(b) Errors of measurement do depend on the accuracy with which R_2 and R_1 can be measured
(c) If the student uses large values of R_2 and R_1 the currents through the arms will be feeble. This will make determination of null point accurately more difficult
(d) Wheatstone bridge is a very accurate instrument and has no errors of measurement

5. In a meter bridge, the point D is a neutral point (figure).
(a) The meter bridge can have no other neutral. A point for this set of resistances
(b) When the jockey contacts a point on meter wire left of D, current flows to B from the wire
(c) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer
(d) When R is increased, the neutral point shifts to left



Very Short Answer Questions

1. Is the motion of a charge across junction momentum conserving? Why or why not?

2. The relaxation time τ is nearly independent of applied E field whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's Law whereas the second fact leads to variation of p with temperature. Elaborate why?

3. What are the advantages of the null-point method in a Wheatstone bridge? What additional measurements would be required to calculate $R_{unknown}$ by any other methmd?

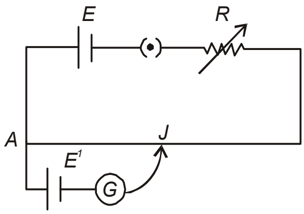
4. What is the advantage of using thick metallic strips to join wires in a potentiometer?

5. For wiring in the home, one uses Cu wires or AL wires. What considerations are involved in this?

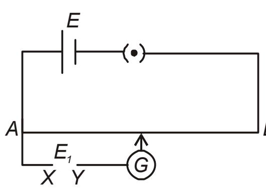
6. Why are alloys used for making standard resistance coils?

7. Power P is to be delivered to a device via transmission cables having resistance R_c. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced.

8. AB is a potentiometer wire (figure). If the value of R is increased, in which direction will the balance point J shift?



9. While doing an experiment with potentiometer (figure) it was found that the deflection is one sided and (i) the deflection decreased while moving from one and A of the wire, to the end R; (ii) the deflection increased, while the jockey was moved towards the end D.



(i) Which terminal positive or negative of the cell E_1 is connected at X in case (i) and how is E_1, related to E?

(ii) Which terminal of the cell E_1 is connected at X in case (1 in 1)?

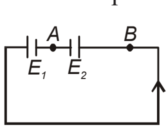
10. A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential differential across R, versus R.

Short Answer Questions

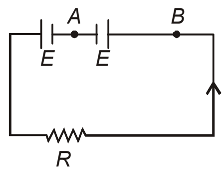
1. First a set of n equal resistors of R each are connected in series to a battery of emf E and internal resistance R, A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 10 times. What is 'n' ?

2. Let there be n resistors $R_1 \ldots R_n$, with $R_{max} = \max(R_1 \ldots R_n)$ and $R_{min} = \min\{R_1 \ldots R_n\}$. Show that when they are connected in parallel, the resultant resistance $R_p = R_{min}$ and when they are connected in series, the resultant resistance $R_s > R_{max}$. Interpret the result physically.

3. The circuit in figure shows two cells connected in opposition to each other. Cell E_1 is of emf 6V and internal resistance 2Ω the cell E_2 is of emf 4V and internal resistance 8Ω. Find the potential difference between the points A and B.



4. Two cells of same emf E but internal resistance r_1 and r_2 are connected in series to an external resistor R (figure). What should be the value of R so that the potential difference across the terminals of the first cell becomes zero?

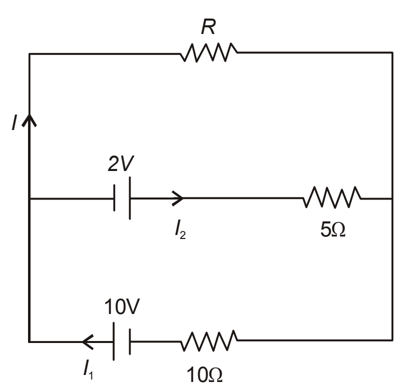


5. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance R_A to R_B.

6. Suppose there is a circuit consisting of only resistances and batteries. Suppose one is to double (or increase it to n-times) all voltages and all resistances. Show that currents are unaltered. Do this for circuit of ExampLes 3,7 in the NCERT Text Book for Class XII.

Long Answer Questions

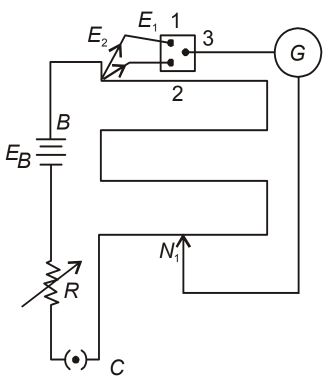
1. Two cells of voltage 10V and 2V and internal resistances 10Ω and 5Ω respectively, are connected in parallel with the positive end of 10V battery connected to negative pole of 2V battery (figure). Find the effective voltage and effective resistance of the combination.



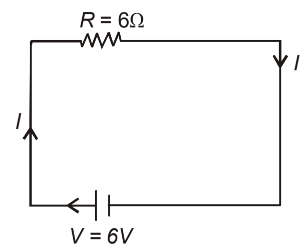
2. A room has AC run for 5 a day at a voltage of 220V. The wiring of the room consists of Cu of 1 mm radius and a length of 10m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires? What would happen if the wiring is made of aluminium of the same dimensions?

$[\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega m,\ \rho_{Al} = 2.7 \times 10^{-8}\ \Omega m]$

3. In an experiment with a potentiometer, $V_B = 10V$. R is adjusted to be 50Ω (figure). A student wanting to measure voltage E_1 of a battery (approx. 8V) finds no null point possible. He then diminishes R to 10Ω and is able to locate the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.



4. (a) Consider circuit in figure. How much energy is absorbed by electrons from the initial state of no current (Ignore thermal motion) to the state of drift velocity ?



(b) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would number associate with energy in problem (a)? n = number of electron/volume = $10^{29}/m^3$. Length of circuit = 10 cm cross-section = A = $(1\ mm)^2$.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(b)** As we know, electric current per unit area I/A, is called current density j i.e., $j = \dfrac{I}{A}$

The SI units of the current density are A/m^2. The current density is also directed along E and is also a vector and the relationship is

$$j = \sigma E$$

Current density changes due to electric field produced by charges accumulated on the surface of wire.

2. **(a)** As we know the equivalent emf (ε_{eq}) in the parallel combination

$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 + \varepsilon_1 r_2}{r_1 + r_2}$$

So according to formula the equivalent emf ε_{eq} of the two cells in parallel combination is between ε_1 and ε_2. Thus ($\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$).

3. **(c)** Adjusting the blance point near the middle of the bridge, i.e.. when l_1 is close to 50 cm. requires a suitable choice of S, R is unknown resistance :

Since, $$\frac{R}{S} = \frac{Rl_1}{R(100 - l_1)}$$

$$\frac{R}{S} = \frac{l_1}{100 - l_1} \quad \text{or} \quad R = S\left[\frac{l_1}{100 - l_1}\right]$$

$$R = S\left[\frac{2.9}{97.1}\right]$$

So, here, R : S = 2.9 : 97.1 implies that the S is nearly 33 times to that of R. In orded to make this ratio 1 : 1 it is necessary to reduce the value of S nearly $\dfrac{1}{33}$ times i.e., nearly 3 Ω,

4. **(b)** The potential drop across wires of potentiometer should be more than emfs of primary cells. Here, values of emfs of two cells are given as 5V and 10V, so the potential drop along the potentiometer wire must be more than 10V. So battery should be of 15V and about 4V potential is droped by using variable resistance.

5. **(a)** As we known that the resistance of wire is

$$R = \rho\frac{l}{A}$$

For maximum value of R, l must be higher and A should be lower and it is possible only when the battery is connected across area of cross section

$$= 1\text{cm} \times \left(\frac{1}{2}\right)\text{cm}.$$

6. **(a)** We know that the relationship between current and drift speed is

$$I = ne\,Av_d$$

Where, I is the current and V_d is the drift velocity.

So, $$I \propto V_d$$

Hence, only drift velocity determines the current in a conductor.

Multiply Choice Questions (More Than One Options)

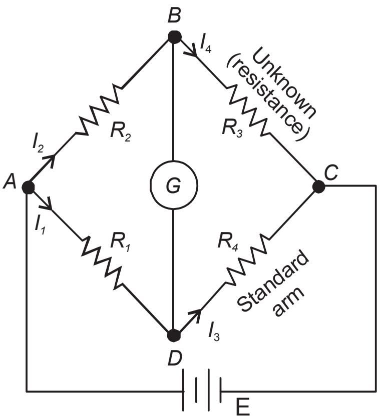
1. **(b, d)**
According to Kirchhoff's junction rule or current law the algebraic sum of the currents flowing towards any point in an electric network is zero i.e., charges are conserved in an electric network, and no any charges accumulate at junction as the sum of entering and outgoing charge are equal at any time interval.
So, Kirchhoff's junction rule is the reflection of conservation of charge.

2. **(a, d)**
In parallel combination of resistances, the equivalent resistance is smaller than smallest resistance (R) present in combination.
Here, the potential drop is taking place across AB and r. Since the equivalent resistance of parallel combination of R and R' is always less than R, so, $I \geq \dfrac{V}{r+R}$ always

3. **(a, b)**
As we know that, the resistivity of a metallic conductor is $(\rho) = \dfrac{m}{ne^2\tau}$
where n is number of charge carriers per unit volume which can change with temperature T and τ is time interval between two successive collisions which decreases with the increase of temperature, so it will affect (ρ) resistivity.

4. **(b, c)** As given that,
Resistance for first student, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 5\Omega$
Resistance for second student, $R_1 = 500\Omega$, $R_2 = 1000\Omega$, $R_3 = 5\Omega$
Now, according to Wheatstone bridge balanced condition,

$$\frac{R_2}{R} = \frac{R_1}{R_3} \quad \text{or} \quad R = R_3 \times \frac{R_2}{R_1}$$



Now putting all the values in Eq. (i), we get $R = 10\Omega$ for both students. Thus, we can analyse that the Wheatstone bridge is most sensitive and accurate if resistances are of same value. So the results of both students depend on the accuracy of resistances used.
When R_2 and R_1 are larger, the currents through galvanometer becomes very weak. It can make the determination of null point accurately more complex.

5. **(a, c)** As we know that, the principal of meter bridge

$$\frac{R}{S} = \frac{L}{(100-L)}$$

When Jockey is at D, the current does not flow through galvanometer. So the potential at B and D are equal. The point D is unique to get null point.
At neutral point, potential at B and neutral point are same. When jockey is placed at to the right of D, the potential drop across AD is more than potential drop across AB, which brings the potential of point D less than that of B, hence current flows from B to D.

Very Short Answer Questions

1. When a free electron approaches to a junction, in addition to the uniform electric field E facing it normally that keep thedrift velocity fixed. And the drift velocity depend on E, e, τ, m by the relation drift velocity

$$V_d = \frac{eE\tau}{m}$$

Then the result into accumulation of charges on the surface of wires at the junction. Which will affect the drift velocity because, it produce additional electric field. These fields change the direction of momentum.

Hence, the motion of a charge across junction is not momentum conserving.

2. Relaxation time (τ) is inversely proportional to the velocities electrons and ions. The applied electric field produces the insignificant change in drift velocities of electrons at the order of 1mm/s, whereas the change in temperature (T), affects velocities at the order of 10^2m/s.

As, the drift velocity increase, this decreases the relaxation time considerably in metals and consequently resistivity of conductor increases as

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

3. As we know that the necessary and sufficient condition for balanced Wheatstone bridge is

$$\frac{P}{Q} = \frac{R}{S}$$

where P and Q are ratio arms and R is known resistance and S is unknown resistance.

The main advantage of null point method in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and the internal resistance of a galvanometer.

We can calculate the $R_{unknown}$ by ohm's law in which we need to calculate the least counts and readings of ammeter and voltmeter.

The $R_{unknown}$ can be calculated applying Kirchhoff's rules to the circuit. We would need additional accurate measurement of all the currents in resistances and galvanometer and internal resistance of the galvanometer.

4. As the area of cross-section of metallic strips in potentiometer is larger than a single wire.

So the resistance of strip is much small.

$$\left(R = \rho\frac{l}{A}\right)$$

In potentiometer, the thick metallic strips are used as they have negligible resistance and need not to be counted in the length l_1 of the null point of potentiometer. Which makes easy to take reading and calculation. It is also for the convenience of experimenter as he measures only their lengths along the straight wires each of lengths 1 m. This measurements is done with the help of centimetre scale or metre scale with accuracy.

5. The main considerations are involved in this process are cost of metal used in electric circuits decreases from Ag, Cu, Al, Fe.

The Cu wires or Al wires are used for wiring in the home because low cost, but Ag is more costly. Al and Cu are good conductors.

6. Alloy has less value of temperature coefficient of resistance with less temperature sensitivity. So, the resistance of the wire almost constant even in small temperature change. The alloys also has high resistivity $\left(R = \rho\frac{l}{A}\right)$.

Hence high resistance, because for given length and cross-section area of conductor (l and A are constant). Due to this reason alloys are used to make standard resistance coils.

7. As we know that the power consumption in transmission lines is $P = i^2 R_c$, where R_c is the resistance of transmission lines, so the power is P = VI.

Thus power can be transmitted in two ways namely (i) at low voltage and high current or (ii) high voltage and low current.

In power transmission at low voltage and high current more power loss as $P \propto i^2$ whereas power transmission at high voltage and low current facilitates the power transmission with minimal power wastage.

The power loss can be reduced by transmitting power at high voltage, but need thicker insulation during transmission.

8. If R increases then, the current in main circuit decreases which in turn, decreases the potential difference across AB and hence potential gradient (K) across AB decreases.

Since, at neutral point, for given emf of cell, I increases as potential gradient (K) across AB has decreased because.

$$E = kI \qquad \left[\because K = \frac{V}{(AB)}\right]$$

So balance potential accross AB equal to potential of secondary circuit the length (AJ) must be longer then earlier.

Thus, with increase of I, the balance point neutral point will shift towards B.

9. (i) One side deflection in galvanometer decreased, when jockey moving from one end 'A' of the wire to the end 'B', thus imply that current in auxiliary circuit decreases, while potential difference A and jockey increases.

This is positive only when positive terminal of the cell E_1 is connected at X and negative at Y. So $E_1 > E$.

(ii) One side deflection in galvanometer increased when jockey moving from one end A of the wire to the end B, this imply that current in auxiliary circuit. increases, while potential difference across A and jockey increases.

This is possible only when negative terminal of the cell E_1, is connected at Y and negative at X. So $E_1 < E$.

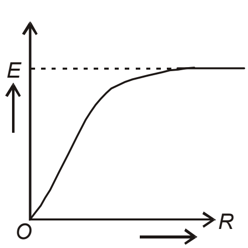
10. When the cell of emf E and internal resistance r is connected across an external resistance R, the relationship between the voltage across R is

$$V = \frac{E}{1+\frac{r}{R}}$$

Thus, $V \propto R$

With the increase of R, V approaches closer to E and when E is infinite, V reduces to zero.

Then the graphical representation between voltage and the resistance R is as shown in figure.



Short Answer Questions

1. When n resistors each of resistance R are in series combination then current I is given by

$$I = \frac{E}{R+nR} \quad \text{..... (i)}$$

Now if n resistors are in parallel combination current 10 I is

$$\frac{E}{R+\frac{R}{n}} = 10I \quad \text{..... (ii)}$$

So, $$\frac{E}{R+\frac{R}{n}} = \frac{10E}{R+nR}$$

$$\frac{1+n}{1+\frac{1}{n}} = 10 \quad \text{or} \quad 10 = \left(\frac{1+n}{n+1}\right)n$$

$\Rightarrow \ n = 10$

So there are 10 resistors in combination.

2. Let R_{min} and R_{max} are the minimum and maximum resistances.

When all resistors are connected in parallel, the resultant resistance R_P is

$$\frac{1}{R_P} = \frac{1}{R_1} + \ldots + \frac{1}{R_n}$$

On multiplying both sides by R_{min} we get,

$$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \ldots + \frac{R_{min}}{R_n}$$

Here, in RHS, there exist one term $\frac{R_{min}}{R_{min}} = 1$ and other terms are positive, so we have

$$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \ldots + \frac{R_{min}}{R_n} > 1$$

i.e., the resultant resistance $(R_p < R_{min})$.

So, in parallel combination, the equivalent resistance of resistors is less than the minimum resistance available in combination of resistors.

Now, in series combination, the equivalent resistant is

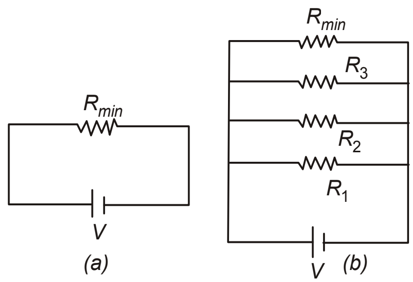
$$R_s = R_1 + \ldots + R_n$$

Here, in RHS, there exist one term having resistance R_{max}.

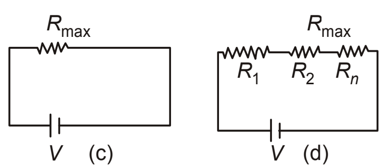
So, we get

$$R_s = R_1 + \ldots + R_{max} \ldots + \ldots + R_n$$
$$R_s = R_1 + \ldots R_{max} + R_n$$
$$= R_{max} + \ldots (R_1 + \ldots +)R_n$$
$$R_s \geq R_{max}$$
$$R_s = R_{max} (R_1 + \ldots R_n)$$

So, in series combination, the equivalent resistance is always greater than the maximum resistance (R_{max}) available in combination of resistors. Physical interpretation



In Fig. (b), R_{min} provides an equivalent route as in Fig (a) for current. But in addition there are (n – 1) routes by the remaining (n – 1) resistors. Current in Fig (b) is greater than current in Fig. (a). Effective resistance in Fig. (b) < R_{min}. Second circuit evidently affords a greater resistance.



In the above figure, R_{max} provides an equivalent route as in Fig. (c) for current. Current in Fig.(d) < current in Fig. (c). Effective resistance in Fig. (d) > R_{max}. Second circuit evidently affords a greater resistance.

3. Applying Ohm's law, (V = IR)

The Effective resistance $r_{eff} = r_1 + r_2$ and effective emf of two cells $E_{eff} = E_1 + E_2$, so the electric current is along anti-clockwise direction, since $E_1 > E_2$.

$$I = \frac{E_1 + E_2}{r_1 + r_2} \quad (\therefore E_1 = 6, E_2 = -4 \therefore r_1 = 2, r_2 = 8)$$

So, $$I = \frac{6-4}{2+8} = 0.2\,A$$

The direction of flow of current is always from high potential to low potential Therefore

$$V_B > V_A$$

So, $V_B - 4V - (0.2) \times 8 = V_A$

So, $V_B - V_A = 5.6V$

So, potential between A and B = 5.6 volt.

4. Applying Ohm's law,

$$V = IR,$$

The Effective resistance = $R + r_1 + r_2$ and effective emf of two cells = E + E = 2E, so the electric current I following in the circuit is

$$I = \frac{E+E}{R + r_1 + r_2}$$

The net potential difference across the terminals of the 1st cell and putting it equal to zero.

$$V_1 = E - Ir_1 \qquad (\because V_1 = 0) \text{ given}$$

$$= E - \frac{2E}{r_1 + r_2 + R} r_1 = 0$$

or, $$E = \frac{2Er_1}{r_1 + r_2 + R}$$

$$1 = \frac{2r_1}{r_1 + r_2 + R}$$

or, $$r_1 + r_2 + R = 2r_1$$

$$R = r_1 - r_2$$

This is the required condition for the potential difference across 1st cell to be zero.

5. We know that, the resistance of wire is

$$R = \rho \frac{l}{A}$$

where A is cross-sectional area of conductor as given that :

$$l_1 = l$$

$$r_1 = \frac{1}{2} \text{ mm}$$

$$= 0.5 \times 10^{-3} \text{m}$$

and $$l_2 = l$$

$$r_2 = \frac{2}{2} \text{ mm}$$

$$= 1 \times 10^{-3} \text{ m}$$

The resistance of first conductor

$$R_A = \frac{\rho l_1}{\pi r_1^2}$$

$$R_A = \frac{\rho l}{\pi \left(10^{-3} \times 0.5\right)^2}$$

The resistance of second conductor,

$$R_B = \frac{\rho l_2}{\pi \left(r_2^2 - r_1^2\right)}$$

$$R_B = \frac{\rho l}{\pi \left[\left(10^{-3}\right)^2 - \left(0.5 \times 10^{-3}\right)^2\right]}$$

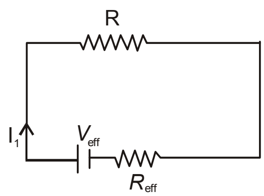
Now, the ratio of two resistances is

$$\frac{R_A}{R_B} = \frac{\left(10^{-3}\right)^2 - \left(0.5 \times 10^{-3}\right)^2}{\left(0.5 \times 10^{-3}\right)^2} = \frac{0.75}{0.25}$$

$$R_A : R_B = 3 : 1$$

6. Consider the effective internal resistance of the battery is R_{eff}, the effective external resistance R and the effective voltage of the battery is V_{eff}.
Applying Ohm's law, (V = IR)
Then current through R is

$$I_1 = \frac{V_{eff}}{R_{eff} + R}$$



If all the resistances and the effective voltage are increased n-times, then we get

$$V_{eff}^{new} = nV_{eff},\ R_{eff}^{new} = nR_{eff}$$

and $$R^{new} = nR$$

Then, the new current in circuit is :

$$I_2 = \frac{nV_{eff}}{nR_{eff} + nR}$$

$$= \frac{n(V_{eff})}{n(R_{eff} + R)}$$

$$= \frac{(V_{eff})}{(R_{eff} + R)} = I_1$$

Hence, current remains the same.

Long Answer Questions

1. As we know that, by Ohm's law,

$$V = IR$$

Applying Kirchhoff's junction rule,

$$I_1 = I + I_2$$

Applying Kirchhoffs II law/loop rule applied in outer loop containing 10V cell and resistance R, we have

$$10 = IR + 10I_1 \quad \text{... (i)}$$

Applying Kirchhoff II law/loop rule applied in outer loop containing 2V cell and resistance R, we have

$$2 = 5I_2 - RI$$
$$= 5(I_1 - I) - RI$$

or, $$4 = 10I_1 - 10I - 2RI \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), gives

$$6 = 3R1 + 10I$$

$$2 = I\left(R + \frac{10}{3}\right)$$

Let the effective potential difference due to both batteries is V_{eq}.

$$V_{eq} = I(R + R_{eff})$$

On comparing, we have V = 2V and effective internal resistance

$$(R_{eff}) = \left(\frac{10}{3}\right)\Omega$$

Since, the effective internal resistance (R_{eff}) of two cells is $\left(\frac{10}{3}\right)\Omega$, being the parallel combination of 5Ω and 10Ω

2. As given that the power consumption P = VI in (5hrs) in a day = 10 unit = 10 kWh
power consumption per hour = 2units = 2kWh
= 2000 J/s
Voltage = 220 V
Also, we know that power consumption in resistor,

$$P = V \times I$$
$$2000W = 220V \times I \text{ or } I \approx 9\,A$$

Now, the resistance of wire is given by $R = \rho\frac{l}{A}$
where, A is cross-sectional area of conductor.
Power consumption in first current carrying wire is

$$P = I^2R \qquad \left(\because\ R = \rho\frac{l}{A}\right)$$

$$P_{Cu} = \left(\rho\frac{l}{A}\right)I^2$$

$$= 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81\ J/s \approx 4\ J/s$$

So loss of energy in wiring ≅ 4 j/sec.
The fractional loss due to heating in first wire is

$$= \frac{4}{2000} \times 100 = 0.2\%$$

Power loss in Al wire $P_{Al} = 4\frac{\rho_{Al}}{\rho_{Cu}}$

∴ $(l_{Al} = l_{Cu})$ and $(A_{Al} = A_{Cu})$

∴ $$P_{Cu} = 4\ j/S$$
$$= 1.6 \times 4$$
$$= 6.4\ J/s$$

The fractional loss due to the joule heating in second wire

$$= \frac{6.4}{2000} \times 100 = 0.32\%$$

3. Consider R' be the resistance of the potentiometer wire and given variable resistor (R = 50Ω)

Then the effective resistance of potentiometer

$= 50\Omega + R'$

Effective voltage applied across potentiometer $= 10\,V$

The current through the main circuit,

$$I = \frac{V}{50\Omega + R'}$$

$$= \frac{10}{50\Omega + R'}$$

Potential difference across wire of potentiometer,

$$IR' = \frac{10R'}{50\Omega + R'}$$

As, with 50Ω resistor, null point can not obtained by 8 volt it's possible only when

$$\frac{10 \times R'}{50 + R'} < 8$$

So, $10R' < 400 + 8R'$

$2R' < 400$ or $R' < 200\,\Omega$

Similarly with 10Ω resistor, null point is obtained at $V' > 8$ its possible only when

$$\frac{10 \times R'}{10 + R'} > 8$$

So, $2R' > 80$, $R' > 40$

As the null point is obtained on 4th segment or at $\left(\frac{3}{4}\right)$ of total length so at $\left(\frac{3}{4}R'\right)$.

$$\frac{10 \times \frac{3}{4}R'}{10 + R'} < 8 \quad \text{(At balance point)}$$

So, $7.5R' < 80 + 8R'$

$R' > 160$

$$\boxed{160 < R' < 200}$$

Any R' between 160Ω and 200Ω will achieve.

Since, the null point on the last (4th) segment of the potentiometer, therefore potential drop across 400 cm of wire > 8V.

This imply that potential gradient

$k \times 400\text{ cm} > 8\,V$ (at balance point)

or $k \times 4\text{ m} > 8\,V$

$$\boxed{k > 2\,V/m}$$

Similarly, potential drop across 300 cm wire < 8 V

$k \times 300\text{ cm} < 8\,V$

$k \times 3\text{ m} < 8\,V$

$$k < \frac{8}{3}\,V/m$$

Hence, $\boxed{\frac{8}{3}V/m > k > 2V/m}$

4. (a) By Ohm's law,

$V = IR,$

$$I = \frac{V}{R}$$

$I = 6V/6\Omega = 1A$

The current in a conductor and drift velocity of electrons are related as $i = neAv_d$

where V_d is drift speed of electrons and n is number density of electrons.

$I = ne\,Av_d$

So, drift velocity $v_d = \frac{i}{neA}$

As given that,

n (number of electron/volume) $= 10^{29}/m^3$

length of circuit = 10cm, cross-section area (A) $= (1mm)^2$, $m_e = 9.1 \times 10^{-31}$

On substituting the values

$$v_d = \frac{1}{10^{29} \times 1.6 \times 10^{-19} \times 10^{-6}}$$

$$= \frac{1}{1.6} \times 10^{-4}\text{ m/s}$$

So, the energy absorbed in the form of KE is

$$KE = \frac{1}{2}m_e v_d^2 \times nAl$$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times \frac{1}{2.56} \times 10^{29} \times 10^{-8} \times 10^{-6} \times 10^{-1}$$

$K.E. = 1.78 \times 10^{-17}\,J$

(b) Power loss

$P = I^2R$

$= 6 \times 1^2$

$= 6W = 6\,J/s$

We know that,

$$\text{(Power loss) } P = \frac{E\,(K.E.)}{t\,(\text{time})}$$

So, $t = \frac{E}{P}$

$$= \frac{1.78 \times 10^{-17}}{6}$$

$= 0.29 \times 10^{-17}$

$\cong 0.3 \times 10^{-17}$

$= 3 \times 10^{-18}$ second

Chapter Four

MOVING CHARGES AND MAGNETISM

EXERCISES

4.1 A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field **B** at the centre of the coil?

4.2 A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire?

4.3 A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

4.4 A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

4.5 What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?

4.6 A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

4.7 Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

4.8 A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre.

4.9 A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

4.10 Two moving coil meters, M_1 and M_2 have the following particulars:

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

4.11 In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.5 \times 10^{-19}$ C, $m_e = 9.1\times10^{-31}$ kg)

4.12 In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

4.13 (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60°

with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

ADDITIONAL EXERCISES

4.14 Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

4.15 A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the numbe of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

4.16 For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{3/2}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by,

$$B = 0.72\frac{\mu_0 NI}{R}, \text{ approximately.}$$

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

4.17 A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

4.18 Answer the following questions:

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

4.19 An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30° with the initial velocity.

4.20 A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^{-5} V m^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique?

4.21 A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) $g = 9.8$ m s^{-2}.

4.22 The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

4.23 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction,

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

4.24 A uniform magnetic field of 3000 G is established along the positive *z*-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?

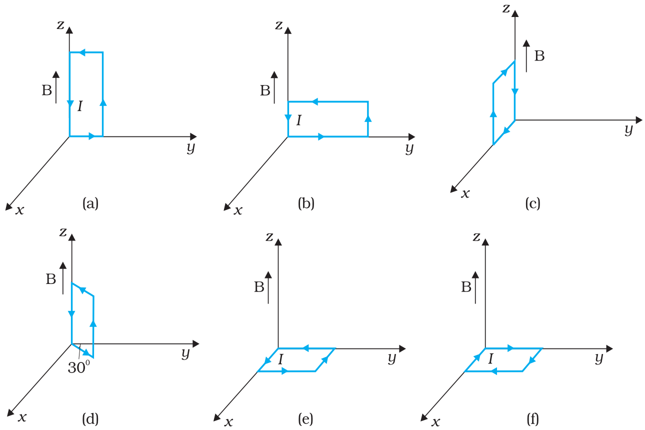


FIGURE 4.28

4.25 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}.)

4.26 A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? g = 9.8 m s^{-2}.

4.27 A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

4.28 A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Chapter 4:- Moving Charges

Multiple Choice Questions (MCQs)

1. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$.

(a) They have equal z-components of momenta

(b) They must have equal charges

(c) They necessarily represent a particle, anti-particle pair

(d) The charge to mass ratio satisfy

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

2. Biot-Savart law indicates that the moving electrons (velocity v) produce a magnetic field **B** such that

(a) B is perpendicular of

(b) B is parallel to v

(c) it obeys inverse cube law

(d) it is along the line joining the electron and point of observation.

3. A current carrying circular loop of radius R is placed in the x-y plane with centre at the origin. Half of the loop with x > 0 is now bent so that it now lies in the y-z plane.

(a) The magnitude of magnetic moment now diminishes

(b) The magnetic moment does not change

(c) The magnitude of B at (0, 0, z), z > R increases

(d) The magnitude of B at (0, 0, z), z >> R is unchanged

4. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true?

(a) The electron will be accelerated along the axis

(b) The electron path will be circular about the axis

(c) The electron will experience a force at 45° to the axis and hence execute a helical path

(d) The electron will continue to move with uniform velocity along the axis of the solenoid

5. In a cyclotron, a charged particle

(a) undergoes acceleration all the time

(b) speeds up between the dees because of the magnetic field

(c) speeds up in a dees

(d) slows down within a dee and speeds up between dees

Multiple Choice Questions (MCQs) (More than one option correct)

1. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field **B.** The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(a) MB

(b) $\sqrt{3}\frac{MB}{2}$

(c) $\frac{MB}{2}$

(d) zero

2. The gyro-magnetic ratio of an electron in an H-atom, according to Bohr model, is

(a) independent of which orbit it is in

(b) negative

(c) positive

(d) increases with the quantum number n.

3. Consider a wire carrying a steady current, I placed in a uniform magnetic field B perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do no work. This implies that,

(a) motion of charges inside the conductor is unaffected by **B**, since they do not absorb energy

(b) some charges inside the wire move to the surface as a result of **B**

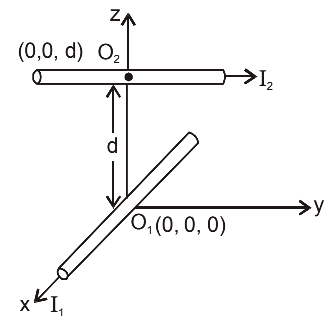
(c) If the wire moves under the influence of **B,** no work is done by the force

(d) if the wire moves under the influence of **B,** no work is done by the magnetic force on the ions, assumed fixed within the wire.

4. Two identical current carrying coaxial loops, carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,
   (a) $\oint B.dl = m\mu_0 I$
   (b) the value off $\oint B.dl = +2\mu_0 I$ is independent of sense of C
   (c) there may be a point on C where, B and dl are perpendicular
   (d) B vanishes everywhere on C
5. A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity v and a positron enters via opposite face with velocity – v. At this instant,
   (a) the electric forces on both the particles cause identical accelerations
   (b) the magnetic forces on both the particles cause equal accelerations
   (c) both particles gain or loose energy at the same rate
   (d) the motion of the Centre of Mass (CM) is determined by **B** alone
6. A charged particle would continue to move with a constant velocity in a region wherein,
   (a) $E = 0, B \neq 0$ (b) $E \neq 0, B \neq 0$
   (c) $E \neq 0, B = 0$ (d) $E = 0, B = 0$

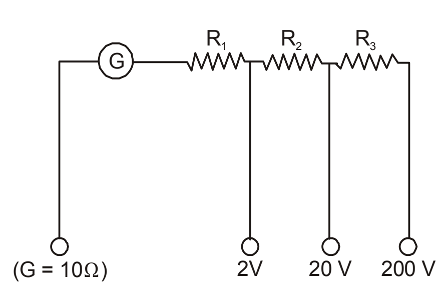
Very Short Answer Questions

1. Verify that the cyclotron frequency $\omega = eB/m$ has the correct dimensions of $[T]^{-1}$.
2. Show that a force that does no work must be a velocity dependent force.
3. The magnetic force depends on v which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?
4. Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency (rf) field were doubled.
5. Two long wires carrying current I_1 and I_2 are arranged as shown in figure. The one carrying current I_1 is along is the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by x = 0 and z = d. Find the force exerted at o_2 because of the wire along the x-axis.

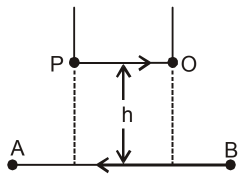


Short Answer Questions

1. A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x – y, y – z and z – x planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.
2. A charged particle of charge e and mass m is moving in an electric field E and magnetic field B. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.
3. An electron enters with a velocity $v = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes) in which there are uniform electric and magnetic fields. The orbit of the electron is found to spiral down inside the cube in plane parallel to the x-y plane. Suggest a configuration of fields E and B that can lead to it.
4. Do magnetic forces obey Newton's third law. Verify for two current elements $dl_1 = dl\,\hat{i}$ located at the origin and $dl_2 = dl\,\hat{j}$ located at (0, R, 0). Both carry current I.
5. A multirange voltmeter can be constructed by using a galvanometer circuit as shown in figure. We want to construct a voltmeter that can measure 2 V, 20 V and 200 V using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.

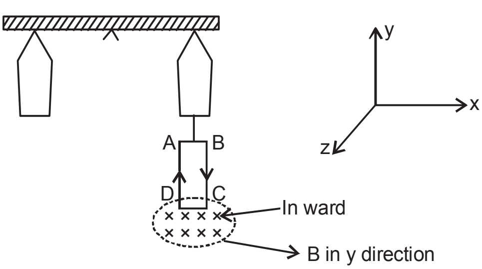


6. A long straight wire carrying current of 25A rests on a table as shown in figure. Another wire PQ of Length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?



Long Answer Questions

1. A 100 turn rectangular coil ABCD (in X-Yplane) is hung from one arm of a balance figure. A mass 500 g is added to the other arm to balance the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in x-z plane) is switched on such that only arm CD of length 1 cm lies in the field. How much additional mass m must be added to regain the balance?



2. A rectangular conducting loop consists of two wires on two opposite sides of length l joined together by rods of length d. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in uniform a magnetic field B at 45° to its plane. Find τ, the torque exerted by the magnetic field on the loop about an axis through the centres of rods.

3. An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5R) respectively, in a uniform magnetic field $B = B_0\hat{i}$, each with an equal momentum of magnitude p = eBR. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?

4. Consider a circular current-carrying loop of radius R in the x-y plane with centre at origin. Consider the line integral

$$\Im(L) = \left|\int_{-L}^{L} B.dl\right|$$

taken along z-axis.

(a) Show that $\Im(L)$ monotonically increases with L

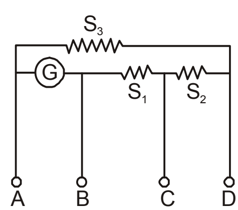
(b) Use an appropriate amperian loop to show that $\Im(\infty) = \mu_0 I$. where I is the current in the wire

(c) Verify directly the above result

(d) Suppose we replace the circular coil by a square coil of sides R carrying the same current I.

What can you say about $\Im(L)$ and $\Im(\infty)$?

5. A multirange current meter can be constructed by using a galvanometer circuit as shown in figure. We want a current meter that can measure 10 mA, 100 mA and 1mA using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1mA. Find S_1, S_2 and S_3 that have to be used.

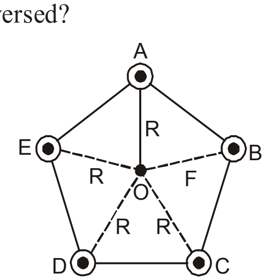


6. Five long wires A, B, C, D and E, each carrying current I are arranged to form edges of a pentagonal prism as shown in figure. Each carries current out of the plane of paper.

(a) What will be magnetic induction at a point on the axis 0? Axis is at a distance R from each wire.

(b) What will be the field if current in one of the wires (say A) is switched off?

(c) What if current in one of the wire (say A) is reversed?



NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(d)** As we know that the uniqueness of helical path is determined by its pitch

$$P(\text{Pitch}) = \frac{2\pi mv\cos\theta}{Bq}$$

Where θ is angle of velocity of charge particle with x-axis

For the given pitch d correspond to charge particle, we have

$$\frac{q}{m} = \frac{2\pi v\cos\theta}{BP} = \text{constant}$$

If motion is not helical, $(\theta = 0)$

As charged particles traverse identical helical paths in a completely opposite direction in a same magnetic field **B**, LHS for two particles should be same and of opposite sign.

$$\therefore \left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

2. **(a)** By Biot-Savart law

$$dB = \frac{Idl\sin\theta}{r^2} = \left(\frac{I\times dl}{r}\right)$$

In Biot-Savat's law, magnetic field **B**||idl × **r** and *idl* due to flow of electron is in opposite direction of v and by direction of cross product of two vectors

$$B \perp V$$

So, the magnetic field is ⊥ to the direction of flow of charge.

3. **(a)** As the direction of magnetic moment of circular loop of radius R placed in the x-y plane is along z-direction and given by M = I (πr^2), when half of the loop with x > 0 is now bent so that it now lies in the y-z plane, the magnitudes of magnetic field moment of each semicircular loop of radius R lie in the x-y plane and the y-z plane is M' = I(πr^2)/4 and the direction of magnetic field moments are along z-directlon and x-direction respectively.

Then resultant is :

$$M_{net} = \sqrt{M'^2 + M'^2} = \sqrt{2}\,M' = \sqrt{2}\,I(\pi r^2)/4$$

So, $M_{net} < M$ or M diminishes.

Hence, the magnitude of magnetic moment is now diminishes.

4. **(d)** Magnetic Lorentz force :

$$F = qVB\sin\theta$$

Magnetic Lorentz force electron is projected with uniform velocity along the axis of a current carrying long solenoid F = –qvB sin 180° = 0(θ = 0°) as magnetic field and velocity are parallel and electric field is zero (E = 0) due to this magnetic field (B) perpendicular to the direction of motion (V). So it will not affect the velocity of moving charge particle. So the electron will continue to move with uniform velocity along the axis of the solenoid

5. **(a)** There is crossed electric and magnetic field between dees so the charged particle accelerates by electric field between dees towards other dees.

So, the charged particle undergoes acceleration as

(i) speeds up between the dees because of the oscillating electric field.

(ii) speed remain the same inside the dees because of the magnetic field but direction undergoes change continuously.

Hence, the charge particle accelerates inside and between Dees always.

Multiple Choice Questions (More Than One Options)

1. **(b, d)**

When the axis of rotation of the loop is along B by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field. So, the work done to rotate the loop is zero.

$$W = MB\cos 90° = 0$$

When rotation not along B.

The work done to rotate the loop in uniform magnetic field $W = MB(\cos\theta_1 - \cos\theta_2)$, where signs are as usual. So the workdone is

$W = MB\cos\theta$ ($\theta = 30°$ given)

$$= MB\cos 30° = \frac{MB\sqrt{3}}{2}$$

2. **(a, b)**

The gyro-magnetic ratio of an electron in an H-atom is equal to the ratio of the magnetic moment and the angular momentum of the electron

$$\mu_e = \frac{\text{magnetic moment of } e(M_e)}{\text{angular momentum of } e(L_e)}$$

If I is the magnitude of the angular momentum of the electron about the central nucleus (orbital angular momentum). Vectorially,

$$\mu_e = \frac{-evr}{2m_e vr} \quad \left(\because\ M_e = \frac{-evr}{2}, L_e = m_e vr\right)$$

$$\boxed{\mu_e = \frac{-e}{2m_e}}$$

So it is independent of velocity or orbit of e depends only on charge.

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

3. **(b, d)**

Magnetic forces on a current carrying conductor, consider wire carrying a steady current I, placed in a uniform magnetic field B, perpendicular to its length is $F = IlB\sin 90° = IlB$

The direction of force is get by Fleming's left hand rule and F is perpendicular to the direction of magnetic field B. Hence, work done by the magnetic force on the ions is 0.

4. **(b, c)**

Loops are identical placed coaxially and carrying same current in opposite sense.

By Ampere's circuital law, we get

$$\oint B.dl = \mu_0(I - I) = 0$$

As the magnetic field inside the loop, there may be a point on C where B and dl are perpendicular to the direction of plane of loop.

So, $\oint B.dl = 0$

or, $|B.d\vec{l}|\cos 90° = 0$

5. **(b, c, d)**

The magnetic forces on a charge particle, $F = q(v \times B)$, is either zero or F is perpendicular to V velocity and magnetic field, which in turn revolves particles on circular path perpendicular to both $(\vec{B}, \vec{V})$ with uniform speed. In both the cases magnetic force on charge particles have equal accelerations.

Both the particles gain or loss energy at the same rate as both are subjected to the same electric force ($F = qE$) in opposite direction, because magnitudes of charges are constant.

As, there is no change of the Centre of Mass of the change particles. Hence, the motion of the Centre of Mass is determined by B alone.

6. **(a, b, d)**

The Lorentz force is experienced by the single moving charge in space is filled with some uniform electric and magnetic fields is

$$F_L = (F_E + F_m)$$
$$F_L = qE + q(v \times B)$$

The, force on charged particle due to electric field $F_E = qE$.

Force on charged particle due to magnetic field. $F_m = q(v \times B)$

The velocity V of charge particle q in magnetic field (B) and electric field (E) will be constant. If Lorentz force $(F_L) = qE + q(v \times B) = ((F_E) + (F_m))$ on charge q.

Now, $F_E = 0$ if $E = 0$ and $F_m = 0$ if $\sin\theta = 0$ or $\theta° = 0°$ or $180°$. Hence, $B \neq 0$.

Also, $E = 0$ and $B = 0$ and the resultant force $qE + q(v \times B) = 0$. In this case $E \neq 0$ and $B \neq 0$

Very Short Answer Questions

1. In a circular motion of charge particle, the charge particle moving perpendicular to the magnetic field, the magnetic Lorentz forces provides necessary centripetal force for revolution.

Contripental force $\left(\frac{mv^2}{R}\right)$ is balanced by

magnetic force $(F_m) = qvB \sin 90° = qvB$

then, $\frac{mv^2}{R} = qvB$

By solving, we get

So, $\frac{qB}{m} = \frac{v}{R} = \omega \quad (\therefore V = \omega R \text{ and } q = e)$

Then the dimensional formula of angular frequency is

$$\therefore \quad [\omega] = \left[\frac{eB}{m}\right] = \left[\frac{V}{R}\right] = [T]^{-1}$$

that is the required expression.

2. Consider no work done by a force, so

$$dW = F.dl = 0$$

$$\Rightarrow \quad F.\frac{dl}{dt}\cdot dt = 0 \quad \left(\therefore V = \frac{dl}{dt}\right)$$

$$\Rightarrow \quad F.v\,dt = 0$$

$$\Rightarrow \quad F.v = 0,\ dt \neq 0$$

$$Fv = 0$$

$$Fv\cos\theta = \cos 90°$$

$$\theta = 90°$$

So, F must be velocity dependent which implies that angle θ between F and v is 90°. If v changes its direction then direction of F should also change, so that F is dependent on V to make work done zero.

3. The magnetic force change from inertial frame to frame. The magnetic force is frame dependent on frame of refrence.

Therefore, the net acceleration which comes into existence out of this is however, frame independent for inertial frames.

4. In cyclotron, frequency of charged particle is equal to the radio frequency. When the frequency of the radio frequency (rf) field were doubled, the time period of the radio frequency (rf) field become half, so, the duration in which particle completes half revolution inside the dees, radio frequency completes the cycle.

So, a charge particle will accelerate and decelerate alternatively and its moves in circular path between the dees during motion in Dees. Hence, the radius of path in the dees will remain same.

5. By Biot-Savart law, magnetic field B is parallel to idl × r and idl have its direction along the direction of flow of current, then the force

$$(F) = B\,idl\sin\theta = Bidl \quad (\because \theta = 90°)$$

For the direction of magnetic field, at O_2, due to wire carrying I_1 current is parallel to y-axis.

$$B \,||\, \text{parallel } idl \times r \text{ or } \hat{i}\times\hat{k}, \text{ but } \hat{i}\times\hat{k} = -\hat{j}$$

Therefore, the direction at O_2 is along Y-direction. The direction of magnetic force exerted at O_2 because of the wire along the, x-axis.

$$F = Il \times B = \hat{j}\times(-\hat{j}) = 0$$

Now, the consider wire of current I_2 is parallel to y-axis. So I_2 and B_1 are also along y-axis, i.e., angle between I_2 and B_1 is zero, so magnetic force F_2 on wire of current I_2 is $F_2 = B_1 I_2\, dl\sin 0° = 0$.

Hence, force on O_2 due to wire of current I_1 is zero.

Short Answer Questions

1. The magnetic field due to arc of current carrying coil which subtends an angle θ at centre is given by

$$B = \frac{\mu_0}{4\pi}\frac{I\theta}{R}$$

The magnetic field for the current carrying loop quarter circles of radius R, lying in the positive quadrants of the x-y plane

when $\left(\theta = \frac{\pi}{2}\right)$

$$B_1 = \frac{\mu_0}{4\pi}\frac{I\left(\frac{\pi}{2}\right)}{R}\hat{k} = \frac{\mu_0}{4\pi}\frac{I}{2R}\hat{k} = \frac{\mu_0 I}{8\pi R}\hat{k}$$

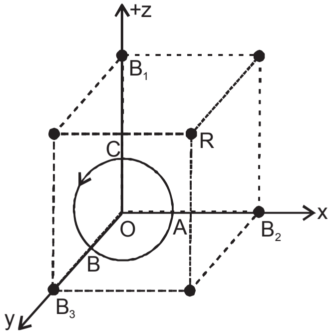
The magnetic field for the current carrying loop quarter circles of radius R. lying in the positive quadrants of the y-z plane

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{2R}\hat{i} = \frac{\mu_0 I}{8\pi R}\hat{i}$$

The magnetic field for the current carrying loop quarter circles of radius R, lying in the positive quadrants of the z-x plane.

$$B_3 = \frac{\mu_0}{4\pi}\frac{I}{2R}\hat{j} = \frac{\mu_0 I}{8\pi R}\hat{j}$$

Current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-y and z-z planes with their centres at the origin, joined together is equal to the vector sum of magnetic field due to each quarter is



$$B = \frac{1}{4\pi}\left(\hat{i}+\hat{j}+\hat{k}\right)\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8\pi R}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

2. When a charge particle in an electric and magnetic field its motion will be circular.

The charge particle moving perpendicular to the magnetic field, the magnetic Lorentz forces provides necessary centripetal force for revolution.

Centripetal force = $\frac{mv^2}{R}$ is balanced by magnetic force

$$(F_m) = qvB \sin 90 = qvB$$

So, $$\frac{mv^2}{R} = qvB$$

By solving, we get

$$\therefore \quad \frac{qB}{m} = \frac{v}{R} = \omega,$$

$$V = \omega R \text{ and } q = e$$

Then dimensional formula of angular velocity (ω)

$$\therefore \quad [\omega] = \left[\frac{eB}{m}\right] = \left[\frac{v}{R}\right] = [T]^{-1}$$

This is the required expression.

3. Let, magnetic field $B = B_0\hat{k}$ in y-axis and moving electron enters with the velocity $v = v_0\hat{i}$ into a cubical region along x-axes.

Then the force on electron, using magnetic Lorentz force, is

$$F = -e\left(v_0 i \times B_0\hat{k}\right) = ev_0B_0\hat{i}$$

which revolves the electron in x-y plane.

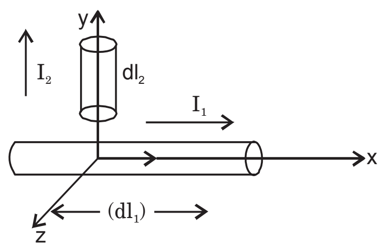
The electric force $F = -eE_0\hat{k}$ accelerates electron along z-axis which in turn increases the radius of circular path. Hence particle traversed on spiral path.

The magnetic field revolves the charge particle in uniform circular motion in x-y plane and electric field along x-direction increases the speed, which in turn increases the radius of circular path and hence, particle traversed on spiral path.

4. By Biot-Savart's law, magnetic field B is parallel to idl × r and idl have its direction along the direction of flow of current.

For the direction of magnetic field, at dl_2, located at (0, R, 0) due to wire d_1, is given by $B \parallel idl \times r$ or $\hat{i} \times \hat{j}$ (because point (0, R, 0) lies on y-axis), but $\hat{i} \times \hat{j} = \hat{k}$

Therefore, the direction of magnetic field at d_2 is along z-direction.



The direction of magnetic force exerted at d_2 because of the first wire along the x-axis.

$F = i(I \times B)$ i.e., $F \| (i \times k)$ or along $-\hat{j}$ direction

So, force due to dl_1 on dl_2 is non-zero.

$$F = B_1 I_2 dl_2 \sin 90° = B_1 I_2 dl_2$$

Now, for the direction of magnetic field, At d_1, located at (0, 0, 0) due to wire d_2 is given by $B \| idl \times r$ or $\hat{j} \times -\hat{j}$ (because origin lies on y-direction w.r.t. point (0, R, 0),), but $j \times -j = 0$.

So, the magnetic field at d_1 does not exist.

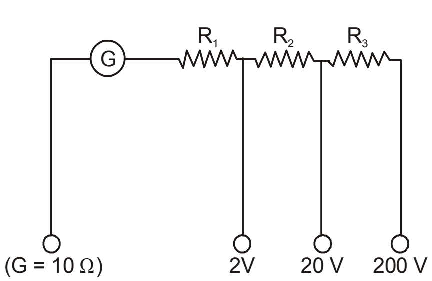
Force due to dl_2 on dl_1 is zero.

$$F = B_2 I_1 dl_1 \sin 0° = 0$$

Hence, magnetic forces do not obey Newton's third law.

5. A galvanometer can be converted into voltmeter by connecting a very high resistance wire connected in series with galvanometer.

The relationship is given by $I_g(G+R) = V$ where I_g is range of galvanometer. G is resistance of galvanometer and R is resistance of wire connected in series with galvanometer.



The resistance of galvanometer G, R_1, R_2 and R_3 are in series and G can tolerate

$$i_G = 1 \times 10^{-3} \text{ Amp}$$

Applying expression in different situations

For $i_G(G + R_1) = 2$ ... (i) for 2 V range

For $i_G(G + R_1 + R_2) = 20$ ... (ii) for 20 V range

and for $i_G(G + R_1 + R_2 + R_3) = 200$ ... (iii) for 200 V range

By putting the given value of G in equaiton (i).

where, $G = 10\,\Omega$

$i_G = 10^{-3}$ Amp

We get, $R_1 = 1990\,\Omega$

$R_2 = 18\,k\Omega$

$R_3 = 180\,k\Omega$

6. When the force applied on PQ by long straight wire current carrying of 25A rests on a table must balance the weight of small current carrying wire. Then the magnetic field produced by long straight wire carrying current of 25A rests on a table on small wire

$$B = \frac{\mu_0 I}{2\pi h}$$

The magnetic force on small conductor is

$F = BIl \sin\theta$ $(\because \theta = 90°, \sin 90° = 1)$

$= BIl(\sin 90°) = Bil$

Consider the wire is balanced at hight h thus, the force applied on PQ with the weight of small current carrying wire.

$$F = mg = \frac{\mu_0 I^2 l}{2\pi h} \qquad (\therefore I_1 = I_2 = I = 25A)$$

$$h = \frac{\mu_0 I^2 l}{2\pi mg} = \frac{4\pi \times 10^{-7} \times 25 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8} = 51 \times 10^{-4}$$

$$h = 0.51 \text{ cm}$$

Long Answer Questions

1. The magnetic field is perpendicular to arms BC and AD, so torque acts on CD and AB arms which rotate coil.

For equilibrium the net torque should also be equal to zero.

When the field is off $\Sigma t = 0$ considering the separation of each hung from mid-point be l.

$$Mgl = W_{coil}\, l$$
$$500gl = W_{coil}\, l$$
$$W_{coil} = 500 \times 9.8\,N$$

Taking moment of force about mid-point, we get the weight of coil

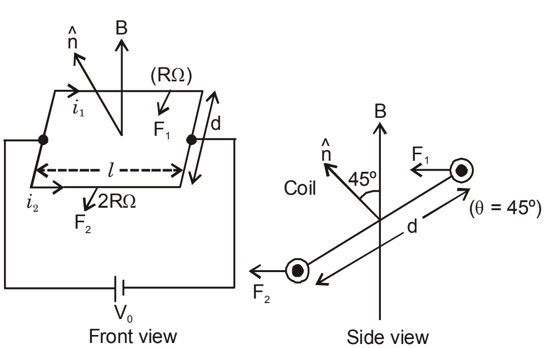
When the magnetic field is switched on and the weight (mg) is added on other side of beam balance to balance the coil,

$$Mgl + mgl = W_{coil}\, l + IBL \sin 90°\, I$$
$$mgl = BIL\, l$$

$$m = \frac{BIL}{g} = \frac{0.2 \times 4.9 \times 1 \times 10^{-2}}{9.8} = 10^{-3}\,kg = 1\,g$$

Hence, 1g of additional mass must be added to regain the balance.

2. As we know that $R = \rho l/A$, consider the thicker wire has a resistance R then the thinner wire has a resistance 2R as the wires are of the same material and same length but with cross-sections differing by a factor 2 is given.



By Ohm's law

$$(V = IR) \quad (\because V_1 = V_2 = V_0)$$

So, for current $(I_1) = \frac{V_1}{R_1} = \frac{V_0}{R}$

and for current $(I_2) = \frac{V_2}{R_2} = \frac{V_0}{2R}$

Now, the force and torque on first wire is

$$F_1 = i_1 lB \sin 45°$$

$$F_1 = \frac{V_0}{R}\frac{lB}{\sqrt{2}}$$

$$\tau_1 = (d).F_1 = \frac{V_0 ldB}{\sqrt{2}R}$$

Similarly, the force and torque on thinner wire is

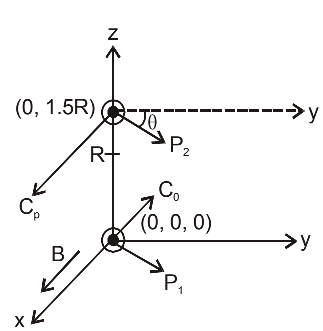
$$F_2\, i_2 lB \sin 45° = \frac{V_0}{2R}\frac{lB}{\sqrt{2}}$$

$$\tau_2 = F_2 = \frac{V_0 ldB}{2\sqrt{2}R}$$

Hence resultant torque is $\tau = \tau_1 - \tau_2$

$$\tau = \frac{1}{2\sqrt{2}}\frac{V_0 ldB}{R}$$

3. As $B = B_0 i$, so magnetic field B is along the x-axis, for a circular orbit the momenta of both particles are in the y-z plane. Let p_1 and p_2 be the momentum of the electron and positron, respectively. Both traverse a circle of radius R of opposite sense. Let p_1 make an angle θ with the y-axis p_2 must make the same angle.



The centres of the respective circles must be perpendicular to the momenta and at a distance R. Let the centre of the electron be at C_e and of the positron at C_p. The coordinates of C_e is

$$C_e \equiv (0 + R\sin\theta, R\cos\theta)$$

The coordinates of C_p will be if the planes of circular path are in y-z plane.

$$C_p \equiv (0, -R\sin\theta, \frac{3}{2}R - R\cos\theta)$$

The circles of the two shall not overlap if the distance between the two centers are greater than 2R.

Consider d be the distance between C_p and C_e.

If (id > d > 2R)

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

By putting all vlaue in above equation,

$$\text{So, } d^2 = (2R\sin\theta)^2 + \left(\frac{3}{2}R - 2R\cos\theta\right)^2$$

$$= 4R^2\sin^2\theta + \frac{9^2}{4}R - 6R^2\cos\theta + 4R^2\cos^2\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta$$

As, d has to be greater than 2R

$$d^2 > 4R^2$$

$$\text{So, } 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2,$$

$$3R^2\left[\frac{3}{4} - 2\cos\theta\right] > 0$$

$$\text{So, } \frac{9}{4} > 6\cos\theta$$

$$\text{or, } \cos\theta < \frac{3}{8}$$

Hence, $\left(\cos\theta < \frac{3}{8}\right)$ is the condition that two circular paths do not intersect; θ is angle of momentum of electron or positron with y-axis.

4. By using Ampere circuital law.

(a) B (z) points in the same direction on z-axis and hence, J(L) is a monotonical function of L. As B and dl are along the same direction, so $\vec{B}.d\vec{l} = B.dl\cos\theta = B.\,dl\cos 0° = B.\,dl$

(b) J(L) + contribution from large distance on contour $C = \mu_0 I$

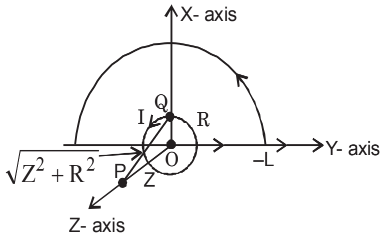
As, $L \to \infty$

Contribution from large distance→0 as

$$B \propto 1/r^3$$

$$J(\infty) - \mu_0 I$$

(c) The magnetic field due to circular current-carrying loop of radius R in the x-y plane with centre at origin at any point lying at a distance a from origin.



Let, a point P is in z-axis at distacne z, i.e, OP = Z and angle between R and QP = θ then the magnetic field at P due to loop is

$$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$

$$\int_{-\infty}^{\infty} B_z dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} dz$$

$$\left(\because \tan\theta = \frac{z}{R}\right)$$

So, $z = R\tan\theta$

$\Rightarrow$ $dz = R\sec^2\theta d\theta$

By putting the value of z,

$$\text{So, } \int_{-\infty}^{\infty} B_z dz = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \mu_0 I$$

(d) $B(z)_{\text{square loop}} <$ loop $B(z)_{\text{circular loop}}$

$\therefore$ $(L)_{\text{square loop}}, = (L)_{\text{circular loop}}$

By using arguments as in (b)

$$(\infty)_{\text{square loop}} = (\infty)_{\text{circular loop}}$$

5. A galvanometer can be converted into ammeter by connecting a very low resistance (shunt S) connected in parallel with galvanometer. The relationship is given by $I_g G = (I - I_g) S$ where I_g is range of galvanometer, G is resistance of galvanometer.

Here,

$I_G.G = (I_1 - I_G)(S_1 + S_2 + S_3)$ for $I_1 = 10$ mA, ... (i)

$I_G.(G + S_1) = (I_2 - I_G)(S_2 + S_3)$ for $I_2 = 100$ mA, ... (ii)

and $I_G.(G + S_1 + S_2) = (I_3 - I_G)(S_3)$ for $I_3 = 1$A, ... (iii)

By solving we get,

$S_1 = 1W, S_2 = 0.1$ W

and $S_3 = 0.001W$

6. **(a)** Let, the five wires A, B, C, D and E be perpendicular to the plane of paper at locations as shown in given figure (Question).

Magnetic field Induction due to five wires will be represented by various sides of a ' closed pentagon in one order, lying in the plane of paper. So, its value is zero.

(b) The vector sum of magnetic field. Thus by each wire at O is equal to 0. Therefore, magnetic induction produced by one current carrying wire is equal in magnitude of resultant of four wires and opposite in direction.

So, the field if current in one of the wires (say A) is switched of f is $\frac{\mu_0}{2\pi}\frac{i}{R}$ perpendicular to AO towards left.

(c) If current in wire A is reversed, then total magnetic field induction at O acting perpendicular to AO towards left.

So, total mangetic field induction = Magnetic field induction due to wire A + magnetic field induction due to wires B, C, D and E

Total magnetic field induction

$$= \left(\frac{\mu_0}{4\pi}\frac{2I}{R}\right) + \left(\frac{\mu_0 2I}{\pi R}\right)$$

Hence, total magnetic field induction

$$= \frac{\mu_0 I}{\pi R} \perp \text{AO towards left.}$$

Chapter Five

MAGNETISM AND MATTER

EXERCISES

5.1 Answer the following questions regarding earth's magnetism:

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18º. Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

5.2 Answer the following questions:

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

[**Note:** Exercise 5.2 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]

5.3 A short bar magnet placed with its axis at 30º with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

5.4 A short bar magnet of magnetic moment m = 0.32 JT^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

5.5 A closely wound solenoid of 800 turns and area of cross section 2.5×10^{-4} m^2 carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

5.6 If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

5.7 A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

5.8 A closely wound solenoid of 2000 turns and area of cross-section 1.6×10^{-4} m^2, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

5.9 A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

5.10 A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

5.11 At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

5.12 A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.

5.13 A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null–point (i.e., 14 cm) from the centre of the magnet? (At *null points*, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

5.14 If the bar magnet in exercise 5.13 is turned around by 180°, where will the new null points be located?

5.15 A short bar magnet of magnetic movement 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field on (a) its normal bisector and (b) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

ADDITIONAL EXERCISES

5.16 Answer the following questions:

(a) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?

(b) Why is diamagnetism, in contrast, almost independent of temperature?

(c) If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

(d) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?

(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnet?

5.17 Answer the following questions:

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

(c) 'A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory?' Explain the meaning of this statement.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?

(e) A certain region of space is to be shielded from magnetic fields. Suggest a method.

5.18 A long straight horizontal cable carries a current of 2.5 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (ignore the thickness of the cable)? (At *neutral points*, magnetic field due to a current-carrying cable is equal and opposite to the horizontal component of earth's magnetic field.)

5.19 A telephone cable at a place has four long straight horizontal wires carrying a current of 1.0 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G, and the angle of dip is 35°. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4.0 cm below the cable?

5.20 A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45° with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.

(a) Determine the horizontal component of the earth's magnetic field at the location.

(b) The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

5.21 A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60°, and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

5.22 A monoenergetic (18 keV) electron beam initially in the horizontal direction is subjected to a horizontal magnetic field of 0.04 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm ($m_e = 9.11 \times 10^{-31}$ kg). [**Note:** *Data* in this exercise are so chosen that the answer will give you an idea of the effect of earth's magnetic field on the motion of the electron beam from the electron gun to the screen in a TV set.]

5.23 A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's law)

5.24 A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field **B** in the core for a magnetising current of 1.2 A?

5.25 The magnetic moment vectors $\boldsymbol{\mu}_s$ and $\boldsymbol{\mu}_l$ associated with the intrinsic spin angular momentum **S** and orbital angular momentum **l**, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by:

$\boldsymbol{\mu}_s = -(e/m)\,\mathbf{S}$,

$\boldsymbol{\mu}_l = -(e/2m)\mathbf{l}$

Which of these relations is in accordance with the result expected *classically*? Outline the derivation of the classical result.

Chapter 5:- Magnetism

Multiple Choice Questions (MCQs)

1. A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as xy-plane. Its magnetic moment m
   (a) is non-zero and points in the z-direction by symmetry
   (b) points along the axis of the toroid ($m = m\phi$)
   (c) is zero, otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid
   (d) is pointing radially outwards

2. The magnetic field of the earth can be modelled by that of a point dipole placed at the centre of the earth. The dipole axis makes an angle of 11.3° with the axis of the earth. At Mumbai, declination is nearly zero. Then,
   (a) the declination varies between 11.3° W to 11.3° E
   (b) the least declination is 0°
   (c) the plane defined by dipole axis and the earth axis passes through Greenwich
   (d) declination averaged over the earth must be always negative

3. In a permanent magnet at room temperature.
   (a) magnetic moment of each molecule is zero
   (b) the individual molecules have non-zero magnetic moment which are all perfectly aligned
   (c) domains are partially aligned
   (d) domains are all perfectly aligned

4. Consider the two idealised systems (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length L >> R, radius of cross-section. In (i) E is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental laws as below
   (a) case (i) contradicts Gauss' law for electrostatic fields
   (b) case (ii) contradicts Gauss' law for magnetic fields
   (c) case (i) agrees with $\oint E.dl = 0$.
   (d) case (ii) contradicts $\oint H.dl = I_{en}$

5. A paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be
   (a) $\frac{32}{3} Am^{-1}$ (b) $\frac{2}{3} Am^{-1}$
   (c) $6 Am^{-1}$ (d) $2.4 Am^{-1}$

Multiple Choice Questions (MCQs) (More than one option correct)

1. S is the surface of a lump of magnetic material.
   (a) Lines of B are necessarily continuous across S
   (b) Some lines of B must be discontinuous across S
   (c) Lines of H are necessarily continuous across S
   (d) Lines of H cannot all be continuous across S

2. The primary origin (S) of magnetism lies in
   (a) atomic currents
   (b) Pauli exclusion principle
   (c) polar nature of molecules
   (d) intrinsic spin of electron

3. A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of $\mu_r = 1000$. The core is heated beyond the Curie temperature, T_c.
   (a) The H field in the solenoid is (nearly) unchanged but the B field decreases drastically
   (b) The H and B fields in the solenoid are nearly unchanged
   (c) The magnetisation in the core reverses direction
   (d) The magnetisation in the core diminishes by a factor of about 10^8

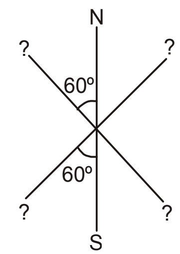
4. Essential difference between electrostatic shielding by a conducting shell and magnetostatic shielding is due to
   (a) electrostatic field lines can end on charges and conductors have free charges
   (b) lines of B can also end but conductors cannot end them
   (c) lines of B cannot end on any material and perfect shielding is not possible
   (d) shells of high permeability materials can be used to divert lines of B from the interior region
5. Let the magnetic field on the earth be modelled by that of a point magnetic dipole at the centre of the earth. The angle of dip at a point on the geographical equator
   (a) is always zero
   (b) can be zero at specific points
   (c) can be positive or negative
   (d) is bounded

Very Short Answer Questions

1. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?
2. A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^6$ A/m. Calculate the magnetisation current I_M.
3. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of $N_2(\sim 5 \times 10^{-9})$ (at STP) and $Cu(\sim 10^{-5})$.
4. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.
5. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.
   (i) In which direction will it move?
   (ii) What will be the direction of its magnetic moment?

Short Answer Questions

1. Verify the Gauss's law for magnetic field of a point dipole of dipole moment m at the origin for the surface which is a sphere of radius R.
2. Three identical bar magnets are rivetted together at centre in the same plane as shown in figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in the figure. Determine the poles of the remaining two.



3. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (i) electric dipole p in an electrostatic field E and (ii) magnetic dipole M in a magnetic field B. Write down a set of conditions on E, B, p, M so that the two motions are verified to be identical. (Assume identical initial conditions).
4. A bar magnet of magnetic moment M and moment of inertia I (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid-point, perpendicular to length, in a magnetic field B. What would be the similar period T' for each piece?
5. Use (i) the Ampere's law for H and (ii) continuity of lines of B, to conclude that inside a bar magnet, (a) lines of H run from the N-pole to 5-pole, while (b) lines of B must run from the 5-pole to N-pole.

Long Answer Questions

1. Verify the Ampere's law for magnetic field of a point dipole of dipole moment $M = M\hat{k}$. Take C as the closed curve running clockwise along
   (i) the z-axis from $z = a > 0$ to $z = R$,
   (ii) along the quarter circle of radius R and centre at the origin in the first quadrant of xz-plane,
   (iii) along the x-axis from $x = R$ to $x = a$, and
   (iv) along the quarter circle of radius a and centre at the origin in the first quadrant of xz-plane

2. What are the dimensions of χ, the magnetic susceptibility? Consider an H-atom. Gives an expression for χ, upto a constant by constructing a quantity of dimensions of χ, out of parameters of the atom e, m, v, R and μ_0. Here, m is the electronic mass, v is electronic velocity, R is Bohr radius. Estimate the number so obtained and compare with the value of $|\chi| \simeq 10^{-5}$ for many solid materials.

3. Assume the dipole model for the earth's magnetic field B which is given by B_V = vertical component of magnetic field $= \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}$

B_H = horizontal component of magnetic field = $\frac{\mu_0}{4\pi}\frac{\sin\theta m}{r^3}$

$\theta = 90°$ – lattitude as measured from magnetic equator.

Find loci of points for which (a) | B | is minimum (b) dip angle is zero and (c) dip angle is 45°.

4. Consider the plane S formed by the dipole axis and the axis of earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the declination and dip angles at P and Q.

5. There are two current carrying planar coil made each from identical wires of length L. C_1 is circular (radius R) and C_z is square (side a). They are so constructed that they have same frequency of oscillation when they are placed in the same uniform B and carry the same current. Find a in terms of R.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. (c) Toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. Thus, in this case magnetic field is only confined inside the body of toroid.

So no magnetic field outside the toroid and magnetic field only inside the toroid.

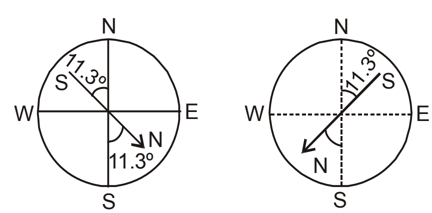
In case of toroid, the magnetic field is in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. This is because the loop encloses no current. Thus, the magnetic moment of toroid is zero.

In other case, if we take r as a large distance outside the toroid, then $m \propto \frac{1}{r^3}$. Which is not possible.

2. (a) Magnetic declination is an angle between angle of magnetic meridian and the geographic meridian.

As the earth's magnetism, the magnetic field lines of the earth resemble that of a hypothetical magnetic dipole located at the centre of the earth.

The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by 11.3° (approx) with respect to geographical of axis earth. This results into two situations as given in the figure.



So, the declination varies between 11.3° W to 11.3° E.

3. (d) We know that a permanent magnet is a substance which at room temperature retain ferromagnetic property for a long period of time. The individual atoms in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume i.e., domain.

Hence, in a permanent magnet at room temperature, domains are all perfectly aligned.

4. (b) The electric field lines, do not form a continuous path while the magnetic field lines form the closed paths.

Gauss's law states that, $\oint_s E.ds = \frac{q}{\varepsilon_0}$ for electrostatic field. So, it does not contradict for electrostatic fields as the electric field lines do not form closed continuous path.

According to Gauss' law in magnetic field,

$$\oint_s E.ds = 0$$

It contradicts for magnetic field, because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed path.

5. **(b)** According to the Curie law, the intensity of magnetisation (I) is directly proportional to the magnetic field induction and inversely proportional to the temperature (t) in kelvin.

So, I magnetisation

$$\propto \frac{\text{B (magnetic field induction)}}{\text{t(temperature in kelvin)}}$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{t_1}{t_2} \quad \text{... (i)}$$

As given that : $I_1 = 8\,Am^{-1}, I_2 = ?$

$$B_1 = 0.6\,T, t_1 = 4K$$

$$B_2 = 0.2\,T, t_2 = 16K$$

by putting the value of B_1, B_2, t_1, t_2 I_1 in equation (i)

So, $\frac{0.2}{0.6} \times \frac{4}{16} = \frac{I_2}{8}$

We get, $I_2 = 8 \times \frac{1}{12}$

$$\boxed{I_2 = \frac{2}{3}\,A/m}$$

Multiple Choice Questions (More Than One Option)

1. **(a, d)** As we know that, the magnetic intensity (H) outside any magnet is $H = B/\mu_0$

For inside the magnet $H = B/\mu_0\mu_r$,

where μ_r is the relative permeability of material.

Magnetic field lines for magnetic induction (B) form continuous lines so, lines of B are necessarily continuous across S.

Also, magnetic intensity (H) to magnetise varies for inside and outside the lump. So, lines of H cannot all be continuous across S.

2. **(a, d)** Motion of charged partical produces magnetism and nature of magnetism depends on the motion of charge particle.

The primary origin of magnetism lies in the fact that the electrons are revolving and spinning about nucleus of an atom, that gives rise to current called atomic current.

This atomic currents gives rise to magnetism. In atom, electrons revolving and spinning about nucleus of an atom is called intrinsic spin of electron.

3. **(a, d)** As we know that, the magnetic field intensity $H = nI$,

where n = number of turns per metre of a solenoid and I = current.

And the magnetic induction

$$B = \mu_0 \mu_r nI \quad \text{...(i)}$$

So, for solenoid $H = nI$

$$H = 1000 \times 1 = 1000\,Am$$

$$(\because \text{given}, n = 1000)$$

So, H is a constant, then it is nearly unchanged.

From (i), $B = \mu_0 \mu_r nI$

$$= (\mu_0 nI)\mu_r = k\mu_r$$

$$(\therefore \text{where}, K = \text{constant})$$

So, $B \propto \mu_r$

Hence, we find that, B varies with the variation in μ_r.

By curies law, when temperature of the iron core of solenoid or ferromagnetic material is raised beyond Curie temperature, then it behave as paramagnetic material, where,

Susceptibility of $(\chi_m)_{Fero1} = 10^3$

Susceptibility of $(\chi_m)_{Para2} = 10^{-5}$

So, $\frac{B_1}{B_2} = \frac{X_1}{X_2} = \frac{(\chi_m)_{Fero1}}{(\chi_m)_{Para2}} = \frac{10^3}{10^{-5}} = 10^8$,

$$B_1 = 10^8 B_2$$

or, $B_2 = 10^{-8} B_1$

4. **(a, c, d)**

Electrostatic shielding is the shielding which block the effects of an electric field.

The conducting shell can block the effects of an external field on its internal content or the effect of an internal field on the outside environment.

As non existence of mono pole magnetic field lines cannot be stopped or shield. So, perfect shielding is not possible.

Magnetostatic shielding is done by using an enclosure made of a high permeability magnetic material to prevent a static magnetic field outside the enclosure from reaching objects inside it or to confine a magnetic field within the enclosure.

5. **(b, c, d)**

Angle of dip is the angle made by the total magnetic field of the earth with the surface of the earth.

The net magnetic field of the earth will be zero and it modelled by a point magnetic dipole at the centre, then it is in the same plane of geographical equator. So, the angle of dip at a specific point on the geographical equator will be zero, it is not zero at all point of geographical equator and it is bounded in a range from positive to negative value.

Very Short Answer Questions

1. The comparison between the spinning of a proton and an electron can be done by comparing their magnetic dipole moment

$$M_p = \frac{eh}{4\pi m_p}$$

and $$M_e = \frac{eh}{4\pi m_e} \quad \left(\because \frac{eh}{4\pi} = \text{constant}\right)$$

$$M \propto \frac{1}{m}$$

or, $$\frac{M_p}{M_e} = \frac{m_e}{m_p}$$

As the mass of proton is 1837 times of electrons so (mp = 1837 me).

$$M_p = \frac{M_e \times m_e}{1837 m_e}$$

$$= \frac{M_e}{1837}$$

So, $$\frac{M_p}{M_e} = \frac{1}{1837} << 1$$

$\Rightarrow$ $M_p << M_e$

Hence, the effect of magnetic moment of proton is neglected as compared to that of electron.

2. As we know that :

$$M \text{ (Magnetic moment)} = \frac{I_m}{l} \quad \text{...(i)}$$

As given that, M (intensity of magnetisation) = 10^6 A/m.

l(length) = 10 cm = 10×10^{-2} m = 0.1 m

I_m (magnetisation current) = ?

From (i) $$M = \frac{I_m}{l}$$

So, $$I_M = M \times l = 10^6 \times 0.1$$

$$I_M = 10^5 \text{ A}$$

3. Magnetic susceptibility is a measure of how a material behaves in external magnetic field.

As we know that :

$$\chi = \frac{\text{Intensity of Magnetisation (M)}}{\text{Magnetic field intensity (H)}}$$

$$= \frac{\text{Magnetic moment (M)/Volume (V)}}{\text{H}}$$

$$= \frac{M}{HV} = \frac{M}{H(\text{mass/density})} = \frac{M\rho}{Hm}$$

where magnetic material are same

$$\left(\because \frac{M}{Hm} = \text{constant}\right)$$

So, $(\chi \propto \rho)$

Hence, $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{\rho_{N_2}}{\rho_{Cu}} \quad \text{...(i)}$$

The density of nitrogen is :

$$\rho_{N_2} = \frac{28g}{22.4L} = \frac{28g}{22400 \text{ ml}} = \frac{28g}{22400} \text{ per cm}^3$$

and, density of copper is $\rho_{Cu} = 8 \text{g/cm}^3$

Now, comparing both densities

$$\frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28}{22400} \times \frac{1}{8} \cong 1.6 \times 10^{-4}$$

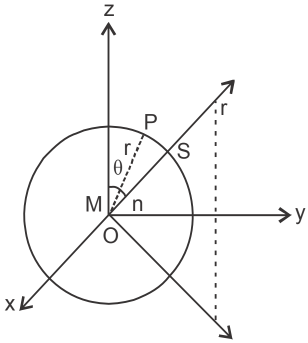
From (i) $$\frac{\chi_{N_2}}{\chi_{Cu}} = 1.6 \times 10^{-4}$$

4. Susceptibility of magnetic material $\chi = \frac{I}{H}$, where I is the intensity of magnetisation induced in the material and H is the magnetising force.
Due to orbital motion of electrons of diamagnetism in an atom developing magnetic moments opposite to applied field. So, the net magnetic moment of the diamagnetic material is zero and hence, the susceptibility χ of diamagnetic material is not much affected by temperature.
The direction of magnetism due to orbital motion of electrons in (paramagnetism and ferromagnetism) the external field applied in the same direction, so net magnetism increased due to temperature. As temperature is raised, the alignment is disturbed, resulting decrease in susceptibility of both with increase in temperature.

5. **(c)** As we know, superconducting material and nitrogen are diamagnetic materials in nature.
When a superconducting material is dipped in liquid nitrogen then superconducting material will again behave as a diamagnetic material. When that diamagnetic material is placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnet or magnetising field.
Hence, it will be repelled and also its direction of magnetic moment will be opposite to the direction of magnetic field.

Short Answer Questions

1. Draw the figure as per given situation,



As we know that :
The gauss's law in magnetisation is :

$$\oint B.dS = 0$$

As given that :
Magnetic moment (M) of dipole at origin O is

$$M = M\hat{k}$$

Consider P be a point at distance r from O and OP makes an angle θ with z-axis. Component of M along OP = Mcosθ.
So, the magnetic field induction at P due to dipole of moment Mcosθ is

$$B = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}\hat{r}$$

According to the diagram, r is the radius of sphere with centre at O lying in yz-plane. Take an elementary area dS of the surface at P. So,

$$dS = r(r\sin\theta d\theta)\hat{r}$$
$$= r^2\sin\theta d\theta\hat{r}$$
$$\left[\therefore\ d\theta = \frac{dS}{r^2} \text{ or } dS = r^2 d\theta\right]$$

$$\oint B.dS = \oint \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}\hat{r}\left(r^2\sin\theta\, d\theta\hat{r}\right)$$

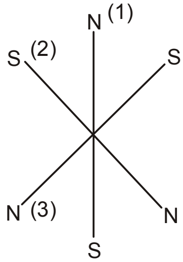
$$= \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi} 2\sin\theta.\cos\theta d\theta = \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi}\sin 2\theta d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{r}\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi}$$

$$= -\frac{\mu_0}{4\pi}\frac{M}{2r}\left[\cos 4\pi - \cos 0\right] = \frac{\mu_0}{4\pi}\frac{M}{2r}[1-1] = 0$$

$$\boxed{\oint B.dS = 0}$$

2. The system will be in stable equilibrium if the net force on the system is zero and net torque on the system is also zero. That is possible only when the poles of the remaining two magnets are as given figure.
The north pole of magnet (1) is equally attracted by south poles of (2) and (3) magnets placed at equal distance.



3. Let us consider the angle between p and E is θ.
Torque on electric dipole of moment p in electric field (E),
So $\tau = pE\sin\theta$.
And the angle between M and B is also θ.
Torque on magnetic dipole moment M in magnetic field B,

$$\tau' = MB\sin\theta$$

Two motions will be identical, if

$$pE\sin\theta = MB\sin\theta$$

$$pE = MB \qquad \text{... (i)}$$

As we know that, the relation between E and B is $E = cB$
By, Putting the value of E in Eq. (i),

$$pcB = MB$$

So, $\boxed{p = \dfrac{M}{c}}$

4. As we know that
The time period of oscillation is,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where,
M = magnetic moment of the magnet
B = uniform magnetic field in which magnet is oscillating.
if a magnet of magnetic moment M is cut into n equal parts then magnetic moment M of all equal parts is nM' = M
Magnetic dipole moment for two parts of magnet (M' = M/2)
So, magnetic moment of each parts of magnet is M' = M/2

Here, moment of inertia $I = \dfrac{ml^2}{12}$.

When magnet is cut into two equal pieces, perpendicular to length, then moment of inertia of each piece of magnet about an axis perpendicular to length passing through its centre is

$$I' = \frac{m'd'^2}{12}$$

$$I' = \frac{m}{2}\frac{(l/2)^2}{12} \qquad \left(\because l' = \frac{l}{2}\right)$$

$$I' = \frac{ml^2}{12}\times\frac{1}{8} = \frac{I}{8} \qquad \left[\because I = \frac{ml^2}{12}\right]$$

So, $I' = \dfrac{I}{8}$

Magnetic dipole moment

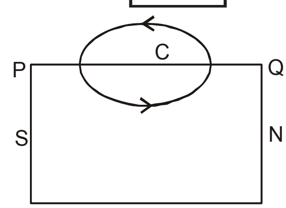
$$M' = M/2$$

Its time period of oscillation is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{(M/2)B}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}}$$

$$\boxed{T' = \frac{T}{2}}$$

5.



Let the magnetic field line of B through the bar magnet must be in a closed loop as shonw in figure above.
Consider C an amperian loop

So, $$\int_Q^P H.dl = \int_Q^P \frac{B}{\mu_0}.dl$$

As the angle between B and dl is less than 90° inside the bar magnet. So, $\cos\theta > 1$ it is positive.

$$\int_Q^P H.dl = \int_Q^P \frac{B}{\mu_0}.dl > 0$$

i.e. positive
Hence, the lines of B must run from south pole(S) to north pole (N) inside the bar magnet. By Ampere's law,

$$\therefore \oint_{PQP} H.dl = 0$$

So, $$\oint_{PQP} H.dl = \int_P^Q H.dl + \int_Q^P H.dl = 0$$

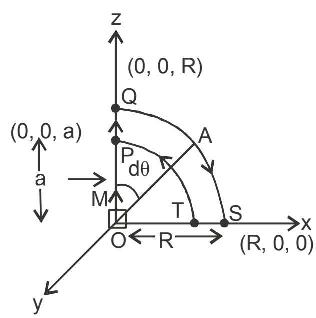
As $\int_Q^P H.dl < 0$ (inside the magnet)

So, $\int_P^Q H.dl > 0$ (outside the magnet)

Hence, it will be so angle between H and dl is more than 90°, so that cos θ is negative. It means the line of H must run from N-pole to S-pole inside the bar magnet.

Long Answer Questions

1. Consider the figure below according to the given situation.



In above figure from P to Q, every point on the z-axis lies at the axial line of magnetic dipole of moment M. Magnetic field induction (B) at a point (0, 0, z) distance z from the magnetic dipole of magnetic moment is

$$|B| = \frac{\mu_0}{4\pi}\frac{2|M|}{z^3} = \frac{\mu_0 M}{2\pi z^3}$$

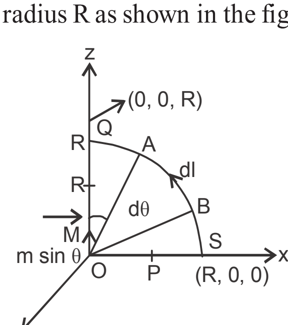
(i) By ampere's law, along z-axis from P to Q

$$\int_P^Q B.dl = \int_P^Q B.dl\cos 0° = \int_a^R B.dz$$

$$= \int_a^R \frac{\mu_0}{2\pi}\frac{M}{z^3}dz = \frac{\mu_0 M}{2\pi}\left(\frac{-1}{2}\right)\left(\frac{1}{R^2}-\frac{1}{a^2}\right)$$

$$= \frac{\mu_0 M}{4\pi}\left(\frac{1}{a^2}-\frac{1}{R^2}\right)$$

(ii) By ampere's law along the quarter circle QS of radius R as shown in the figure below:



The point A can be considered on the equatorial line of the magnetic dipole of moment Msinθ. So, magnetic field at point A on the circular arc is

$$B = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3};\ dl = Rd\theta$$

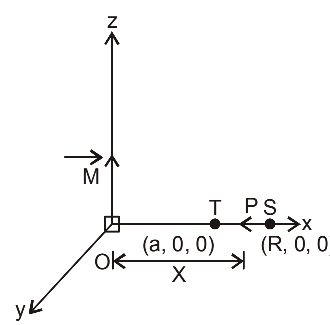
$$\therefore\quad d\theta = \frac{dl}{R}$$

By ampere's law,

$$\int B.dl = \int B.dl\cos\theta = \int_0^{\pi/2}\frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3}Rd\theta$$

$$\text{Circular arc} = \frac{\mu_0}{4\pi}\frac{M}{R}(-\cos\theta)_0^{\pi/2} = \frac{\mu_0}{4\pi}\frac{M}{R^2}$$

(iii) By ampere's law along x-axis from (X = R to X = a) over the path ST, consider the figure here given below:



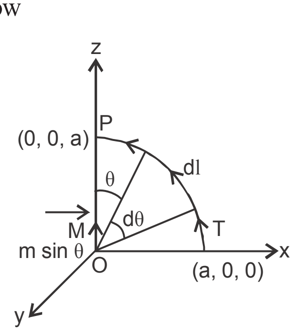
In above figure, every point lies on the equatorial line of magnetic dipole. Magnetic field induction at a point ((P) at a) distance x from the dipole is

$$B = \frac{\mu_0}{4\pi}\frac{M}{x^3}$$

If angle between (–M) and dl is 90°.

$$\text{So,}\quad \int_S^T B.dl = \int_R^a -\frac{\mu_0 M}{4\pi x^3}.dl = 0$$

(iv) By ampere's law along the quarter circule TP of radius a. Shown in the figure given below



From case (ii), we have the line integral of B along the quarter circle TP of radius a is circular arc TP

$$\int B.dl = \int_{\pi/2}^{0} \frac{\mu_0}{4\pi}\frac{M\sin\theta}{a^3} a d\theta = \frac{\mu_0}{4\pi}\frac{M}{a^2}\int_{\pi/2}^{0}\sin\theta d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{a^2}[-\cos\theta]_{\pi/2}^{0} = \frac{\mu_0 M}{4\pi a^2}[-1+0]$$

$$= \frac{-\mu_0}{4\pi}\frac{M}{a^2}$$

$$\therefore \oint_{PQST} B.dl = \int_P^Q B.dl + \int_Q^S B.dl + \int_S^T B.dl + \int_T^P B.dl$$

$$= \frac{\mu_0 M}{4}\left[\frac{1}{a^2} - \frac{1}{R^2}\right] + \frac{\mu_0}{4\pi}\frac{M}{R^2} + 0 + \left(-\frac{\mu_0}{4\pi}\frac{M}{a^2}\right) = 0$$

Hence, $\oint_{PQST} B.dl = 0$

2. Magnetic susceptibility is a measure of how a magnetic material responds to an external field,

So, $\chi_m = \frac{\text{(Intensity of magnetisation)}}{\text{(Magnetising force)}}$

As M and H both have same units and dimensions. Thus, χ has no dimensions. Here, χ is to be related with e, m, v, R and μ_0.

As we know that, the dimensions of μ_0 is $= [MLQ^{-2}]$

By Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

So, $\mu_0 = \frac{4\pi r^2 dB}{Idl\sin\theta} = \frac{4\pi r^2}{Idl\sin\theta} \times \frac{F}{qv\sin\theta}$

$$\left[\because dB = \frac{F}{qv\sin\theta}\right],$$

So, dimensions of μ_0

$$= \frac{L^2 \times (MLT^{-2})}{(QT^{-1})(L)\times 1 \times (Q)(LT^{-1})\times(1)} = [MLQ^{-2}]$$

$\therefore F = MLT^{-2}, I = QT^{-1}, V = LT^{-1}, \theta = 90°$

where Q is the dimension of charge.

χ depends on magnetic moment induced when H is turned on and M and H both have same unit.

So, χ is dimensionless, it should have no involvement of charge Q in its dimensional formula. It will be so if μ_0 and e together should have the value $\mu_0 e^2$, as e has the dimensions of charge.

Consider, dimension of $\chi = \mu_0 e^2 m^a v^b R^c$ ... (i)

where a, b, c are the power of m, v and R respectively, such that relation (i) is satisfied.

Dimensional equation of (i) is

$$[M^0L^0T^0Q^0] = [MLQ^{-2}]\times[Q^2][M^a]\times(LT^{-1})^b\times[L]^c$$
$$= [M^{1+a} + L^{1+b+c}T^{-b}Q^0]$$

Equating the powers of M, L and T, we have,

$0 = 1 + a$

So, $\boxed{a = -1}$

$0 = -b$

So, $\boxed{b = 0}$ $0 = 1 + 0 + c$

$0 = 1 + b + c$

So, $c = -1$

By putting values of a, b, c in Eq. (i), we get

$$\chi = \mu_0 e^2 m^{-1} v^2 R^{-1}$$

$$\chi = \frac{\mu_0 e^2}{mR} \quad \text{...(ii)}$$

As given that; $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$,
$e = 1.6 \times 10^{-19}$ C,
$m = 9.1 \times 10^{-31}$ kg,
$R = 10^{-10}$ m

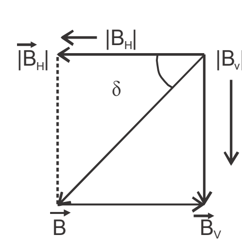
So, $\chi = \frac{(4\pi\times10^{-7})\times(1.6\times10^{-19})^2}{(9.1\times10^{-31})\times10^{-10}} = 10^{-4}$

Hence, $\frac{\chi}{\chi_{\text{(given solid)}}} = \frac{10^{-4}}{10^{-5}} = 10$

3. **(a)** As given that,

$$B_V = \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3} \quad \text{...(i)}$$

$$B_H = \frac{\mu_0}{4\pi}\frac{\sin\theta m}{r^3} \quad \text{...(ii)}$$



Squaring both the equations and adding, we get

$$B^2 = B_V^2 + B_H^2$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{m^2}{r^6}[4\cos^2\theta + \sin^2\theta]$$

$$B = \sqrt{B_V^2 + B_H^2} = \frac{\mu_0}{4\pi}\frac{m}{r^3}[3\cos^2\theta + 1]^{1/2} \quad ...(iii)$$

From Eq. (iii), the value of B is minimum,

if $\cos\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$. So, the magnetic equator is the locus.

(b) For angle of dip δ is

$$\tan\delta = \frac{B_V}{B_H} = \frac{\frac{\mu_0}{4\pi}\cdot\frac{2m\cos\theta}{r^3}}{\frac{\mu_0}{4\pi}\cdot\frac{\sin\theta . m}{r^3}}$$

$$= 2\cot\theta \quad ..... (iv)$$

$$\tan\delta = 2\cot\theta$$

For dip angle is zero, so, that $\delta = 0$,

$$\cot\theta = 0, \theta = \frac{\pi}{2}$$

So locus is magnetic equator.

(c) $$\tan\delta = \frac{B_V}{B_H}$$

If, $(\delta = \pm 45°)$

or, $\frac{B_V}{B_H} = \tan(\pm 45°)$, $\frac{B_V}{B_H} = \tan 45°$

$$\frac{B_V}{B_H} = 1, \boxed{B_V = B_H}$$

$$2\cot\theta = 1 \quad \text{From Eq. (iv)}$$

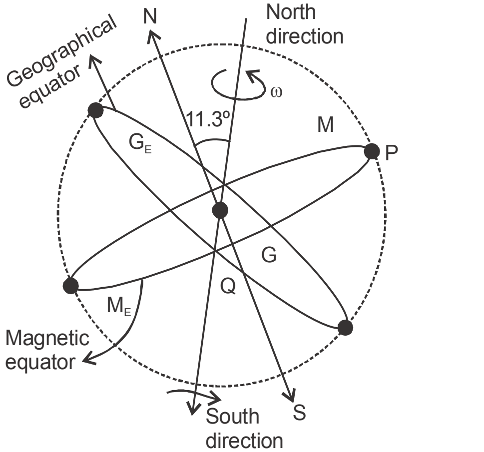
$$\cot\theta = \frac{1}{2}$$

$$\tan\theta = 2$$

So, $\theta = \tan^{-1}(2)$

Hence, $\theta = \tan^{-1}(2)$ is the locus

4. The declination is zero as P is in the plane S, needle is in north,

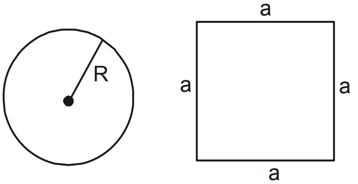


P is lies on the magnetic equator, so the angle of dip = 0, because the value of angle of dip at equator is zero. Q is also on the magnetic equator, thus the angle of dip is zero. As angle between axis of rotation of earth tilted on its magnetic axis by 11.3°, so the declination will be 11.3° between P and Q.

5. As given that C_1 is circular coil of radius R, length L, number of turns per unit length

$$n_1 = \frac{L}{2\pi R}$$

C_2 is square of side a and perimeter L, number of turns per unit length $n_2 = \frac{L}{4a}$



So magnetic moment of C_1

$$m_1 = n_1 I A_1 = \frac{L.I.\pi R^2}{2\pi R}$$

And magnetic moment of C_2

$$m_2 = n_2 I A_2 = \frac{L}{4a}.I.a^2$$

$$m_1 = \frac{LIR}{2} \quad ... (i)$$

$$m_2 = \frac{LIa}{4} \quad \text{...(ii)}$$

Moment of inertia of C_1

$$I_1 = \frac{MR^2}{2} \quad \text{...(iii)}$$

Moment of inertia of C_2

$$\Rightarrow \quad I_2 = \frac{Ma^2}{12} \quad \text{...(iv)}$$

As the frequencies for both coil are given by

$$f_1 = f_2$$

$$\left(\frac{2\pi}{T_1}\right) = \left(\frac{2\pi}{T_2}\right), (T_1 = T_2) \quad \text{... (v)}$$

So, time period of C_1

$$\Rightarrow T_1 = 2\pi\sqrt{\frac{I_1}{m_1 B}}$$

And time period of C_2

$$\Rightarrow \quad T_2 = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

From (v): $T_1 = T_2$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\frac{I_1}{m_1} = \frac{I_2}{m_2}$$

or

$$\frac{m_2}{m_1} = \frac{I_2}{I_1} \quad \text{...(vi)}$$

Putting the values by Eqs. (i), (ii), (iii) and (iv) in (vi)

So,

$$\frac{LIa.2}{4 \times LIR} = \frac{Ma^2.2}{12.MR^2}$$

$$\frac{a}{2R} = \frac{a^2}{6R^2}$$

$$a = 3R$$

Chapter Six

ELECTROMAGNETIC INDUCTION

EXERCISES

6.1 Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).

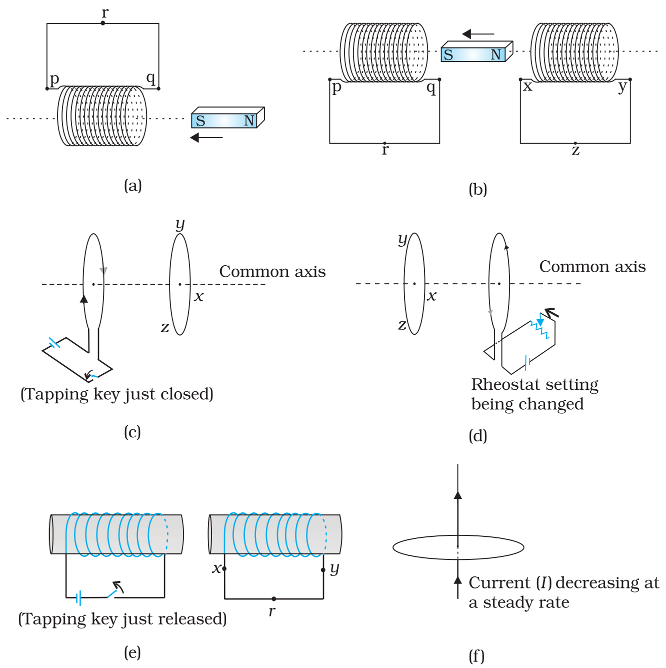


FIGURE 6.18

6.2 Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19:
(a) A wire of irregular shape turning into a circular shape;
(b) A circular loop being deformed into a narrow straight wire.

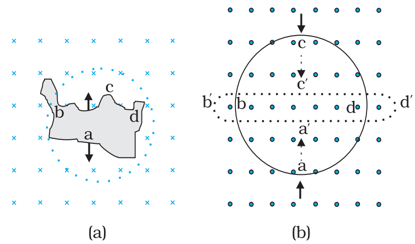


FIGURE 6.19

6.3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

6.4 A rectangular wire loop of sides 8 cm ad 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

6.5 A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

6.6 A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

6.7 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.
(a) What is the instantaneous value of the emf induced in the wire?
(b) What is the direction of the emf?
(c) Which end of the wire is at the higher electrical potential?

6.8 Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

6.9 A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

6.10 A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°.

ADDITIONAL EXERCISES

6.11 Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?

6.12 A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s^{-1} in the positive *x*-direction in an environment containing a magnetic field in the positive *z*-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative *x*-direction (that is it increases by 10^{-3} T cm^{-1} as one moves in the negative *x*-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 mΩ.

6.13 It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

6.14 Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.

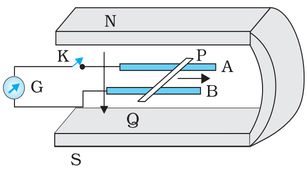


FIGURE 6.20

(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is *no net force* on the electrons in the rod PQ even though they do

experience magnetic force due to the motion of the rod. Explain.
(d) What is the retarding force on the rod when K is closed?
(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 cm s^{-1}) when K is closed? How much power is required when K is open?
(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?
(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

6.15 An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

6.16 (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.
(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s. Calculate the induced emf in the loop at the instant when x = 0.2 m. Take a = 0.1 m and assume that the loop has a large resistance.

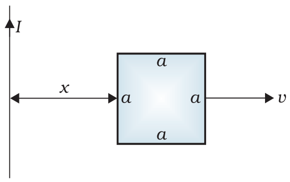


FIGURE 6.21

6.17 A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,

$\mathbf{B} = -B_0\,\mathbf{k}$ ($r \le a$; $a < R$)

$= 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?

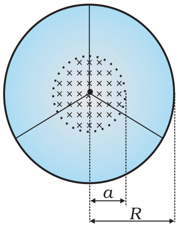


FIGURE 6.22

Chapter 6:- Electromagnetic Induction

Multiple Choice Questions (MCQs)

1. A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $B = B_0\,(2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is

(a) $2B_0L^2$Wb (b) $3B_0L^2$Wb
(c) $4B_0L^2$Wb (d) $\sqrt{29}B_0L^2$Wb

2. A loop, made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C(L, L, 0), D (0, L, 0), E(0, L, L) and F (0, 0, L). A magnetic field $B = B_0\left(\hat{i} + \hat{k}\right)$ T is present in the region. The flux passing through the loop ABCDEFA (in that order) is

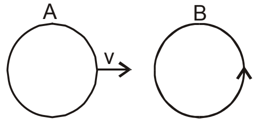
(a) B_0L^2 Wb (b) $2B_0L^2$Wb
(c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$Wb

3. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then,

(a) a direct current flows in the ammeter A
(b) no current flows through the ammeter A
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(d) a time varying non-sinusoidal current flows through the ammeter A.

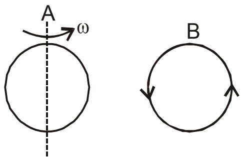
4. There are two coils A and B as shown in figure a current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that

(a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter clockwise direction in A



5. Same as problem 4 except the coil A is made to rotate about a vertical axis (figure). No current flows in B if A is at rest. The current in coil A, when the current in B (at t = 0) is counter-clockwise and the coil A is as shown at this instant, t = 0, is

(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter clockwise
(d) constant current counter clockwise



6. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) both l and A decrease

Multiple Choice Questions (MCQs) (More than one option correct)

1. A metal plate is getting heated. It can be because

(a) a direct current is passing through the plate
(b) it is placed in a time varying magnetic field
(c) it is placed in a space varying magnetic field, but does not vary with time
(d) a current (either direct or alternating) is passing through the plate

2. An emf is produced in a coil, which is not connected to an external voltage source. This can be due to

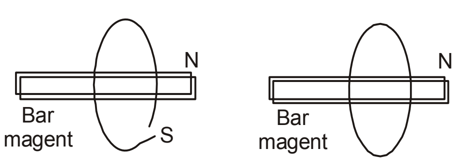
(a) the coil being in a time varying magnetic field
(b) the coil moving in a time varying magnetic field
(c) the coil moving in a constant magnetic field
(d) the coil is stationary in external spatially varying magnetic field, which does not change with time

3. The mutual inductance M_{12} of coil 1 with respect to coil 2
(a) increases when they are brought nearer
(b) depends on the current passing through the coils
(c) increases when one of them is rotated about an axis
(d) is the same as M_{21} of coil 2 with respect to coil 1

4. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because
(a) the magnetic field is constant
(b) the magnetic field is in the same plane as the circular coil and it may or may not vary
(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably
(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction

Very Short Answer Questions

1. Consider a magnet surrounded by a wire with an on/off switch S (figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit? Explain.

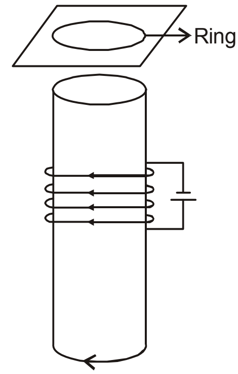


Circuit open Circuit closed

2. A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

3. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

4. Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.



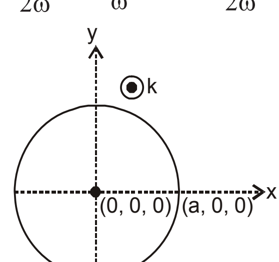
5. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see figure of Question 14). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?

6. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

Short Answer Questions

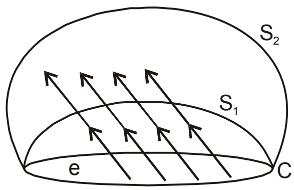
1. A magnetic field in a certain region is $B = B_0 \cos(\omega t)\hat{k}$ and a coil of radius a with resistance R is placed in the x-y plane with its centre at the origin in the magnetic field (figure). Find the magnitude and the direction of the current at (a, 0, 0) at

$$t = \frac{\pi}{2\omega},\ t = \frac{\pi}{\omega} \text{ and } t = \frac{3\pi}{2\omega}$$

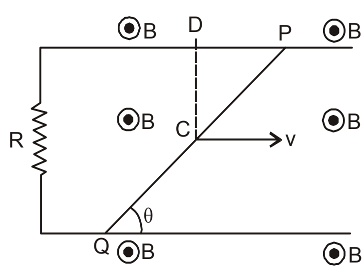


2. Consider a closed loop C in a magnetic field (figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula ϕ

$= B_1\ dA_1,\ B_2\ dA_2$ .... . Now, if we choose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.

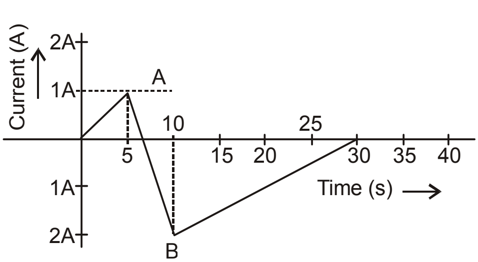


3. Find the current in the wire for the configuration shown in figure. Wire PQ has negligible resistance. B, the magnetic field is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by a distance d.



It is independent of q.

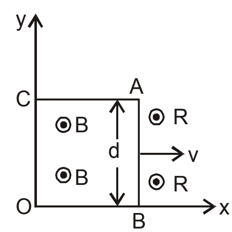
4. A (current versus time) graph of the current passing through a solenoid is shown in figure. For which time is the back electromotive force (u) a maximum. If the back emf at t = 3s is e, find the back emf at t = 7 s, 15s and 40 s. OA, AB and BC are straight line segments.



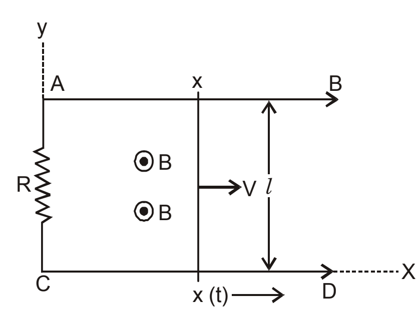
5. There are two coils A and B separated by some distance. If a current of 2A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1A passes through B, what is the flux through A?

Long Answer Questions

1. A magnetic field $B = B_0 \sin((\omega t)\hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (figure). The wires are in the x-y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit. What is the force needed to keep the wire moving at constant velocity?



2. A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $B = B(t)\hat{k}$.



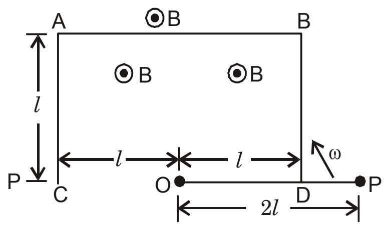
(i) Write down equation for the acceleration of the wire XY.

(ii) If B is independent of time, obtain v(t), assuming $v(0) = u_0$

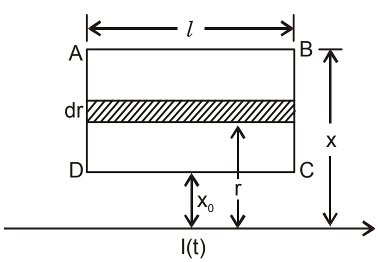
(iii) For (ii), show that the decrease in kinetic energy of XY equals the heat lost in.

3. ODBAC is a fixed rectangular conductor of negligible resistance (CO is not connected) and OP is a conductor which rotates clockwise with an angular velocity ω (figure). The entire system is in a uniform magnetic field B whose direction

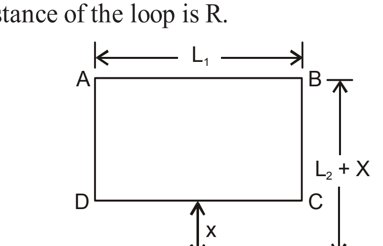
is along the normal to the surface of the rectangular conductor ABDC. The conductor OP is in electric contact with ABDC. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor, as it rotates by 180°.



4. Consider an infinitely long wire carrying a current I(t), with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in the rectangular loop of wire ABCD if its resistance is R (figure).

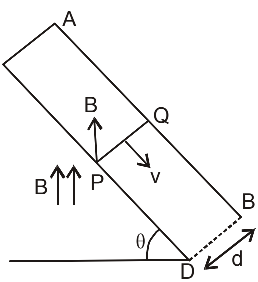


5. A rectangular loop of wire ABCD is kept close to an infinitely long wire carrying a current $I(t) = I_0 (1 - t/T)$ for $0 \le t \le T$ and $I(0) = 0$ for $t > T$ (figure.). Find the total charge passing through a given point in the loop, in time T. The resistance of the loop is R.

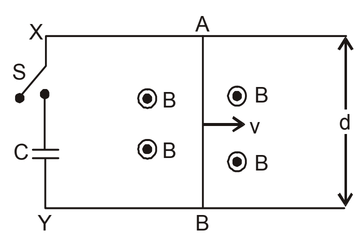


6. A magnetic field B is confined to a region $r \le a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge = Q) of radius b, $b > a$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

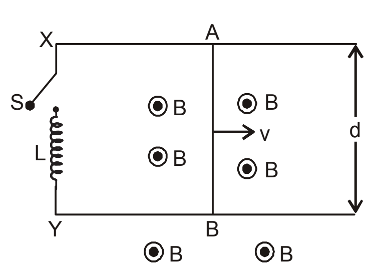
7. A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle 0 with respect to the horizontal (figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.



8. Find the current in the sliding rod AB (resistance = R) for the arrangement shown in figure. B is constant and is out of the paper. Parallel wires have no resistance, v is constant. Switch S is closed at time t = 0.



9. Find the current in the sliding rod AB (resistance = R) for the arrangement shown in figure. B is constant and is out of the paper. Parallel wires have no resistance, v is constant. Switch S is closed at time t = 0.



10. A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of magnetic field is $B_z = B_0(1+\lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g.

11. A long solenoid S has n turns per meter, with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b (where b < a). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as a function of $mt^2 + C$.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(c)** As we know that, the magnetic flux linked with uniform surface of area A in uniform magnetic field is $\phi = B.A$

The direction of A is perpendicular to the plane of square and square line in x-y plane in a region.

$$A = L^2k$$

As given that, $B = B_0(2\hat{i}+3\hat{j}+4\hat{k})$

So, $\phi = B.A = B_0(2\hat{i}+3\hat{j}+4\hat{k}).L^2\hat{k}$

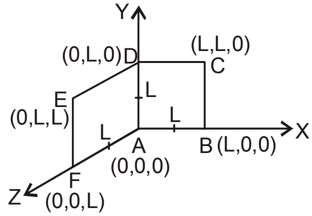
$= 4B_0L^2\,Wb$

2. **(b)** The loop can be considered in two planes, Plane of ABCDA lies x-y plane whose area vector

$A_1 = |A|\hat{k}$, $A_1 = L^2\hat{k}$

whereas plane of ADEFA lies in y-z plane whose area vector $A_2 = |A|\hat{i}$, $A_2 = L^2\hat{i}$.

Then the magnetic flux linked with uniform surface of area A in uniform magnetic field is



$\phi = B.A$

$A = A_1 + A_2 = (L^2\hat{k}+L^2\hat{i})$

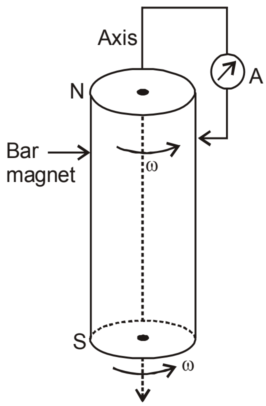
and $B = B_0(\hat{i}+\hat{k})$

Now, $\phi = B.A = B_0(\hat{i}+\hat{k})\cdot(L^2\hat{k}+L^2\hat{i})$

$= 2\,B_0L^2\,Wb$

3. **(b)** Induced current flow only when circuit is complete and there is a variation about circuit this problem is associated with the phenomenon of electromagnetic induction.

If there is a symmetry in magnetic field of cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence, no current flows in the ammeter (A).



4. **(d)** When the coil A stops moving the current in B b ecome ze ro, it possible only if the current in A is constant. If the current in A would be variable, there must be an induced emf (current) in B even if the A stops moving. So there is a constant current in same direction or counter clockwise direction in A as in B by lenz's law.

5. **(a)** By Lenz's law, at (t = 0) the current in B is counter-clockwise and the coil A is considered above to it. The counterclockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A.

When coil A start rotating at t = 0, the current in A is constant along clockwise direction by Lenz's rule. As flux changes across coil A by rotating it near the N-pole formed by flowing current in B, in anticlockwise.

6. **(b)** The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability is given by

$L = \mu_r \mu_0 n^2 Al$

$\therefore \quad n = N/l$

$$L = \mu_r \mu_0 \left[\frac{N^2 . A}{l.l}\right] . l$$

$$L = \mu_r \mu_0 [N^2A/l] \quad \left(L \propto A, L \propto \frac{1}{l}\right)$$

As μ_r and N are constant here so, to increase L for a coil, area A must be increased and l must be decreased.

Multiple Choice Questions (More Than One Options)

1. **(a, b, d)**

A plate or conductor is getting heated when a DC or AC current is passed due to heating effect of electric current ($H = I^2Rt$). Also, when metal plate is subjected to time varying magnetic field, the magnetic flux linked with the plate changes and eddy currents comes into existence which make the heating of plate take place.

2. **(a, b, c)**

emf produced in coil due to change in magnetic flux with time or magnetic flux linked with the isolated coil change when the coil being in a time varying magnetic field, the coil moving in a constant magnetic field or in time varying magnetic field

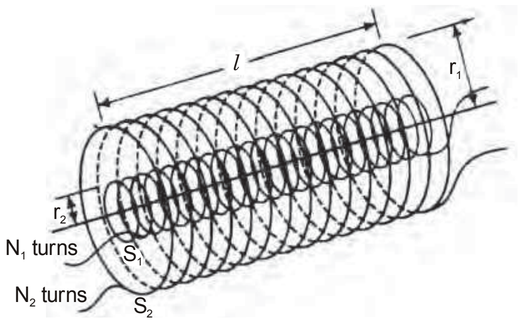
3. **(a, d)**

The mutual inductance M_{12} of coil increases when they are brought nearer and is the same as M_{12} of coil 2 with respect to coil 1.

(M_{21}), mutual inductance of solenoid S_1 with respect to solenoid S_2

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

where $r_1 l$ is the common area of cross-section of coil and common length (l) so M_{12} on passing current and rotation.



So, M_{12} i.e., mutual inductance of solenoid S_2 with respect to solenoid S_1 is

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

So, we get, $\quad M_{12} = M_{21} = M$

4. **(b, c)**

When circular coil expands in the region of magnetic field such that when it is in the same plane as the circular coil or the magnetic field has a perpendicular to the plane of the coil component whose magnitude is decreasing suitably so that the cross product of magnetic field and surface area of plane of coil remain constant at every instant.

Very Short Answer Questions

1. When the switch is thrown from the off position (open circuit) to the on position (closed circuit), then, neither B, nor A nor the angle between B and A change. There is no change in magnetic flux B linked with coil occur, so no electromotive force is produced and consequently no current will flow in the circuit.

2. When the coil is stretched, magnetic flux will leak through the gaps between successive elements of the spiral coil i.e., the wires are pulled apart. According to Lenz's law, the emf produced must oppose this decrease, which can be done by an increase in current. Hence current will increase.

3. If the ferromagnetic iron core is inserted in the current carrying solenoid, the magnetic field will increase due to the magnetisation of iron core and consequently the flux increases.

By Lenz's law, the emf produced must oppose, this increase in flux, which can be done by making decrease in current. So, the current in solenoid will decrease.

4. If the current is switched on, magnetic flux increased across the ring. According to Lenz's law, this increase in flux will be opposed and it can happen if the ring moves away from the solenoid.

This is due to the flux increases will cause a counter clockwise current i.e., opposite direction to that in the solenoid.

That makes the same sense of flow of current in the ring when viewed from the bottom of the ring and solenoid forming same magnetic pole infront of each other. So that, they will repel each other and the ring will move upward.

5. A current was flowing through the solenoid so it behave like a magnet and let S pole is upper side as flux in ring is constant, so no induced current in ring.

When the current is switched off, magnetic flux decrease through the ring. According to Lenz's law, this decrease in flux will be opposed and the ring experience downward force towards the solenoid.

This is due to the flux i decrease will cause a clockwise current, the same direction to that in the solenoid. That makes the opposite sense of flow of current in the ring 'when viewed from the bottom of the ring' and solenoid forming opposite magnetic pole infront of each other.

So that, they will attract each other but as ring is placed at the cardboard it could not be able to move downward.

6. Consider cylindrical bar magnet of radius 0.8 cm is dropped inside a hollow cylindrical metallic pipe with an inner radius of 1 cm, flux linked with the cylinder changes and eddy currents are produced in the metallic pipe. According to Lenz's law, these currents will oppose the motion of the magnet.

So, upward force acts on bar magnet, thus, magnet's downward acceleration will be less than the acceleration due to gravity g. Other wise, an unmagnetised iron bar will not produce eddy currents and will fall with an acceleration due to gravity g.

So, the magnet will take more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe.

Short Answer Questions

1. At any instant, flux passes through the ring is

$$\phi = B.A = BA\cos\theta$$

or, $\phi = B_0(\pi a^2)\cos\omega t$

By Faraday's law of electromagnetic induction., Magnitude of induced emf is

$$\varepsilon = B_0(\pi a^2)\omega\sin\omega t$$

This causes flow of induced current, is

$$I = \frac{\varepsilon}{R}$$

then, $I = B_0(\pi a^2)\,\omega\sin\omega t/R$

Now, finding the value of current at different instants, thus we get current.

At, $t = \frac{\pi}{2\omega}$, $I = \frac{B_0(\pi a^2)\omega}{R}$ (along $\hat{j}$)

Because $\sin\omega t = \sin\left(\omega\frac{\pi}{2\omega}\right) = \sin\frac{\pi}{2} = 1$

At, $t = \frac{\pi}{\omega}$, $I = 0$

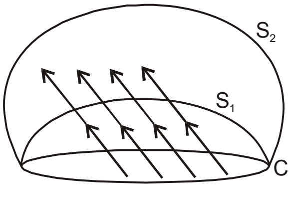
Here, $\sin\omega t = \sin\left(\omega\frac{\pi}{\omega}\right) = \sin\pi = 0,$

At, $t = \frac{3}{2}\frac{\pi}{\omega}$, $I = -\frac{B(\pi a^2)\omega}{R}$ (along $-\hat{j}$)

Because, $\sin\omega t = \sin\left(\omega\frac{3\pi}{2\omega}\right) = \sin\frac{3\pi}{2} = -1$

2. The magnetic flux linked with the surface can considered as the number of magnetic field lines passing through the surface. So, let $d\phi = \vec{B}\vec{A}$ represents magnetic field lines in an area A to magnetic flux B.

By the concept of continuity of lines B cannot end or start in space, so the number of lines passing through surface S_1 must be the same as the number of magnetic lines passing through the surface S_2. So, in both the cases we gets the same magnetic flux.



3. Considering the figure in which emf induced across PQ due to its motion or change in magnetic flux linked with the loop change due to change of enclosed area .

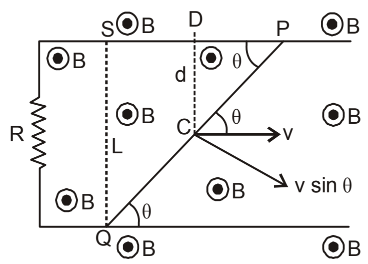
The motional electric field E due to the motion along CD is E = vB, the direction of E be perpendicular to both V and B, (F = q × B).

So, the motional emf along

PQ = (length PQ) × (field along PQ)

= (length PQ) × (vB sinθ)

$$\varepsilon = \left(\frac{d}{\sin\theta}\right)\times(vB\sin\theta) = vBd$$



The induced emf make flow of current in closed circuit of resistance R.

Induced current $I = \frac{\varepsilon}{R} = \frac{vBd}{R}$

It is independent of q.

4. The back electromotive force in solenoid is (u) will be maximum. If there is maximum rate of change of current. This happens in AB part of the graph. Thus maximum back emf will be obtained between 5s < t < 10s.

So, the back emf at t = 3s is e.

Also, the rate of change of current at t = 3, s = slope of OA from t = 0s to t = 5s = 1/5A/s.

So, back electromotive force

$$F = L\times\frac{1}{5} = \frac{L}{5} = e$$

If $t = 3s, \frac{dI}{dt} = 1/5$ (L is a constant).

Applying $e = L\frac{dI}{dt}$

Similarly, we get back emf u_1 between 5 to 10 sec.

For 5s < t < 10s

$$u_1 = -L\frac{3}{5} = -\frac{3}{5}L = -3e$$

So, at t = 7s, $u_1 = -3e$

Back emf between 10 to 30 sec.

For 10s < t < 30s

$$u_2 = L\frac{2}{20} = \frac{L}{10} = \frac{1}{2}e$$

For t > 30s, $u_2 = 0$

Hence, the back emf at t = 7s, 15 s and 40 s are – 3e, e/2 and 0 respectively.

5. Let A current I_1 is passing through the coil A having mutual inductance M_{21} with respect to coil B is,

$$N_2\phi_2 = M_{21}I_1$$

where, M_{21} is called the mutual inductance of coil A with respect to coil B and $M_{21} = M_{12}$ and M_{12} is called the mutual inductance of coil B with respect to coil A.

Total flux through B,

$$M_{21} = \frac{N_2\phi_2}{I_1}$$

∴ Magnetic flux $(N_2\phi_2) = 10^{-2}$ ∴ $I_i = 2$ Amp

So, we get

$$\text{Mutual inductance} = \frac{10^{-2}}{2} = 5\,\text{mH}$$

Now, total flux through A is

$N_1\phi_1 = M_{12}I_2$ = 5mH × 1A = 5 mWb

(∴ $M_{21} = M_{12} = 5$ mH)

Long Answer Questions

1. Consider the parallel wires at y = 0. i.e., along x-axis and y = d.

At t = 0, AB has x = 0, i.e., along y-axis and moves with a velocity v. Let at time t, wire is at $x(t) = vt\hat{i}$,

So, the motional emf across AB is

$$\varepsilon_1 = (B_0\sin\omega t)vd(-\hat{j}) = -(B_0\sin\omega t)vd$$

emf due to change in field (along OBAC)

$$\varepsilon_2 = -B_0\omega\cos\omega t\, x(t)d$$

Total emf in the circuit = emf due to change in field (along OBAC) + the motional emf across AB = –[$B_0\omega\cos\omega t(x(t)d + (B_0\sin\omega t)vd$]

$\varepsilon = -B_0 d[\omega x\cos(\omega t) + v\sin(\omega t)]$

Electric current in clockwise direction is

$$I = \frac{\varepsilon}{R} = \frac{B_0 d}{R}(\omega x\cos\omega t + v\sin\omega t)$$

The force acting on the conductor is

$F = il B\sin 90° = ilB$

Substituting the values, we get

Then force needed along i

$$= \frac{B_0 d}{R}(\omega x\cos\omega t + v\sin\omega t) \times d \times B_0\sin\omega t$$

$$F = \frac{B_0^2 d^2}{R}(\omega x\cos\omega t + v\sin\omega t)\sin\omega t$$

2. Consider the parallel wires along x-axis at y = 0 and y = l, at t = 0, so XY has x = 0 i.e., along y-axis.

(i) Let the wire XY be at x = x (t) at time t = t, and at t = 0 is at x = 0,

The magnetic flux linked with the loop is

$\phi = B.A = BA\cos 0 = BA$

Magnetic flux at any instant t time is

$\phi(t) = B(t)\,[l \times x(t)]$

Total emf in the circuit = emf due to change in field (along XYAC) + the motional emf across XY

$$E = \frac{-d\phi(t)}{dt} = \frac{-dB(t)}{dt}lx(t) - B(t)l\left(\frac{dx(t)}{dt}\right)$$

$$E = -\frac{d\phi(t)}{dt} = -\frac{dB(t)}{dt}lx(t) - B(t)lv(t)$$

By Fleming right hand rule :

Electric current in clockwise direction is

$$I = \frac{E}{R}\alpha$$

The force acting on the conductor is

$F = ilB\sin 90° = ilB(t)$

By putting all the values in above equation, we get

$$\text{Force} = \frac{IB(t)}{R}\left[-\frac{dB(t)}{dt}Ix(t) - B(t)Iv(t)\right]\hat{i}$$

From Newton's second law of motion,

$$\text{So, } m\frac{d^2x}{dt^2} = -\frac{I^2B(t)}{R}\frac{dB}{dt}x(t) - \frac{I^2B^2(t)}{R}\frac{dx}{dt} \quad ..... (i)$$

which is the required equation.

(ii) If B is independent of time i.e., B = Constant (B does not change with time)

$$\therefore \quad \frac{dB}{dt} = 0$$

$B(t) = B$ and $v(t) = v$

Substituting the above value in Eq (i), we get,

$$\frac{d^2x}{dt^2} = \frac{-I^2}{mR}[B + Bv]$$

$$\frac{dv}{dt} + \frac{I^2B^2}{mR}v = 0$$

Integrating using variable separable form of differential equation, we have

$$v = A\exp\left(\frac{-I^2B^2t}{mR}\right)$$

Now, from given conditions,

at t = 0, $v = u_0$

$v(t) = u_0\exp(-I^2B^2t/mR)$

Which is the required equation,

(iii) As we know that, the power consumption is

$P = I^2R$

$$\text{Here, } I^2R = \frac{B^2I^2v^2(t)}{R^2} \times R \qquad \left[\therefore I = \frac{BIVt}{R}\right]$$

$$\text{Power loss} = \frac{B^2I^2}{R}u_0^2\exp(-2I^2B^2t/mR)$$

So, energy consumed in time interval dt is

$= Pdt = I^2Rdt$

Then, the total energy consumed in time t

$$= \int_0^t I^2Rdt$$

$$= \frac{B^2I^2}{R}u_0^2\frac{mR}{2I^2B^2}\left[1 - e^{-2(I^2B^2t/mr)}\right]$$

$$= \frac{mu_0^2}{2}\left[1 - e^{(-2I^2B^2t/mr)}\right]$$

So, total energy consumed in time (t) is

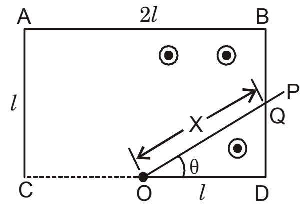
= (initial K.E.) – (Final K.E.)

$$= \left[\frac{m}{2}u_0^2 - \frac{m}{2}v^2(t)\right]$$

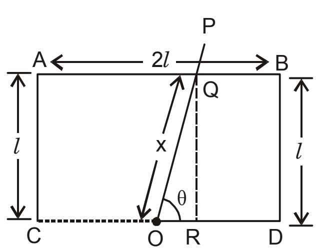
3. Induced emf produced by the pattern of rate of change of area can be considered uniform, it lies between $0 < \theta < \frac{\pi}{4}; \frac{\pi}{4} < \theta < \frac{3\pi}{4}$ and $\frac{3\pi}{4} < \theta < \frac{\pi}{2}$

(i) Consider the position of rotating conductor at time interval and making angle θ lies between 0 to 45°.

$$t = 0 \text{ to } t = \frac{\pi}{4\omega}(\text{or } T/8)$$



(i)



(ii)

So, the rod OP will make contact with the side BD as shown in figure (i). Consider the length OQ of the contact at any instant t such that $0 < t < \frac{\pi}{4\omega}$ or $0 < t < \frac{T}{8}$ be x. The flux through the area ODQ is

Magnetic flux in ΔODQ is

$$\phi = B.A$$

$$\left[\because \frac{QD}{l} = \tan\theta, QD = l\tan\theta\right]$$

The direction of B and A are 0° or 180°.

$$\phi = B\frac{1}{2}QD \times OD = B\frac{1}{2}l\tan\theta \times l$$

$$\phi = \frac{1}{2}Bi^2\tan\theta, \ (\because \theta = \omega t)$$

$$= \frac{1}{2}Bl^2\tan\omega t$$

By applying Faraday's law of EMI,

So, the magnitude of the emf

$$\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\sec^2\omega t$$

And, the current is $I = \frac{\varepsilon}{R}$

where R is the resistance of the rod in contact.

$$I = \frac{Bl^2}{2R}\omega\sec^2\omega t$$

where $R \propto \lambda$

$R = \lambda x$

From figure (i),

$$\cos\theta = \frac{l}{x}; x = \frac{l}{\cos\theta}, (\theta = \omega t)$$

$$\boxed{x = \frac{l}{\cos\omega t}}$$

So, $R = \frac{\lambda l}{\cos\omega t}$

And, $I = \frac{1}{2}\frac{Bl^2\omega}{\lambda l}\sec^2\omega t\cos\omega t$

$$= \frac{Bl\omega}{2\lambda\cos\omega t}$$

(ii) Consider the length OQ of the contact at some time t such that $\frac{\pi}{4\omega} < t < \frac{3\pi}{4\omega}$ or $\frac{T}{8} < t < \frac{3T}{8}$ be x. The rod is in contact with the side AB. The flux through the area OQBD is

$$\phi = B.A$$

$$\text{Area of } \Delta ORQ = \frac{1}{2}yl$$

$$\tan\theta = \frac{l}{y}$$

$$y = \frac{l}{\tan\theta}$$

$$\text{Area} = \frac{1}{2}\frac{l^2}{\tan\theta}$$

$$\phi = \left(l^2 + \frac{1}{2}\frac{l^2}{\tan\theta}\right)B$$

Where, $\theta = \omega t$

So, the magnitude of emf generated in the loop is

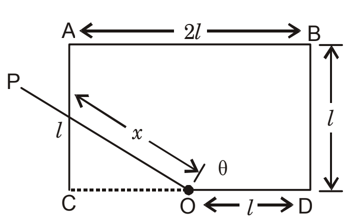
$$\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\frac{\sec^2\omega t}{\tan^2\omega t}$$

The current is $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x}$

$$= \frac{\varepsilon\sin\omega t}{\lambda l} = \frac{1}{2}\frac{Bl\omega}{\lambda\sin\omega t}$$

(iii) Similarly for $\frac{3\pi}{4\omega} < t < \frac{\pi}{\omega}$ or $\frac{3T}{8} < t < \frac{T}{2}$,

the rod will be in touch with AC.



The flux through OQABD is

$$\phi = B\left(2l^2 + \frac{1}{2}ly\right)$$

$$\tan(180-\theta) = \frac{l}{y} \Rightarrow y = \frac{-l}{\tan\theta}$$

$$\text{So, Area} = \frac{-1}{2}\frac{l^2}{\tan\theta} \qquad (\because \theta = \omega t)$$

The flux through OQABD is

$$\phi = \left(2l^2 - \frac{l^2}{2\tan\omega t}\right)B$$

Then the magnitude of emf generated in loop is

$$\varepsilon = \frac{d\phi}{dt} = \frac{B\omega l^2\sec^2\omega t}{2\tan^2\omega t}$$

$$l = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{1}{2}\frac{Bl\omega}{\lambda\sin\omega t}$$

Which are the required expressions.

4. Consider a strip of length l and width dr at a distance r from the surface of infinite long current carrying wire. The magnetic field at strip due to current carrying wire is

$$\text{Field } B(r) = \frac{\mu_0 I}{2\pi r}$$

Where, B(r) is perpendicular to the paper.

So, total flux through the strip is

$$\phi = \frac{\mu_0 I}{2\pi}l\int_{x_0}^{x}\frac{dr}{r} = \frac{\mu_0 I}{2\pi}\ln\left[\frac{x}{x_0}\right] \ldots (i)$$

The induced emf induced can be obtained by differentiating eq. (i) and then applying Ohm's law

$$\varepsilon = \frac{d\phi}{dt}, \Rightarrow IR = \frac{d\phi}{dt} \qquad (\because \frac{\varepsilon}{R} = I)$$

We have, induced current

$$I = \frac{1}{R}\frac{d\phi}{dt} = \frac{1}{R}\frac{d}{dt}\left[\frac{\mu_0 Il}{2\pi}\ln\left(\frac{x}{x_0}\right)\right]$$

$$= \frac{\mu_0 l}{2\pi R}\ln\left(\frac{x}{x_0}\right)\cdot\left[\frac{dI}{dt}\right]$$

$$I = \frac{\mu_0 l}{2\pi}\frac{\lambda}{R}\ln\frac{x}{x_0} \quad \left[\because \frac{dI}{dt} = \lambda \text{ (given)}\right]$$

5. The emf induced can be obtained by differentiating the expression of magnetic flux then

Applying Ohm's law, If I is instantaneous current

$$I = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt}$$

$\because$ $$I(t) = \frac{dQ}{dt}$$

then $$\frac{dQ}{dt} = \frac{1}{R}\frac{d\phi}{dt}$$

or $$dQ = \frac{1}{R}d\phi$$

Integrating the variable separable form of differential equation for finding the charge Q that passed in time t, we get

$$\int_{Q_2}^{Q_1} dQ = \frac{1}{R}\int \phi \, d\phi$$

$$Q(t_1) - Q(t_2) = \frac{1}{R}\left[\phi(t_1) - \phi(t_2)\right]$$

x' veries from x to $(x + L_2)$ so total flux in strip.

$$\left(\therefore Q(t) = \frac{\mu_0 I(t) L_1}{2\pi R}\right)$$

$$\phi(t_1) = L_1 \frac{\mu_0}{2\pi}\int_x^{L_2+x}\frac{dx'}{x'} I(t_1)$$

$$\phi(t_1) = \frac{\mu_0 L_1}{2\pi} I(t_1) \ln\frac{(L_2 + x)}{x}$$

The magnitude of charge,

$$\int_0^Q dQ = \frac{1}{R}\frac{\mu_0 L_1}{2\pi}\ln\left[\frac{L_2 + x}{x}\right]$$

$$= \frac{\mu_0 L_1}{R 2\pi}\ln\frac{L_2 + x}{x}(I_0 - 0) = \frac{\mu_0 L_1}{2\pi R} I_0 \ln\left(\frac{L_2 + x}{x}\right)$$

This is the total charge or required expression.

6. As the magnetic field is reduce to zero in time Δt. So, the magnetic flux linked with the ring also reduces from maximum to zero. That induces an emf in conducting ring by the change of magnetic flux. The induces emf causes the electric field E generation across the ring.

The induced emf in metallic ring = electric field $E \times (2\pi b)$ ... (i)

Applying Faraday's law of EMI, so

The induced emf = rate of change of magnetic flux = rate of change of magnetic field × area

$$= \frac{B\pi a^2}{\Delta t} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$2\pi b E = \text{emf} = \frac{B\pi a^2}{\Delta t}$$

So, the charged ring experienced an electric force = QE

This electric force try to rotate the coil, then the torque is

Torque = b × Force

($\because$ b is the perpendicular distance)

$$= QEb = Q\left[\frac{B\pi a^2}{2\pi b \Delta t}\right]b$$

$$\text{Torque on ring} = Q\frac{Ba^2}{2\Delta t}$$

If ΔL is the change in angular momentum

$$\Delta L = \text{Torque} \times \Delta t = Q\frac{Ba^2}{2\Delta t} \times \Delta t$$

$$\Rightarrow \quad \Delta L = \frac{QBa^2}{2}$$

As, initial angular momentum of ring is zero.

So,

Torque × Δt = Change in angular momentum

Final angular momentum

$$= mb^2\omega = \frac{QBa^2}{2}$$

$$\omega = \frac{QBa^2}{2mb^2}$$

This is the required expression of angular speed.

7. From free body diagram the component of magnetic field (B) perpendicular the plan (PQ) = $B\cos\theta$

Now, the conductor moves with speed v perpendicular to $B\cos\theta$ component of magnetic field.

So, $d\phi = \vec{B}\cdot d\vec{A}$

$$d\phi = (B\cos\theta)(d \times v \times dt)$$

$$\left(\frac{d\phi}{dt} = vBd\cos\theta\right)$$

This causes motional emf across two ends of rod, which is given by = $v(B\cos\theta)d$

$$\left(\therefore \frac{d\phi}{dt} = iR\right)$$

So that it makes flow of induced current

$$i = \frac{v(B\cos\theta)d}{R}$$

where, R is the resistance of rod. Now, current carrying rod experience force F = iBd, (horizontally in backward direction). So, the component of magnetic force parallel to incline plane along upward direction

$$= F\cos\theta = iBd\cos\theta$$

$$F_m = \left(\frac{v(B\cos\theta)d}{R}\right)Bd\cos\theta$$

Also, the component of weight (mg) parallel to incline plane along downward direction = $mg\sin\theta$.

From Newton's second law of motion

$$m\frac{d^2x}{dt^2} = mg\sin\theta - F_m$$

$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{vB\cos\theta d}{R}\times(Bd)\cos\theta$$

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2 v$$

$$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2 v = g\sin\theta$$

Which is the linear differential equation.

After solving the equation we have,

$$v = \frac{g\sin\theta}{\frac{B^2d^2\cos^2\theta}{mR}} + A\exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right) \quad ...(i)$$

Where, A is a constant to be determine by initial conditions,

at, $t = 0, \quad v = 0$

We have $0 = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - Ae^0$

$$\Rightarrow \quad A = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}$$

By putting the value of A in equation (i),

Then the required expression of velocity as a function of time is

$$v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left(1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right)$$

Let us consider a new constant (α) is

$$= \left(\frac{mR}{B^2d^2\cos^2\theta}\right)$$

So, $v = \alpha g\sin\theta\,(1 - e^{-t/\alpha})$

8. Consider the given figure in which the conductor of length d moves with speed v, perpendicular to magnetic field B as shown in figure. This produces motional emf across two ends of rod, which is ($\varepsilon = vBd$). Since, switch S is closed at time t = 0. capacitor is charged by this potential difference. Let Q(t) is charge on the capacitor and current flows from A to B.

So, the net induced current when the capacitor oppose the charge flow is

$$I = I_i - I_c,$$

$$I = \frac{vBd}{R} - \frac{Q}{RC} \quad \left(\therefore I_i = \frac{Bvd}{R}, I_c = \frac{Q}{RC}\right)$$

So, we get

$$\frac{Q}{RC} + \frac{dQ}{dt} = \frac{vBd}{R}$$

Then the solution of linear differential equation is

$$Q=\frac{\left(\frac{vBd}{R}\right)}{\frac{1}{RC}}+Ae^{-t/RC}$$

$$Q=vBdC+Ae^{-t/RC}$$

Initially, at time $t=0$, $Q=0$, so $A=-vBdC$

By putting the value of A in differential equation(i),

$$Q=vBdC\,[1-e^{-t/RC}]$$

Differentiating both side w.r.t. (t), we get

$$\frac{dQ}{dt}=\frac{vBdCe^{-t/RC}}{RC}$$

$$I=\frac{vBd}{R}e^{-t/RC}$$

Which is the required expression of current.

9. Consider the conductor (AB) of length d moves with speed v. perpendicular to magnetic field B as shown in figure. So, angle between B and v is 90° this produces motional emf across two ends of rod, which is = vBd, when switch S is closed at time $t=0$ current start growing inductor by the potential difference due to motional emf.

From Kirchhoff's law, we get

$$-L\frac{dI}{dt}+vBd=IR$$

or, $$L\frac{dI}{dt}+IR=vBd$$

That is the linear differential equation

So, $$I=\frac{vBd}{R}+Ae^{-Rt/2}$$

(At $t=0$ $I=0$),

$$0=\frac{vBd}{R}+A,\qquad (\because e^{\circ}=1)$$

So, $$A=-\frac{vBd}{R}$$

$$I=\frac{vBd}{R}(1-e^{-Rt/2}).$$

That is the required expression of current.

10. According to the question, the magnetic flux across the metallic ring of mass 'm' and radius 'l' falling under gravity in a region having a magnetic field whose z-component of magnetic field is $B_z=B_0(1+\lambda z)$ is

$$\phi=B_z.A$$

$$\phi=B_z(\pi l^2)$$

$$\phi=B_0(1+\lambda z)(\pi l^2) \qquad \text{..... (i)}$$

By applying Faraday's law of EMI, we get induced emf is $\varepsilon=\frac{d\phi}{dt}$ = (rate of change of flux)

And the angle between $\vec{B}$ and $\vec{A}$ is 0°

So, $$\varepsilon=\frac{d}{dt}\left[B_0(1+\lambda z)(\pi l^2)\right]$$

$[\therefore$ From eq. (i) $\phi=[B_0(1+\lambda Z)](\pi l^2)$

By Ohm's law,

$$IR=(B_0\pi l^2).\left[0+\lambda\frac{dz}{dt}\right]$$

So, $$B_0(\pi l^2)\lambda\frac{dz}{dt}=IR$$

By rearranging the terms

$$I=\frac{\pi l^2 B_0\lambda}{R}v$$

Energy lost/second = I^2R

$$=\frac{(\pi l^2\lambda)^2B_0^2v^2}{R}$$

The energy must come from rate of change in PE, so

$$PE=mg\frac{dz}{dt}=mgv$$

As the kinetic energy is constant for v = constant

So, $$mgv = \frac{\left(\pi l^2 \lambda B_0\right)^2 v^2}{R}$$

or, $$v = \frac{mgR}{\left(\pi l^2 \lambda B_0\right)^2}$$

That is the required expression of velocity.

11. According to the question, when verying current is passed through solenoid the verying magnetic field can induce the current in another smaller coil.

So, magnetic field due to a solenoid S_1 is

$$B = \mu_0 n l$$

Magnetic flux in smaller coil

$$\phi = NBA$$

where, $A = \pi b^2$

By applying Faraday's law of EMI, the induced emf in coil due to solenoids verying magnetic field.

$$e = \frac{-d\phi}{dt} \qquad (\because \phi = NBA)$$

So, $$e = \frac{-d}{dt}(NBA) \qquad (\therefore B = \mu_0 Ni)$$

$$e = -N\pi b^2 \mu_0 n \frac{dl}{dt} = -Nn\pi\mu_0 b^2 \frac{d}{dt}\left(mt^2 + C\right)$$

$$= -\mu Nn\pi b^2 \, 2mt$$

Thus, current varies as a function of $(mt^2 + C)$.

So, net emf produced in N turns of smaller coil

$$e = -\mu_0 Nn\pi b^2 \, 2mt$$

EXERCISES

7.1 A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.
(a) What is the rms value of current in the circuit?
(b) What is the net power consumed over a full cycle?

7.2 (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?
(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

7.3 A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

7.4 A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

7.5 In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

7.6 Obtain the resonant frequency ω_r of a series *LCR* circuit with $L = 2.0$H, $C = 32$ μF and $R = 10$ Ω. What is the *Q*-value of this circuit?

7.7 A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

7.8 Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

7.9 A series *LCR* circuit with $R = 20$ Ω, $L = 1.5$ H and $C = 35$ μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

7.10 A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its *LC* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[*Hint:* For tuning, the natural frequency i.e., the frequency of free oscillations of the *LC* circuit should be equal to the frequency of the radiowave.]

7.11 Figure 7.21 shows a series *LCR* circuit connected to a variable frequency 230 V source. $L = 5.0$ H, $C = 80$μF, $R = 40$ Ω.

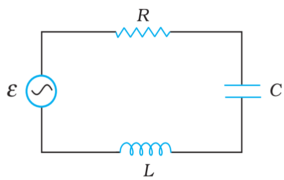


FIGURE 7.21

(a) Determine the source frequency which drives the circuit in resonance.
(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the *LC* combination is zero at the resonating frequency.

ADDITIONAL EXERCISES

7.12 An *LC* circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.

(a) What is the total energy stored initially? Is it conserved during *LC* oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored
(i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

7.13 A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

7.14 Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

7.15 A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

7.16 Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

7.17 Keeping the source frequency equal to the resonating frequency of the series *LCR* circuit, if the three elements, *L*, *C* and *R* are arranged in parallel, show that the total current in the parallel *LCR* circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

7.18 A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

7.19 Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

7.20 A series *LCR* circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\ \Omega$ is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the *Q*-factor of the given circuit?

7.21 Obtain the resonant frequency and *Q*-factor of a series *LCR* circuit with $L = 3.0$ H, $C = 27\ \mu$F, and $R = 7.4\ \Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

7.22 Answer the following questions:

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across *C* and the ac signal across *L*.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why ca we not use an ordinary resistor instead of the choke coil?

7.23 A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

7.24 At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 m^3s^{-1}. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8$ ms^{-2}).

7.25 A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

7.26 Do the same exercise as above with the replacement of the earlier transformer by a 40,000-220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Chapter 7:- Alternating Currents

Multiple Choice Questions (MCQs)

1. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is
   (a) $5\sqrt{2}$A (b) $5\sqrt{3/2}$ A
   (c) 5/6 A (d) $5/\sqrt{2}$A

2. An alternating current generator has an internal reactance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to
   (a) zero (b) X_g
   (c) $-X_g$ (d) R_g

3. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means
   (a) input voltage cannot be AC voltage, but a DC voltage
   (b) maximum input voltage is 220 V
   (c) the meter reads not v but $< v^2 >$ and is calibrated to read $\sqrt{<v^2>}$
   (d) The pointer of the meter is stuck by some mechanical defect

4. To reduce the resonant frequency in an L-C-R series circuit with a generator
   (a) the generator frequency should be reduced
   (b) another capacitor should be added in parallel to the first
   (c) the iron core of the inductor should be removed
   (d) dielectric in the capacitor should be removed

5. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication?
   (a) R = 20 Ω, L = 1.5 H, C = 35μF
   (b) R = 25Ω, L = 2.5 H, C = 45 μF
   (c) R = 15Ω, L = 3.5 H, C = 30 μF
   (d) R = 25Ω, L = 1.5 H, C = 45 μF

6. An inductor of reactance 1Ω and a resistor of 2Ω are connected in series to the terminals of a 6V (rms) AC source. The power dissipated in the circuit is
   (a) 8 W (b) 12 W
   (c) 14.4 W (d) 18 W

7. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is

(a) $1/\sqrt{2}$ A (b) $\sqrt{2}$ A

(c) 2 A (d) $2\sqrt{2}$ A

Multiple Choice Questions (MCQs) (More than one option correct)

1. As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?
   (a) Inductor and capacitor
   (b) Resistor and inductor
   (c) Resistor and capacitor
   (d) Resistor, inductor and capacitor

2. In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?
   (a) Only resistor
   (b) Resistor and an inductor
   (c) Resistor and a capacitor
   (d) Only a capacitor

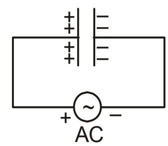
3. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?
   (a) For a given power level, there is a lower current
   (b) Lower current implies less power loss
   (c) Transmission lines can be made thinner
   (d) It is easy to reduce the voltage at the receiving end using step-down transformers

4. For a L-C-R circuit, the power transferred from the driving source to the driven oscillator is

$$P = I^2 Z \cos\phi$$

   (a) Here, the power factor $\cos\phi \geq 0$, $P \geq 0$
   (b) The driving force can give no energy to the oscillator (P = 0) in some cases
   (c) The driving force cannot syphon out (P < 0) the energy out of oscillator
   (d) The driving force can take away energy out of the oscillator

5. When an AC voltage of 220 V is applied to the capacitor C
   (a) the maximum voltage between plates is 220 V
   (b) the current is in phase with the applied voltage
   (c) the charge on the plates is in phase with the applied voltage
   (d) power delivered to the capacitor is zero

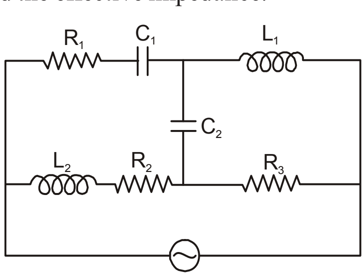


6. The line that draws power supply to your house from street has
   (a) zero average current
   (b) 220 V average voltage
   (c) voltage and current out of phase by 90°
   (d) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$

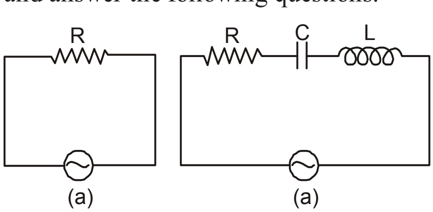
Very Short Answer Questions

1. If a L-C circuit is considered analogous to a harmonically oscillating springblock system, which energy of the L-C circuit would be analogous to potential energy and which one analogous to kinetic energy?

2. Draw the effective equivalent circuit of the circuit shown in figure, at very high frequencies and find the effective impedance.



3. Study the circuits (a) and (b) shown in figure and answer the following questions.

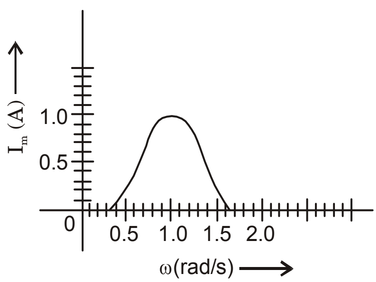


(a) Under which conditions would the rms currents in the two circuits be the same?

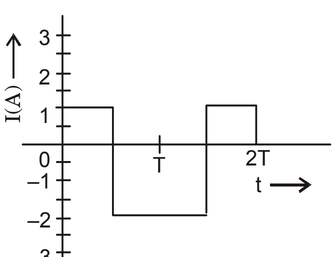
(b) Can the rms current in circuit (b) be larger than that in (a)?

4. Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative?

5. In series LCR circuit, the plot of I_{max} versus ω is shown in figure. Find the bandwidth and mark in the figure.



6. The alternating current in a circuit is described by the graph shown in figure. Show rms current in this graph.



7. How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an L-C-R series circuit, change as the supply frequency is gradually increased from very low to very high values.

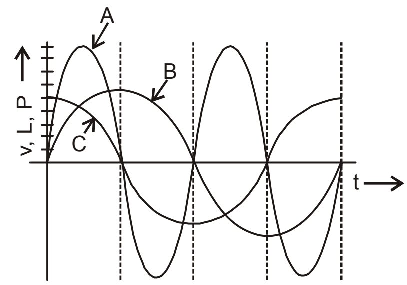
Short Answer Questions

1. A device 'X' is connected to an AC source. The variation of voltage, current and power in one complete cycle is shown in figure.

(a) Which curve shows power consumption over a full cycle?

(b) What is the average power consumption over a cycle?

(c) Identify the device X.



2. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

3. A coil of 0.01H inductance and 1Ω resistance is connected to 200 V, 50Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current.

4. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of tansformer being used.

5. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.

6. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

Long Answer Questions

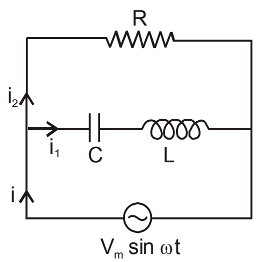
1. An electrical device draws 2 kW power from AC mains (voltage 223 V (rms) = $\sqrt{50000}$ V). The current differs (lags) in phase by $\phi\left(\tan\phi = \frac{-3}{4}\right)$ as compared to voltage. Find (a) R, (b) $X_C - X_L$ and (c) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?

2. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if

(i) power is transmitted at 220V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage to 11000V, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{cu} = 1.7 \times 10^{-8}$ SI unit)

3. Consider the L-C-R circuit shown in figure. Find the net current i and the phase of i. Show that $i = \frac{V}{Z}$. Find the impedance Z for this circuit.



4. For a L-C-R circuit driven at frequency ω, the equation reads

$$L\frac{di}{dt} + Ri + \frac{q}{C} = V_i = V_m \sin \omega t$$

(a) Multiply the equation by i and simplify where possible.

(b) Interpret each term physically.

(c) Cast the equation in the form of a conservation of energy statement.

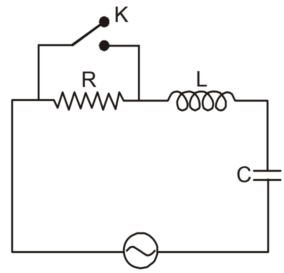
(d) Intergrate the equation over one cycle to find that the phase difference between V and i must be acute.

5. In the L-C-R circuit, shown in figure the AC driving voltage is $V = V_m \sin \omega t$.

(a) Write down the equation of motion for q(t).

(b) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.

(c) Describe subsequent motion of charges.



NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(b)** As given that, $\nu = 50$ Hz, $I_{rms} = 5A$

$$t = \frac{1}{300} s$$

As we know that $I_{rms} = \frac{I_0}{\sqrt{2}}$

I_0 = Peak value $= \sqrt{2}.I_{rms} = \sqrt{2} \times 5$

$I_0 = 5\sqrt{2}A$

$$\text{at, } t = \frac{1}{300}\text{sec}, \; I = I_0 \sin\omega t = 5\sqrt{2}\sin 2\pi\nu t$$

$$= 5\sqrt{2}\sin 2\pi \times 50 \times \frac{1}{300}$$

$$I = 5\sqrt{2}\sin\frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{3/2} \text{ Amp}$$

$$\left(\therefore \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right)$$

$$I = \left(5\sqrt{\frac{3}{2}}\right) \text{Amp}$$

2. (c) To deliver maximum power from the generator to the load, total internal reactance must be equal to conjugate of total external reactance.

So, $X_{int} = X_{ext}$

$X_g = (X_L) = -X_L$

Hence, $X_L = -X_g$

(Reactance in external circuit)

3. **(c)** As we know that,

The voltmeter in AC reads rms values of voltage

$$I_{rms} = \sqrt{2}I_0 \text{ and } V_{rms} = \sqrt{2}v_0$$

The voltmeter in AC circuit connected to AC mains reads mean value ($\langle v^2 \rangle$) and is calibrated in such a way that it gives rms value of $\langle v^2 \rangle$, which is multiplied by form factor √2 to give rms value V_{rms}.

4. **(b)** As we know that,

The resonant frequency in an L-C-R series circuit is

$$v_0 = \frac{1}{2\pi\sqrt{LC}}$$

So, to reduce ν_0 either increase L or increase C. To increase capacitance, another capacitor must be connect in parallel with the first capacitor.

5. **(c)** As we know that, Quality factor (Q) of an L-C-R circuit must be higher so Q is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is resistance, L is inductance and C is capacitance of the circuit.

So, for higher Q, L must be large, and C and R should be low.

Hence, option (c) is verify.

6. **(c)** As given that,

$X_L = 1\Omega, R = 2\Omega, E_{rms} = 6V, P_{av} = ?$

The average power dissipated in the L, R, series circuit with AC source

Then $$P_{av} = E_{rms} I_{rms} \cos\phi \quad ...(i)$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_{rms}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{4+1} = \sqrt{5}$$

$$I_{rms} = \frac{6}{\sqrt{5}} A$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

By putting the value of I_{rms}, E_{rms}, $\cos\phi$ in equation (i), then,

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{72}{\sqrt{5}\sqrt{5}}$$

$$= \frac{72}{5} = 14.4 \text{ watt}$$

7. **(a)** As given that,

Secondary voltage (V_S) is :

$$V_S = 24 \text{ Volt}$$

Power associated with secondary is :

$$P_S = 12 \text{ Watt}$$

As we know that $P_S = V_S I_S$

$$I_S = \frac{P_S}{V_S} = \frac{12}{24} = \frac{1}{2} A = 0.5 \text{ Amp}$$

Peak value of the current in the secondary

$$I_0 = I_S\sqrt{2} = 0.5\sqrt{2}$$

$$= \frac{5}{10}\sqrt{2} \left[I_0 = \frac{1}{\sqrt{2}} \text{ Amp} \right]$$

Multiple Choice Questions
(More Than One Options)

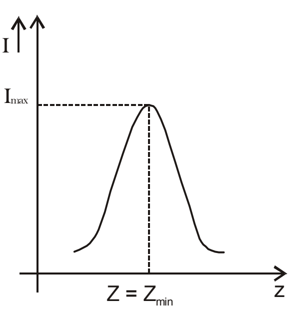
1. **(a, d)**

In a circuit, current will be maximum when reactance is minimum.

Reactance of an inductor of inductance L is,

$$X_L = 2\pi\nu L$$

where ν is frequency of the AC circuit.



X_c = Reactance of the capacitive circuit

$$= \frac{1}{2\pi f C}$$

On increasing frequency ν, clearly X_L increases and X_C decreases

For a L-C-R circuit,

Z = Impedance of the circuit

$$= \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

As frequency (ν) increases, Z decreases and at certain value of frequency know as resonant frequency (ν_0), impedance Z is minimum that is Z_{min} = R current varies inversely with impedance and at Z_{min} current is maximum.

2. **(c, d)**

As per question on increasing frequency, the current increases.

So, the reactance of the circuit must be decreases as increasing frequency.

For a capacitive circuit, when f is frequency of the AC circuits.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Clearly when frequency increases, X_C decreases, to increase current capacitors

For R-C circuit, $X = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

So, option (c) and (d) are verify.

3. **(a, b, d)**

Power loss due to transmission lines having resistance (R) and rms current I_{rms} is

$$\text{Power loss} = I_{rms}^2 R$$

So to decrease power loss I_{rms} and R must be lowers for a constant power supply and energy transmit over large distances at high alternating voltages, so current flowing through the wires will be low because for a given power (P).

So, $P = E_{rms} I_{rms}$, I_{rms} is low when E_{rms} is high.

$$\text{Power loss} = I^2_{rms} R = \text{low} \quad (\because I_{rms} \text{ is low})$$

Now, at the receiving end high voltage is reduced by using step-down transformers.

For step-up transformer,

Output power =Input power

$$V_S I_S = V_P I_P \quad (\because V_S > U_P \text{ so } I_P \geq I_S)$$

So, loss of power during transmission become lower.

4. **(a, b, c)**

As given that :

$$P = I^2 Z \cos\phi \quad \text{...(i)}$$

where I is the current, Z = Impedance, cos ϕ power factor

As we know that,

$$P = I^2 R \quad \text{...(ii)}$$

Compare (i) & (ii) equation,

So, (power factor)

$$(\cos\phi = \frac{R}{Z})$$

where $R > 0$ and $Z > 0$

So, $\cos\phi = \frac{R}{Z}$ is +ve,

$$\cos\phi > 0$$

$$P > 0$$

5. **(c, d)**

When the AC voltage (220 V) is applied to the capacitor.

The plate connected to positive terminal with positive charge will be at higher potential and the plate connected to negative terminal with negative charge will be at lower potential. So, that the charge is in phase with the applied voltage.

Power applied lo a circuit is

$$P_{av} = V_{rms} I_{rms} \cos\phi$$

For pure capacitive circuit

If $\phi = 90°$

then, $\cos\phi = 0$

So, Power delivered $P_{av} = 0$

6. **(a, d)**

In house, we are using AC supply, so AC currents are used which are having zero average value over a cycle. In houshald circuit L and C are connected, so R, Z cannot be equal.

So, the line is having some resistance so power factor $\cos\phi = \frac{R}{Z} \neq 0$

So, $\phi \neq \frac{\pi}{2} \Rightarrow \phi < \frac{\pi}{2}$

Hence, phase angle of voltage & current lies between 0 and $\frac{\pi}{2}$.

Very Short Answer Questions

1. An L-C circuit analogous to a harmonically oscillating spring block system.

The electrostatic energy $\frac{1}{2}CV^2$ is analogous to potential energy due to charging of capacitor and energy associated with moving charges (current) that is magnetic energy $\left(\frac{1}{2}LI^2\right)$ is analogous to kinetic energy, due to the motion of charge partical.

2. As we know that, the inductive reactance

$$X_L = 2\pi fL$$

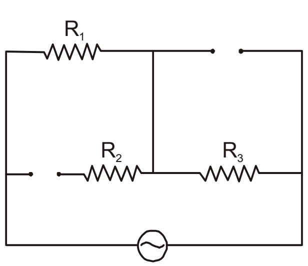
As the reactance is high when the frequency (f) and inductance L are high then the circuit is open.

And the capacitive reactance $X_C = \frac{1}{2\pi fL}$ at high frequency X_C will be low circuit is short or closed.

So, for very high frequencies ($f \to \infty$, $X_L \to \infty$ and $X_C \to 0$)

When reactance of a circuit is infinite it will be considered as open circuit. When reactance of a circuit is zero it will be considered as short circuited.

Thus, $C_1, C_2 \to$ shorted and $L_1, L_2 \to$ opened.



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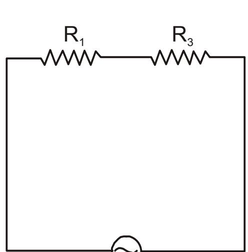


Hence, effective impedance $= R_{eq} = R_1 + R_3$

3. As we know that, the relation between V_{rms} and I_{rms} is

$$I_{rms} = \frac{V_{rms}}{R}$$

Consider, $(I_{rms})_a$ = rms current in circuit (a)

$(I_{rms})_b$ = rms current in circuit (b)

So, rms current in circuit (a) is

$$(I_{rms})_a = \frac{V_{rms}}{R} = \frac{V}{R}$$

And rms current in circuit (b) is

$$(I_{rms})_b = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(a) When $(I_{rms})_a = (I_{rms})_b$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

Squaring both side

$$R^2 = R^2 + (X_L - X_C)^2$$

or, $(X_L - X_C)^2 = 0$

So, $X_L = X_C$, resonance condition

(b) As $Z \geq R$,

$$\frac{(I_{rms})_a}{(I_{rms})_b} = \frac{\sqrt{R^2 + (X_L - X_C)^2}}{R}$$

$$\sqrt{R^2 + (X_L - X_C)^2} < R$$

Squaring both side

$$R^2 + (X_L - X_C)^2 < R^2$$

$$(X_L - X_C)^2 < 0$$

So, squaring of any value can not be negative.

So, $(I_{rms})_a \geq (I_{rms})_b$

Hence, the rms current in circuit (b), cannot be larger than that in circuit (a).

4. Consider the applied emf

$$E = E_0 \sin(\omega t)$$

current developed is

$$I = I_0 \sin(\omega t \pm \phi)$$

Instantaneous power output of the AC source

$$P = EI = (E_0 \sin \omega t)\ [I_0 \sin(\omega t \pm \phi)]$$

$$= E_0 I_0 \sin \omega t . \sin(\omega t + \phi)$$

On solving ,

So, $P = \frac{E_0 I_0}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$ ... (i)

Average power $P_{av} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$

$$= \frac{V_0 I_0}{2}\cos\phi$$

or, $= V_{rms} I_{rms}\cos\phi$ ... (ii)

where ϕ is the phase difference.

From Eq. (i)

when $\cos\phi < \cos(2\omega t + \phi)$ $(\therefore P_{av} < 0)$

So, the instantaneous power output of an AC source can be negative

From Eq. (ii), $P_{av} > 0$

$\therefore$ $\cos\phi = \frac{R}{Z} > 0$ (Always positive)

So, R, Z the reactance are always positive.

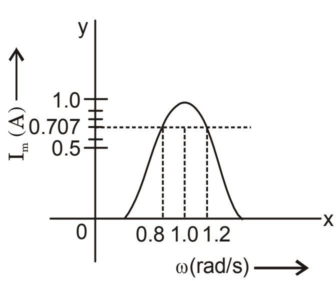
Thus, the average power output of an AC source cannot be negative.

Hence, P_{av} never be negative.

5. Let ω_1 and ω_2 are two frequencies at which magnitude of current is $\frac{1}{\sqrt{2}}$ times of maximum value for LCR circuit.

i.e., $I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0.7$ Amp

The value of current is maximum at resonant frequency.



From the above diagram ω_1 and ω_2 at 0.7 amp the corresponding frequencies are 0.8 rad/s and 1.2 rad/s. So,

For the given diagram,

Bandwidth $(\Delta\omega) = \omega_2 - \omega_1 = 1.2 - 0.8$

$= 0.4$ rad/sec

6. Consider the given graph,

As we know that,

The rms current is $\left[I_{rms} = \frac{I_0}{\sqrt{2}}\right]$

From the graph : $I_0 = \sqrt{I_1^2 + I_2^2} = \sqrt{1^2 + 2^2}$

So, $I_{rms} = \sqrt{\frac{1^2+2^2}{2}} = \sqrt{\frac{5}{2}} = 1.58$ A

Hence, the rms value of the current (I_{rms})

$\simeq 1.6$ Amp.

7. When the voltage leads the current in L-C-R series circuit so that the phase angle (ϕ) is : where $(X_L > X_C)$

So, $\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$

If ν is small so, $\left[2\pi\nu L - \frac{1}{2\pi\nu C}\right]$ is negative.

So, $\tan\phi < 0$ (for $v < v_0$)

And, $\tan\phi > 0$ (for $v > v_0$)

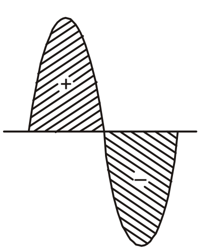
Short Answer Questions

1. **(a)** As we know that,

Power = P = VI

The curve of power will be having maximum amplitude, equals to multiplication of amplitudes of voltage (V) and current (I) curve. So, the curve will be represented by A. It shows the power consumption over a cycle, the power of a circuit will become zero, when V = 0 or I = 0 or both become zero, which is clear from graph 'A' is zero at this position.

(b) In the shown by shaded area in the diagram, the full cycle of the graph consists of one positive and one negative symmetrical area are equal. So net area of power graph is zero. So, power consumption in circuit is zero.



So, that average power of AC circuit over a cycle is zero.

(c) As the average power is zero, hence the device may be inductor (L) or capacitor (C) or the series combination of L and C. So power factor $\cos\phi = 0$, ($\phi = 90°$)

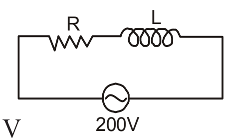
2. For a Direct Current (DC), one ampere' is equal to one coulomb charge flowing per sec.

$$1 \text{ ampere} = 1 \text{ coulomb/sec}$$

So, an AC current changes direction with the source frequency and the attractive force would average to zero. So that the AC ampere must be defined in terms of some property that is independent of the direction of current.

Joule's heating effect is such property and hence it is used to define rms value of AC. According to joule's heating effect, one ampere current in AC is the current which can produce same quantity of heat per second as the direct current can produce in one ohm resistance.

3. As given that,



inductance $L = 0.01$ H

resistance $R = 1\Omega$,

voltage (V) = 200 V

and frequency (f) = 50 Hz

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{1^2 + (2\times3.14\times50\times0.01)^2}$$

or $Z = \sqrt{10.86} = 3.3\,\Omega$

For phase angle (ϕ)

$$\tan\phi = \frac{Z}{R} = \frac{XL}{R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$

$$= \frac{2\times3.14\times50\times0.01}{1} = 3.14$$

$$\phi = \tan^{-1}(3.14) \approx 72°$$

Phase difference $\phi = \dfrac{72\times\pi}{180}$ radian

$$= 1.20 \text{ radian.}$$

Time lag between alternating voltage and current, $\phi = \omega t$

So, $\Delta t = \dfrac{\phi}{\omega} = \dfrac{72\pi}{180\times2\pi\times50} = \dfrac{1}{250}$ sec.

4. As given that,

$$P_S = 60\text{W}$$

$$I_S = 0.54\text{A}$$

Primary current $I_p = ?$

Taking line voltage as 220 V ($V_P = 220$ V)

$$P_S = V_S I_S$$

$$P_S = 60\text{W}, I_S = 0.54\text{A}$$

$$V_S = \frac{60}{0.54} = 110 \text{ Volt.}$$

Voltage in the secondary (V_S) is less than voltage in the primary (V_P).

So, the transformer is step down transformer.

Since, the transformation ratio factor is

$$r = \frac{\text{Secondary voltage}}{\text{Primary voltage}}$$

$$r = \frac{V_S}{V_P}$$

$\therefore$ For transformer $\dfrac{V_S}{V_P} = \dfrac{I_P}{I_S}$

Substituting the values,

$$\frac{110\text{V}}{220\text{V}} = \frac{I_P}{0.54\text{ A}}$$

$$\Rightarrow \quad I_P = 0.27 \text{ Ampere}$$

5. A capacitor does not allow a direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this charging and discharging by AC voltage or current.

So, a capacitor will pass more current through it if the voltage is changing at a faster rate, i.e. if the frequency of supply is higher. It implies that the reactance offered by a capacitor is less with increasing frequency.

So, the reactance of capacitor is $X_C = \dfrac{1}{\omega C}$

6. When current in an inductor change the direction of induced emf. The inductor opposes flow of current through it by developing a back emf according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and vice-versa.

Thus, the induced emf is proportional to the rate of change of current. So inductor provide greater reactance to the flow of current through it if the rate of change is faster, i.e., if the frequency is higher. So the reactance of an inductor, is directly proportional to the frequency. Mathematically, the reactance offered by the inductor is

$$X_L = (2\pi\nu)L = \omega L$$

Long Answer Questions

1. As given that, the power is

$$P = 2kW = 2000\ W$$

The voltage when current lags to the voltage

$$V = 223\ V,$$

$$\tan\phi = -\frac{3}{4}$$

$$I_M = I_0 = ?$$

$$X_C - X_L = ?$$

$$R = ?$$

As we know that,

$$\text{Power (P)} = \frac{V^2}{Z}$$

So, $$Z = \frac{V^2}{P} = \frac{223 \times 223}{2 \times 10^3} = 25$$

Impedance Z = 25 Ω

$$\text{Impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad 25 = \sqrt{R^2 + (X_L - X_C)^2}$$

or $$625 = R^2 + (X_L - X_C)^2 \quad \text{... (i)}$$

And, $$\tan\phi = \frac{X_L - X_C}{R} = \frac{3}{4}$$

or $$X_L - X_C = \frac{3R}{4} \quad \text{... (ii)}$$

By putting the value of $(X_L - X_C) = \frac{3R}{4}$ in Eq. (i), then,

$$625 = R^2 + \left(\frac{3R}{4}\right)^2 = R^2 + \frac{9R^2}{16}$$

or $$625 = \frac{25R^2}{16}$$

(a) Resistance $R = \sqrt{25 \times 16} = \sqrt{400}$
$= 20\ \Omega$

(b) $$X_L - X_C = \frac{3R}{4} = \frac{3}{4} \times 20 = 15\ \Omega$$

(c) Main current

$$I_M = \sqrt{2}I = \sqrt{2}\frac{V}{Z}$$

$$= \frac{223}{25} \times \sqrt{2} = 12.6\ A$$

So, R, X_C, X_L are all doubled, $\tan\phi = \left(\frac{X_C - X_L}{R}\right)$ does not change. Z will be doubled, then the current $\left(I = \frac{V}{Z}\right)$ become halved. As I become half (P = VI) power is also halved as voltage remain same.

2. **(i)** According to the question, the town is 10 km away, length of pair of copper wires used,

$$L = 20\ km = 20000\ m$$

Resistance of Cu wires,

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi (r)^2}$$

As given that, $\rho_{Cu} = 1.7 \times 10^{-8}; r = 0.5\ cm$

$$= \frac{1.7 \times 10^{-8} \times 20000}{3.14(0.5 \times 10^{-2})^2} = 4\Omega$$

As given that, $P = 1\ MW = 10^6$ watt
So, I at 220 V, $P = VI = 10^6$ W,

$$I = \frac{10^6}{220} = 0.45 \times 10^4\ A$$

Power loss $P = RI^2$
$= 4 \times (0.45)^2 \times 10^8$ W
$= 8.26 \times 10^7$ W

Power loss in heating
$= 82.6$ MW
as 82.6 MW > 10^6 W

So, this method cannot be used for transmission.

(ii) When power $P = 10^6$ W is transmitted at 11000 V.

$$V'I' = 10^6 W = 11000\, I'$$

Current drawn, $I' = \frac{1}{1.1} \times 10^2$

$\therefore \quad R_{Cu} = 4\,\Omega$

$$(P) \text{ Power loss} = RI^2 = \frac{1}{1.21} \times 4 \times 10^4$$

$$P = 3.3 \times 10^4 \text{ watt}$$

So, Fraction of power loss $= \frac{3.3 \times 10^4}{10^6} = 0.033$

Hence, (Power loss) = 3.3%

3. According to the given L-C-R circuit the total current i from the source. It is divided into two parts i_1 through R and i_2 through series combination of C and L.

So, $\quad i = i_1 + i_2$

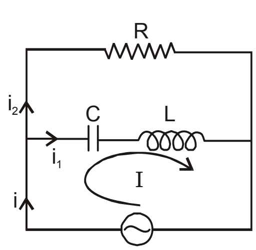
From given figure,

$$V_m \sin \omega t = Ri_1$$

$$i_1 = \frac{V_m \sin \omega t}{R} \quad \text{... (i)}$$

If q_2 is charge on the capacitor at any time t, then for series combination of C and L.

Let I current move in first half of loop shown in figure below :



$V_m \sin \omega t$

By applying KVL,

$$V_C + V_L = V_m \sin \omega t$$

$$\frac{q_2}{C} + \frac{L di_2}{dt} - V_m \sin \omega t = 0$$

then, $\frac{q_2}{C} + \frac{L d_2 q_2}{dt^2} = V_m \sin \omega t$

$$\left[\because i_2 = \frac{dq_2}{dt}\right] \quad \text{... (ii)}$$

Consider, $\quad q_2 = q_m \sin(\omega t + \phi) \quad$ ... (iii)

Differentiate both side w.r.t (t)

Then, $\quad i_2 = \frac{dq_2}{dt} = q_m \omega \cos(\omega t + \phi)$

Again differenciate both side w.r.t. (t)

Then, $\quad \frac{d^2 q_2}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi)$

By putting these values in Eq. (ii), so we have

$$q_m \left[\frac{1}{C} + L(-\omega^2)\right] \sin(\omega t + \phi) = V_m \sin \omega t$$

when $\phi = 0$ and $\left(\frac{1}{C} - L\omega^2\right) > 0$,

$$q_m \left(\frac{1}{C} - L\omega^2\right) \sin \omega t = V_m \sin \omega t$$

then $\quad q_m = \frac{V_m}{\left(\frac{1}{C} - L\omega^2\right)} \quad$ ... (iv)

From Eq. (iii), $\quad i_2 = \frac{dq_2}{dt}$

$$i_2 = \omega q_m \cos(\omega t + \phi) \quad \text{... (v)}$$

Now by putting q_m value from (iv) to equation (v),

$$i_2 = \frac{\omega V_m \cos(\omega t + \phi)}{\frac{1}{C} - L\omega^2}$$

Taking $\phi = 0$, $i_2 = \frac{V_m \cos(\omega t)}{\left(\frac{1}{\omega C} - L\omega\right)} \quad$ ... (vi)

From Eqs. (i) and (v), we find that i_1 and i_2 are out of phase by $\frac{\pi}{2}$.

Now, $\quad i_1 + i_2 = \frac{V_m \sin \omega t}{R} + \frac{V_m \cos \omega t}{\left(\frac{1}{\omega C} - L\omega\right)}$

Put $\quad \frac{V_m}{R} = A = C \cos \phi \quad$ ... (vii)

and $\quad \frac{V_m}{\left(\frac{1}{\omega C} - L\omega\right)} = B = C \sin \phi \quad$ ... (viii)

$$\therefore \quad i_1 + i_2 = C \cos \phi \sin \omega t + C \sin \phi \cos \omega t$$

$$i = C \sin(\omega t + \phi)$$

Squaring and adding (vii), (viii)

$$A^2 + B^2 = C^2(\cos^2\phi + \sin^2\phi)$$

then, $$C = \sqrt{A^2 + B^2}$$

and $$\phi = \tan^{-1}\left(\frac{B}{A}C\right)$$

$$= \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)}\right]^{1/2}$$

and $$\phi = \tan^{-1}\frac{R}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Hence, $$i = i_1 + i_2$$

$$= \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2} \sin(\omega t + \phi) \quad \text{...(ix)}$$

$$\phi = \tan^{-1}\left[\frac{R}{\frac{I}{\omega C} - \omega L}\right],$$

As we know that,

$$I = \frac{V}{R} \quad [\because I = i \quad R = z/\sin(\omega t + \phi)]$$

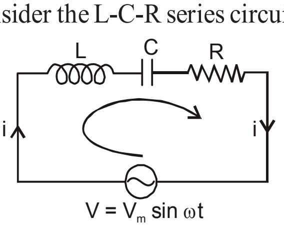
$$i = \frac{V}{Z}\sin(\omega t + \phi) \quad \text{...(x)}$$

Comparing (ix) & (x)

or $$\frac{i}{V_m} = \frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2}$$

Hence, that is the required impedance Z of the circuit.

4. Let us consider the L-C-R series circuit,



Applying KVL in the loop, then,

$$V_L + V_C + V_R = V_m \sin \omega t$$

So, $$L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t \quad \text{...(i)}$$

Multiplying both sides by i, in equation (i) then,

$$Li\frac{di}{dt} + \frac{q}{C}i + i^2R = (V_m i)\sin \omega t \quad \text{...(ii)}$$

where $$Li\frac{di}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$$

It represent the rate of change of energy stored in an inductor (L).

$P = Ri^2$ = represent the joule heating loss

$$\boxed{\frac{q}{C}i = \frac{d}{dt}\left(\frac{q^2}{2C}\right)}$$

It represent the rate of change of energy stored in the capacitor at dt time.

Vi represent rate at which driving force pours in energy. It goes into (i) ohmic loss and (ii) increase of stored energy.

Thus, Eq. (ii) is in the form of conservation of energy statement. Integrating both sides of Eq. (ii) with respect to time over one full cycle ($0 \to T$) so,

$$\int_0^T \frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{q^2}{2C}\right)dt + \int_0^T Ri^2 dt = \frac{1}{2}\int_0^T Vi\, dt$$

$$(\because V = V_m \sin \omega t)$$

$$\left[0 + \frac{1}{2}i^2RT\right] = \frac{1}{2}\int_0^T Vi\, dt$$

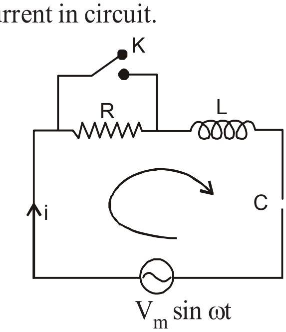
So, $$\left[0 + \frac{1}{2}(i^2RT)\right] = \frac{1}{2}\int_0^T Vi\, dt$$

$$(\because \text{ as } i^2RT \text{ is } +\text{ve})$$

$$0 + (+\text{ve}) = \int_0^T Vi\, dt$$

$\int_0^T Vi\, dt > 0$ is positive, which is possible if phase difference between V and i is a constant and the angle is acute.

5. **(a)** Consider the given figure of L-C-R circuit, when tapping key K to short circuit R and let i be the current in circuit.



As given that, $V = V_m \sin \omega t$

By applying KVL (when key K is often)

$$V_R + V_L + V_C = V_m \sin \omega t$$

$$iR + L\frac{di(t)}{dt} + \frac{q(t)}{C} - V_m \sin \omega t = 0 \quad \text{...(i)}$$

$$(\because i(t) = i = i_m \sin(\omega t + \phi))$$

$$i = \frac{dq(t)}{dt}$$

Differentiate (i) w.r.t. (t),

$$\frac{di(t)}{dt} = \frac{d^2q(t)}{dt^2}$$

From Eq. (i)

$$\frac{dq(t)}{dt}R + L\frac{d^2q(t)}{dt^2} + \frac{q(t)}{C} = V_m \sin \omega t$$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = V_m \sin \omega t$$

That is the required equation of variation (motion) of charge with respect to time.

(b) Let time dependent charge in circuit is at phase angle with voltage then,

$$q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$$

$$i = \frac{dq}{dt} = \omega q_m \sin(\omega t + \phi)$$

As we know that

$$i_m = \frac{V_m}{Z}$$

$$i_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

at $(t = t_0)$,

$$\therefore \quad i = i_m \sin(\omega t_0 + \phi)$$

$$\text{So,} \quad i_m = \frac{i}{\sin(\omega t_0 + \phi)}$$

$$(i) = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \sin(\omega t_0 + \phi)$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

The energy is stored in L and C, when R is short circuited at $t = t_0$.

$$U_L = \frac{1}{2}Li^2$$

$$U_L = \frac{1}{2}L\left[\frac{V_m}{\sqrt{(R^2 + X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

$$\text{and} \quad U_C = \frac{1}{2} \times \frac{q^2}{C}$$

$$= \frac{1}{2C}\left[q^2 m \cos^2(\omega t_0 + \phi)\right]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2$$

$$U_C = \frac{1}{2C} \times \left(\frac{i_m}{\omega}\right)^2 \cos^2(\omega t_0 + \phi)$$

$$= \frac{i^2}{2C\omega^2}\cos^2(\omega t_0 + \phi)$$

$$[\because i_m = q_m \omega \text{ or } q_m = \frac{i_m}{\omega}]$$

$$\text{So } U_C = \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{\cos^2(\omega t_0 + \phi)}{\omega^2}$$

$$U_C = \frac{1}{2C\omega^2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \cos^2(\omega t_0 + \phi)$$

(c) The circuit becomes an L-C oscillator when R is short circuited. The capacitor will go on discharging and all energy will transfer to L and back and forth. So, there is oscillation of energy from electrostatic to magnetic and vice-versa.

Chapter Eight

ELECTROMAGNETIC WAVES

SUMMARY

1. Maxwell found an inconsistency in the Ampere's law and suggested the existence of an additional current, called displacement current, to remove this inconsistency. This displacement current is due to time-varying electric field and is given by

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

and acts as a source of magnetic field in exactly the same way as conduction current.

2. An accelerating charge produces electromagnetic waves. An electric charge oscillating harmonically with frequency ν, produces electromagnetic waves of the same frequency ν. An electric dipole is a basic source of electromagnetic waves.

3. Electromagnetic waves with wavelength of the order of a few metres were first produced and detected in the laboratory by Hertz in 1887. He thus verified a basic prediction of Maxwell's equations.

4. Electric and magnetic fields oscillate sinusoidally in space and time in an electromagnetic wave. The oscillating electric and magnetic fields, **E** and **B** are perpendicular to each other, and to the direction of propagation of the electromagnetic wave. For a wave of frequency ν, wavelength λ, propagating along z-direction, we have

$$E = E_x(t) = E_0 \sin (kz - \omega t)$$

$$= E_0 \sin \left[2\pi\left(\frac{z}{\lambda} - \nu t\right)\right] = E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

$$B = B_y(t) = B_0 \sin (kz - \omega t)$$

$$= B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \nu t\right)\right] = B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

They are related by $E_0/B_0 = c$.

5. The speed c of electromagnetic wave in vacuum is related to μ_0 and ε_0 (the free space permeability and permittivity constants) as follows:

$c = 1/\sqrt{\mu_0 \varepsilon_0}$. The value of c equals the speed of light obtained from optical measurements.

Light is an electromagnetic wave; c is, therefore, also the speed of light. Electromagnetic waves other than light also have the same velocity c in free space.

The speed of light, or of electromagnetic waves in a material medium is given by $v = 1/\sqrt{\mu \varepsilon}$

where μ is the permeability of the medium and ε its permittivity.

6. Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields.

Electromagnetic waves transport momentum as well. When these waves strike a surface, a pressure is exerted on the surface. If total energy transferred to a surface in time t is U, total momentum delivered to this surface is $p = U/c$.

7. The spectrum of electromagnetic waves stretches, in principle, over an infinite range of wavelengths. Different regions are known by different

names; γ-rays, X-rays, ultraviolet rays, visible rays, infrared rays, microwaves and radio waves in order of increasing wavelength from 10^{-2} Å or 10^{-12} m to 10^{6} m.

They interact with matter via their electric and magnetic fields which set in oscillation charges present in all matter. The detailed interaction and so the mechanism of absorption, scattering, etc., depend on the wavelength of the electromagnetic wave, and the nature of the atoms and molecules in the medium.

POINTS TO PONDER

1. The basic difference between various types of electromagnetic waves lies in their wavelengths or frequencies since all of them travel through vacuum with the same speed. Consequently, the waves differ considerably in their mode of interaction with matter.
2. Accelerated charged particles radiate electromagnetic waves. The wavelength of the electromagnetic wave is often correlated with the characteristic size of the system that radiates. Thus, gamma radiation, having wavelength of 10^{-14} m to 10^{-15} m, typically originate from an atomic nucleus. X-rays are emitted from heavy atoms. Radio waves are produced by accelerating electrons in a circuit. A transmitting antenna can most efficiently radiate waves having a wavelength of about the same size as the antenna. Visible radiation emitted by atoms is, however, much longer in wavelength than atomic size.
3. The oscillating fields of an electromagnetic wave can accelerate charges and can produce oscillating currents. Therefore, an apparatus designed to detect electromagnetic waves is based on this fact. Hertz original 'receiver' worked in exactly this way. The same basic principle is utilised in practically all modern receiving devices. High frequency electromagnetic waves are detected by other means based on the physical effects they produce on interacting with matter.
4. Infrared waves, with frequencies lower than those of visible light, vibrate not only the electrons, but entire atoms or molecules of a substance. This vibration increases the internal energy and consequently, the temperature of the substance. This is why infrared waves are often called *heat waves*.
5. The centre of sensitivity of our eyes coincides with the centre of the wavelength distribution of the sun. It is because humans have evolved with visions most sensitive to the strongest wavelengths from the sun.

EXERCISES

8.1 Figure 8.6 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.

(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

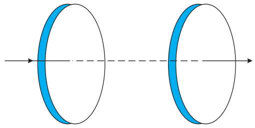


FIGURE 8.6

8.2 A parallel plate capacitor (Fig. 8.7) made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of **B** at a point 3.0 cm from the axis between the plates.

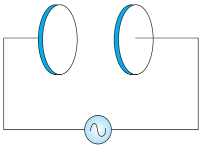


FIGURE 8.7

8.3 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radiowaves of wavelength 500m?

8.4 A plane electromagnetic wave travels in vacuum along *z*-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

8.5 A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

8.6 A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

8.7 The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0 = 510 nT. What is the amplitude of the electric field part of the wave?

8.8 Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine, B_0, ω, k, and λ. (b) Find expressions for **E** and **B**.

8.9 The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

8.10 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0 × 10^{10} Hz and amplitude 48 V m^{-1}.

(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the **E** field equals the average energy density of the **B** field. [$c = 3 \times 10^8$ m s^{-1}.]

ADDITIONAL EXERCISES

8.11 Suppose that the electric field part of an electromagnetic wave in vacuum is $\mathbf{E} = \{(3.1 \text{ N/C}) \cos [(1.8 \text{ rad/m})\, y + (5.4 \times 10^6 \text{ rad/s})t]\}\,\hat{\mathbf{i}}$.
(a) What is the direction of propagation?
(b) What is the wavelength λ ?
(c) What is the frequency ν ?
(d) What is the amplitude of the magnetic field part of the wave?
(e) Write an expression for the magnetic field part of the wave.

8.12 About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation
(a) at a distance of 1m from the bulb?
(b) at a distance of 10 m?
Assume that the radiation is emitted isotropically and neglect reflection.

8.13 Use the formula $\lambda_m T$ = 0.29 cmK to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

8.14 Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.
(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).
(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].
(d) 5890 Å - 5896 Å [double lines of sodium]
(e) 14.4 keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mössbauer spectroscopy)].

8.15 Answer the following questions:
(a) Long distance radio broadcasts use short-wave bands. Why?
(b) It is necessary to use satellites for long distance TV transmission. Why?
(c) Optical and radiotelescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?
(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?
(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?
(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

Chapter 8:- Electromagnetic Waves

Multiple Choice Questions (MCQs)

1. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in
(a) visible region
(b) infrared region
(c) ultraviolet region
(d) microwave region

2. A linearly polarised electromagnetic wave given as $E = E_0\hat{i}\cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at z = a. Assuming that the material of the wall is optically inactive, the reflected wave will be given as
(a) $E_r = E_0\hat{i}(kz - \omega t)$
(b) $E_r = E_0\hat{i}\cos(kz + \omega t)$
(c) $E_r = -E_0\hat{i}\cos(kz + \omega t)$
(d) $E_r = E_0\hat{i}\sin(kz - \omega t)$

3. Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2, the total momentum delivered (for complete absorption) during 30 min is
(a) 36×10^{-5} kg-m/s (b) 36×10^{-4} kg-m/s
(c) 108×10^{4} kg-m/s (d) 1.08×10^{7} kg-m/s

4. The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is
(a) $\frac{E}{2}$ (b) 2E
(c) $\frac{E}{\sqrt{2}}$ (d) $\sqrt{2}E$

5. If E and B represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along
(a) E (b) B
(c) $B \times E$ (d) $E \times B$

6. The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is
(a) c : 1 (b) $c^2 : 1$
(c) 1 : 1 (d) $\sqrt{c} : 1$

7. An EM wave radiates outwards from a dipole antenna, with E_0 as the amplitude of its electric

field vector. The electric field E_0 which transports significant energy from the source falls off as

(a) $\frac{1}{r^3}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r}$ (d) remains constant

8. An electromagnetic wave travels in vacuum along z-direction $E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t)$. Choose the correct options from the following
   (a) The associated magnetic field is given as $B = \frac{1}{c}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t)$
   (b) The associated magnetic field is given as $B = \frac{1}{c}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t)$
   (c) The given electromagnetic field is circularly polarised
   (d) The given electromagnetic wave is plane polarised

Multiple Choice Questions (MCQs) (More than one option correct)

1. An electromagnetic wave travelling along z-axis is given as $E = E_0 \cos(kz - \omega t)$. Choose the correct options from the following
   (a) The associated magnetic field is given as $B = \frac{1}{c}\hat{k} \times E = \frac{1}{\omega}(\hat{k} \times E)$
   (b) The electromagnetic field can be written in terms of the associated magnetic field as $E = c(B \times \hat{k})$
   (c) $\hat{k} \cdot E = 0, \hat{k} \cdot B = 0$
   (d) $\hat{k} \times E = 0, \hat{k} \times B = 0$
2. A plane electromagnetic wave propagating along x-direcrion can have the following pairs of E and B.
   (a) E_x, B_y (b) E_y, B_z
   (c) B_x, E_y (d) E_z, B_y
3. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. The electromagnetic waves produced
   (a) will have frequency of 10^9 Hz
   (b) will have frequency of 2×10^9 Hz
   (c) will have wavelength of 0.3 m
   (d) fall in the region of radiowaves
4. The source of electromagnetic waves can be a charge.
   (a) moving with a constant velocity
   (b) moving in a circular orbit
   (c) at rest
   (d) falling in an electric field
5. An EM wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?
   (a) Radiation pressure is $\frac{I}{c}$ if the wave is totally absorbed
   (b) Radiation pressure is $\frac{I}{c}$ if the wave is totally reflected
   (c) Radiation pressure is $\frac{2I}{c}$ if the wave is totally reflected
   (d) Radiation pressure is in the range $\frac{I}{c} < p < \frac{2I}{c}$ for real surfaces

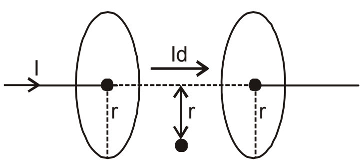
Very Short Answer Questions

1. Why is the orientation of the portable radio with respect to broadcasting station important?
2. Why does microwave oven heats up a food item containing water molecules most efficiently?
3. The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi vt$. The plates are very large and close together (area = A, separation = d). Neglecting the edge effects, find the displacement current through the capacitor.
4. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?
5. The magnetic field of a beam emerging from a filter facing a floodlight is given by $B_0 = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T. What is the average intensity of the beam?
6. Poynting vectors S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propogation. Mathematically, it is given by $S = \frac{1}{\mu_0} E \times B$. Show the nature of S versus t graph.

7. Professor CV Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

Short Answer Questions

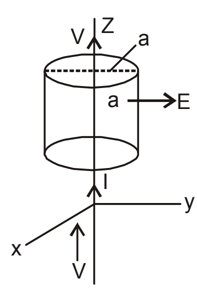
1. Show that the magnetic field B at a point in between the plates of a parallel plate capacitor during charging is $\frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt}$ (symbols having usual meaning).



2. Electromagnetic waves with wavelength
(i) λ_1 is used in satellite communication.
(ii) λ_2 is used to kill germs in water purifies.
(iii) λ_3 is used to detect leakage of oil in underground pipelines.
(iv) λ_4 is used to improve visibility in runways during fog and mist conditions.
(a) Identify and name the part of electromagnetic spectrum to which these radiations belong.
(b) Arrange these wavelengths in ascending order of their magnitude.
(c) Write one application of each.
3. Show that average value of radiant flux density S over a single period T is given by $S = \frac{1}{2c\mu_0}E_0^2$.
4. You are given a 2μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?
5. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{C}$.
6. What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of room, its intensity essentially remain constant. What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?
7. Even though an electric field E exer ts a force qE on a charged particle yet electric field of an EM wave does not contribute to the radiation pressure (but transfers energy). Explain.

Long Answer Questions

1. An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the z-axis (figure). The wire is set into motion along its length with a uniform velocity $v = v\hat{k}_z$.
Calculate the pointing vector $S = \frac{1}{\mu_0}(E \times B)$.



2. Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon = 80\varepsilon_0$, permeability $\mu = \mu_0$ and resistivity $\rho = 0.25$ m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_0 \sin(2\pi\nu t)$. What fraction of the conduction current density is the displacement current density?
3. A long straight cable of length l is placed symmetrically along z-axis and has radius $a(<< l)$. The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_0 \sin(2\pi\nu t)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance s from the wire inside the cable is

$$E(s, t) = \mu_0 I_0 \nu \cos(2\pi\nu t) \ln\left(\frac{s}{a}\right)\hat{k}.$$

(i) Calculate the displacement current density inside the cable.
(ii) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I^d.

(iii) Compare the conduction current I_0 with the displacement current I_0^d.

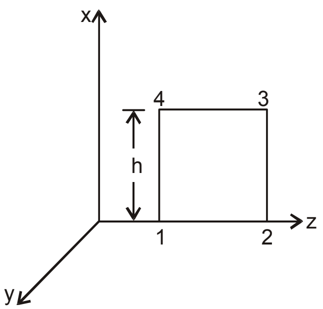
4. A plane EM wave travelling in vacuum along z-direction is given by $E = E_0 \sin(kz - \omega t)\hat{i}$ and $B = B_0 \sin(kz - \omega t)\hat{j}$.

(i) Evaluate $\int E.dl$ over the rectangular loop 1234 shown in figure.

(ii) Evaluate $\int B.ds$ over the surface bounded by loop 1234.

(iii) Use equation $\int E.dl = \frac{-d\phi_B}{dt}$ to prove $\frac{E_0}{B_0} = c$.

(iv) By using similar process and the equation $\int B.dl = \mu_0 I + \varepsilon_0 \frac{d\phi_E}{dt}$, prove that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$



5. A plane EM wave travelling along z-direction is described by $E = E_0 \sin(kz - \omega t)\hat{i}$ and $B = B_0 \sin(kz - \omega t)\hat{j}$. Show that

(i) the average energy density of the wave is given by $u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$

(ii) the time averaged intensity of the wave is given by $I_{av} = \frac{1}{2} c\varepsilon_0 E_0^2$.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(c)** As we know that,

$$E = h\nu$$

As given that $h = 6.62 \times 10^{-34}$ J-s

$E = 11\text{ eV} = 11 \times 1.6 \times 10^{-19}$

$$\nu = ?$$

$$11\text{ eV} = h\nu$$

So, $\nu = \frac{11 \times 1.6 \times 10^{-19}}{h} J = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} J$

$$= 2.65 \times 10^{15}\text{ Hz}$$

So, that frequency radiation belongs to ultraviolet region.

2. **(b)** The type of wave doesn't change when a wave is reflected from denser medium but only its phase changes by 180°. As E is along positive x-axis so reflected ray will be along negative x-axis and its component will also be opposite to earlier in (–z) direction and phase will change.

For the reflected wave $\hat{z} = -\hat{z}, \hat{i} = -\hat{i}$ and additional phase of π in the incident wave.

As given that the incident electromagnetic wave is, $E = E_0 \hat{i} \cos(kz - \omega t)$

So, the reflected electromagnetic wave is

$$E_r = E_0(-\hat{i})\cos(k(-z) - \omega t + \pi)$$
$$= -E_0 \hat{i} \cos(-(kz + \omega t) + \pi)$$
$$= -E_0 \hat{i} \cos(\pi - (kz + \omega t))$$
$$= E_0 \hat{i} \cos(kz + \omega t)$$

3. (b) As we know that the momentum of incident light

$$= \frac{U(\text{total energy})}{c}$$

As given that the energy flux $\phi = 20\text{W/cm}^2$

Surface are $A = 30\text{ cm}^2$

Time for total momentum delivered

$t = 30\text{ min} = 30 \times 60\text{ sec}$

So, total energy falling in time t sec is

$U = \phi At = 20 \times 30 \times (30 \times 60)$ J

Momentum of the incident light

$$= \frac{U}{c} \qquad (\because c = 3 \times 10^8)$$

Momentum of incident light

$$= \frac{20 \times 30 \times (30 \times 60)}{3 \times 10^8} = 36 \times 10^{-4}\ \text{kg-ms}^{-1}$$

As no reflection from the surface and for comidete absorption.

Momentum of the reflected light = 0

Hence, momentum delivered to the surface

= Charge in momentum. $= (p_f - f_i)$

$= 36 \times 10^{-4} - 0 = 36 \times 10^{-4}$ kg-ms^{-1}

4. **(c)** As we know that the electric field intensity on a surface due to incident radiation is,

$$I_{av} \propto E_0^2$$

$$\frac{P_{av}}{A} \propto E_0^2 \qquad \text{(A is constant)}$$

Here, $P_{av} \propto E_0^2$

So, $E_0 \propto \sqrt{P_{av}}$

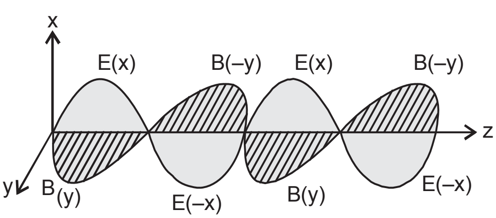
$$\therefore \quad \frac{(E_0)_1}{(E_0)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}}$$

$$\frac{(E_0)_1}{(E_0)_2} = \sqrt{\frac{100}{50}} = \left[\frac{\sqrt{2}}{1}\right]$$

$$(E_0)_2 = (E_0)_1 \sqrt{2}$$

5. **(d)** The direction of propagation of electromagnetic wave is perpendicular to both electric field E and magnetic field B, i.e., in the direction of E × B by right thumb rule.

The diagram given below



So, electromagnetic wave is along the z-direction which is give the cross product of E and B direction is perpendicular to E and B from $\vec{E}$ to $\vec{B}$. i.e., (E × B) in z-direction.

6. **(c)** Intensity in terms of electric field

$$U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$$

Intensity in terms of magnetic field

$$U_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

We also know that the relationship between E and B is $\quad E_0 = cB_0$

So the average energy by electric field is

$$(U_{av}) = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0 (cB_0)^2$$

$$= \frac{1}{2}\varepsilon_0 \times c^2 B^2 \qquad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$\therefore \quad (U_{av})_{\text{Electric field}} = \frac{1}{2}\varepsilon_0 \times \frac{1}{\mu_0 \varepsilon_0} B_0^2$$

$$= \frac{1}{2}\frac{B_0^2}{\mu_0} = (U_{av})_{\text{Magnetic field}}$$

So, the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector.

Then, the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is

$$\text{Ratio} = \frac{(U_{av})_{\text{electric field}}}{(U_{av})_{\text{Magnetic field}}} = 1 : 1$$

7. **(c)** As we know that, the electric field is inversly proportional to r, so $\left(E_0 \propto \frac{1}{r}\right)$

From a diode antenna, an electromagnetic waves are radiated outwards from dipole antenna with the amplitude of electric field vector (E_0) which transports significant energy from the source falls off intensity inversely as the distance (r) from the antenna, i.e.,

radiated energy $\left(E_0 \propto \frac{1}{r}\right)$

8. **(d)** In electromagnetic wave, the electric field vector is

$$E = \left(E_1\hat{i} + E_2\hat{j}\right)\cos(kz - \omega t)$$

and the associated magnetic field vector,

$$B = \frac{E}{c} = \frac{E_1\hat{i} + E_2\hat{j}}{c}\cos(kz - \omega t)$$

So, E and B are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to E as well as B, so the electromagnetic wave plane polarised.

Multiple Choice Questions (More Than One Options)

1. **(a, b, c)**

Let, the elctromagnetic wave is travelling along negative z-direction. Then electric field vector

$$E = E_0 \cos(kz - \omega t)$$

which is perpendicular to z-axis. It acts along negative y-direction.

and the associated magnetic field B in electromagnetic wave is along x-axis i.e., along $\hat{k} \times E$ is given by

As, $B_0 = \frac{E_0}{c}$

So, $B = \frac{1}{c}(\hat{k} \times E)$ (along x-axis)

The associated electric field in terms of magnetic fleld can be represented as

$$E = c(B \times \hat{k})$$

If angle between $\hat{k}$ and E is 90°

Then, $\hat{k}E = E_0 \cos 90° = 0$

and if the angle between $\hat{k}$ and $\hat{B}$ also 90° then

$$\hat{k} \cdot B = E_0 \cos 90° = 0$$

So, $\left\{\begin{matrix} \hat{k} \cdot E = 0 \text{ is true if k.i=0} \\ \hat{k} \cdot B = 0 \text{ is true if } k.\hat{j} = 0 \end{matrix}\right\}$ at $\theta = 90°$

2. **(b, d)**

In the electromagnatic wave electric and magnetic field vectors E and B are perpendicular to each other as well as perpendicular to the direction of propagation of electromagnetic wave.

Here electromagnetic wave is plane polarised and propagating along x-direction. So, electric and magnetic field vectors should be either y-direction or z-direction.

3. **(a, c, d)**

As given that, the frequency of the charged particles oscillates about its mean equilibrium position = 10^9 Hz.

Vibrating particle produces electric and magnetic field.

So, frequency of electromagnetic waves produced by the charged particle is

$$\nu = 10^9 \text{ Hz.}$$

$$\text{Wavelength } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = 0.3 \text{ m}$$

Since the range of radiowaves is between 10 Hz to 10^{12} Hz and hence 10^9 Hz lies in region of readio waves.

4. **(b, d)**

In circular orbit, the direction of the motion of charge particle is changing continuously, thus it is an accelerated motion.

And when a charge particle starts acceleration, then it falls in an electric field, because the velocity of charge particle changes so its motion become accelerated and can produce EM wave.

5. **(a, c, d)**

As we know that, the radiation pressure (P) is the force exerted by charge particle in electromagnetic wave on unit area of the surface, i.e., rate of change of momentum per unit area of the surface.

Then, the momentum per unit area per unit time

$$= \frac{\text{Intensity}}{\text{Speed of wave}} = \frac{I}{c}$$

The change in momentum per unit time per unit area $= \frac{\Delta I}{c} =$ radiation pressure $p = \frac{\Delta I}{c}$

Momentum of incident wave per unit area per second $= \frac{I}{c}$

When wave is fully absorbed by the surface, the momentum of the reflected wave per unit time per unit area = 0.

Radiation pressure (p) = change in momentum per unit area per second $= \frac{\Delta I}{c} = \frac{I}{c} - 0 = \frac{I}{c}$.

When wave is totally reflected, then momentum of the reflected wave per unit area per time $= -\frac{I}{c}$, Radiation pressure $p = \frac{I}{c} - \left(-\frac{I}{c}\right) = \frac{2I}{c}$.

So variation of radiation pressure P lies between the range $\frac{I}{c} < p < \frac{2I}{c}$ for real surface.

Very Short Answer Questions

1. The orientation of the portable radio with respect to broadcasting station is important because the electromagnetic waves are plane polarised so the receiving antenna should be parallel to the vibration of the electric or magnetic field of the wave.

2. In microwave oven heats up the food items containing water molecules most efficiently because the frequency of microwaves matches the resonant frequency of water molecules, which further causes increase in temperature.

3. As we know that,

   As the displacement current through the capacitor is, $I_d = I_c = \frac{dq}{dt}$ ... (i)

   As given that, $q = q_0 \cos 2\pi\nu t$

   Putting this value of q in Eq (i), we get

   So, $I_d = I_c = \frac{d}{dt}(q_0 \cos 2\pi\nu t)$

   $I_d = I_c = -q_0 \sin 2\pi\nu t \times 2\pi\nu$

   $I_d = I_c = 2p\nu q_0 \sin 2p\nu t$

4. The capacitive reactance (X_c) is inversely proportional to the conduction current

   i.e., $(X_c \propto \frac{1}{I})$

   and capacitive reaction is also given by

   $X_c = \frac{1}{2\pi fC}$,

   or, $X_c \propto \frac{1}{f}$ and $I_c = \frac{V}{X_c}$

   When frequency decreases, X_c increases and the conduction current is also decreases as $I \propto \frac{1}{X_c}$.

   On the other hand displacement current also decreases as frequency decreases thus the conduction current is equal to the displacement current.

5. As we know that,

   The standard equation of magnetic field

   $B = B_0 \sin(kx - \omega t)$

   And, the given equation is

   $B = 12 \times 10^{-6} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)T.$

   On comparing this equation with standard equation (i), we have.

   $B_0 = 12 \times 10^{-8}$

   So, the average intensity of the beam

   $$I_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c = \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$$

   $= 1.72\ W/m^2$

6. In an electromagnetic waves. Let electric field (E) of EM wave varying along y-axis, B is along z-axis and propagation of wave be along x-axis. Then E × B will give the direction of flow of energy in electromegnetic wave, along x-axis.

   Let $E = E_0 \sin(\omega t - kx)\hat{j}$

   $B = B_0 \sin(\omega t - kx)\hat{k}$

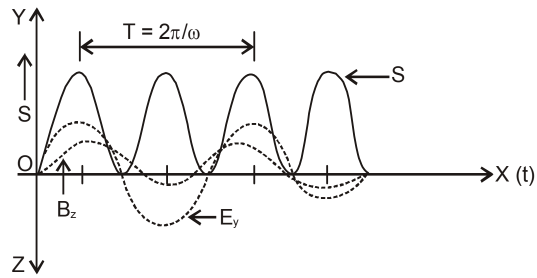
   Poynting vector $S = \frac{1}{\mu_0}(E \times B)$

   $= \frac{1}{\mu_0} E_0 B_0 \sin^2(\omega t - kx)[\hat{j} \times \hat{k}]$

   $= \frac{E_0 B_0}{\mu_0} \sin^2(\omega t - kx)\hat{i}$

   So, $S = \frac{1}{\mu_0}(\sin^2(\omega t - kx))\hat{i}$ $(\because E_0 B_0 = 1)$

   Then the variation of |S| with time (t) will be as shown in figure :



7. An electromagnetic wave carries energy and momentum like other waves.

   So, it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. So the property of electromagnetic waves helped professor CV Raman to surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. The tails of the comets are also due to radiation pressure.

Short Answer Questions

1. According to the given figure

Consider I_d be the displacement current in the region between two plates of parallel plate capacitor, shown in figure.

The magnetic field induction at a point in a region between two plates of capacitor at a perpendicular distance r from the axis of plates is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r} I_d = \frac{\mu_0}{2\pi r} \times \left(\varepsilon_0 \frac{d\phi_E}{dt}\right)$$

$$\left[\because I_d = \frac{E_0 d\phi_E}{dt}\right]$$

$$= \frac{\mu_0 \varepsilon_0}{2\pi r}\left[\frac{d}{dt}(E\pi r^2)\right] = \frac{\mu_0 \varepsilon_0}{2\pi r} \pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} \qquad [\because \phi_E = E\pi r^2]$$

Hence Proved.

2. **(a)**

(i) Microwave is also used in satellite communications.

So, λ_1 is the wavelength of microwave and it is used in microwave oven.

(ii) Ultraviolet rays are used to kill germs in water purifier. So, λ_2 is the wavelength of UV rays.

(iii) X-rays are used to detect leakage of oil in underground pipelines. So, λ_3 is the wavelength of X-rays region. It is used to detect fracture of bones in body.

(iv) Infrared rays λ_4 is used to improve visibility on runways during fog and mist conditions. So, the wavelength of infrared waves is larger of low scattering.

(b) The arrangement of wavelength in assending order are :

$$\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$$

(c) The application all the above equipment are

(i) Microwave is used in radar.

(ii) UV is used in LASER eye surgery.

(iii) X-ray is used in radio therapy to cure untracable skin diseases and malignant growths.

(iv) Infrared is used in optical communication.

3. As we know that

The Radiant flux density or Poynting vector

$$S = \frac{1}{\mu_0}(E \times B) = c^2 \varepsilon_0 (E \times B) \quad \left[\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right]$$

Let electromagnetic waves be propagating along x-axis. So its electric field vector of electromagnetic wave be along y-axis and magnetic field vector be along z-axis. So that,

$$E_0 = E_0 \cos(kx - \omega t)$$

and $$B = B_0 \cos(kx - \omega t)$$

$$E \times B = (E_0 . B_0)\cos^2(kx - \omega t)$$

$$S = c^2 \varepsilon_0 (E . B)$$

$$= c^2 \varepsilon_0 (E_0 . B_0) \cos^2(kx - \omega t)$$

So, average value of the magnitude of radiant flux density over complete cycle is

$$S_{av} = c^2 \varepsilon_0 | E_0 \cdot B_0 | \frac{1}{T}\int_0^T \cos^2(kx - \omega t)\, dt$$

$$= c^2 \varepsilon_0 E_0 B_0 \times \frac{1}{T} \times \frac{T}{2}$$

$$\left[\because \int_0^T \cos^2(kx - \omega t)\, dt = \frac{T}{2}, c = \frac{E_0}{B_0}\right]$$

So, $$S_{av} = \frac{c^2}{2}\varepsilon_0 E_0 \left(\frac{E_0}{c}\right)$$

$$S_{av} = \frac{c}{2}\varepsilon_0 E_0^2 = \frac{c}{2} \times \frac{1}{c^2 \mu_0} E_0^2$$

$$\left[\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ or } \varepsilon_0 = \frac{1}{c^2 \mu_0}\right]$$

So, $$S_{av} = \frac{E_0^2}{2\mu_0 c}$$

Hence Proved.

4. As given that, the capacitance of capacitor

$C = 2\mu F = 2 \times 10^{-6} F$

Displacement current

$I_d = 1\text{ mA} = 10^{-3} A$

As we know that, $q = CV$ $\qquad [\because q = it]$

or $$I_d = C\frac{dV}{dt}$$

$$1 \times 10^{-3} = 2 \times 10^{-6} \times \frac{dV}{dt}$$

or $$\frac{dV}{dt} = \frac{1}{2}\times 10^{+3} = 500 \text{ V/s}$$

Hence, by applying a varying voltage difference of 500 V/s, so we would produce a displacement current of desired value.

5. As we know that,

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

If mass (m) of radiant particle for wave of velocity (C) then $E = mC^2$

Let, $E = U;$

$U = (mC)C$

$$U = PC \qquad [\because P = mC]$$

differentiating both side w.r.t. (t)

So, $$\left(\frac{dv}{dt}\right) = C\left(\frac{dP}{dt}\right) \qquad \text{..... (ii)}$$

Then, the Force is the rate of change of momentum

i.e., $$F = \frac{dp}{dt}$$

By putting the value of $\left(\frac{dp}{dt}\right)$ in equation (ii)

So, $$F = \frac{1}{C}\left[\frac{dv}{dt}\right]$$

From (i) equation, $P = \frac{F}{A}$

So, $$(\text{Pressure})\ (P) = \frac{1}{A}\cdot\frac{dU}{C.dt}$$

$$\left[\because I = \text{Intensity} = \frac{dU}{A.dt}\right]$$

$$P = \frac{I}{C}$$

6. Bulb spreads its light in all around spherically and symmetrically. So, if the distance is doubled, the area of spherical region ($4\pi r^2$) will become four times.

So, the intensity becomes one fourth the initial value in straight line for spherical source ($I \propto 1/4\pi r^2$) but in case of laser it does not spread in all directions, so its intensity remain almost same. Geometrical characteristic of LASER beam which is responsible for the constant intensity are :

(i) Unidirection (ii) Monochromatic

(iii) Coherent light (iv) Highly collimated

These characteristic are missing in the case of light from the bulb.

7. In electromagnetic wave, the electric field of an EM wave is an oscillating field, so the electric force caused by its on a charged particle whenever electric force averaged over an integral number of cycle is zero, since its direction changes every half cycle.

Hence, electric field is not responsible for radiation pressure.

Long Answer Questions

1. Let us consider a cylindrical surface in such a way that the axis of cyclinder lies on wire. So electric field intensity due to long straight wire at a distance a and charge density λ c/m.

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{j})$$

$$B = \frac{\mu_0 i}{2\pi a}\hat{i} = \frac{\mu_0\lambda v}{2\pi a}(\hat{i}) \qquad \left[\because i = \frac{q}{t} = \frac{\lambda l}{t} = \lambda v\right]$$

$$\therefore \quad S = \frac{1}{\mu_0}[E\times B] = \frac{1}{\mu_0}\left[\frac{\lambda(\hat{j})}{2\pi\varepsilon_0 a}\times\frac{\mu_0}{2\pi a}\lambda v(\hat{i})\right]$$

$$\left[\because \hat{j}\times\hat{i} = -\hat{k}\right]$$

$$= \frac{\lambda^2 v}{4\pi^2\varepsilon_0 a^2}(j\times\hat{i})$$

$$S = -\frac{\lambda^2 v}{4\pi^2\varepsilon_0 a^2}\hat{k}$$

2. Let distance between the parallel plates of capacitor is d and applied voltage $V(t) = V_0\sin(2\pi vt)$

So, electric field

$$E = \frac{V(t)}{d} = \frac{V_0}{d}\sin(2\pi vt)$$

By applying Ohm's law, conduction current density is

$$J_c = \frac{1}{\rho}\frac{V_0}{d}\sin(2\pi vt) = \frac{V_0}{\rho d}\sin(2\pi vt)$$

Let, $$J_0^c = \frac{V_0}{\rho d} \qquad \text{..... (i)}$$

Then $\quad J_c = J_0^c \sin 2\pi\nu t$

Now the displacement current density is

$$J_d = \varepsilon \frac{\delta E}{dt} = \frac{\varepsilon\delta}{dt}\left[\frac{V_0}{d}\sin 2\pi\nu t\right]$$

$$\left[\because E = \left[\frac{V_0}{d}\sin(2\pi\nu t)\right]\right]$$

$$= \frac{\varepsilon 2\pi\nu V_0}{d}\cos(2\pi\nu t)$$

Let, $J_0^d = \frac{2\pi\nu\varepsilon V_0}{d} = J_0^d\cos(2\pi\nu t)$ ..... (ii)

Dividing equation (ii) to equation (i)

So, $\frac{J_0^d}{J_0^c} = \frac{2\pi\nu\varepsilon V_0}{d}\cdot\frac{\rho d}{V_0} = 2\pi\nu\varepsilon\rho$

$$[\because \text{Given } \varepsilon = 80\varepsilon_0]$$

$$= 2\pi \times 80\varepsilon_0 \nu \times 0.25 = 4\pi\varepsilon_0 \nu \times 10$$

$$\frac{J_0^d}{J_0^c} = \frac{10\nu}{9\times10^9}\quad\left[\because 4\pi\varepsilon_0 = \frac{1}{9\times10^9}\right]$$

$$\left[\because \text{Given } \nu = 4\times10^8\right]$$

$$= \frac{10\times4\times10^8}{9\times10^9} = \left(\frac{4}{9}\right)$$

3. We know that,

Displacement current density

$$J_d = \varepsilon_0\frac{dE}{dt}$$

(i) The induced electric field E(s, t) at a distance (s < radius of co-axial cable) from the wire inside the cable is given as :

$$E(s,t) = \mu_0 I_0 \nu\cos(2\pi\nu t)\ln\left(\frac{s}{a}\right)\hat{k}$$

Now, displacement current density, (J_d) is given by,

$$J_d = \varepsilon_0\frac{dE}{dt} = \varepsilon_0\frac{d}{dt}\left[\mu_0 I_0 \nu\cos(2\pi\nu t)\ln\left(\frac{s}{a}\right)\hat{k}\right]$$

$$= \varepsilon_0\mu_0 I_0 \nu\frac{d}{dt}[\cos 2\pi\nu t]\ln\left(\frac{s}{a}\right)\hat{k}\quad\left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$$

$$= \frac{1}{c^2}I_0\nu^2 2\pi[-\sin 2\pi\nu t]\ln\left(\frac{s}{a}\right)\hat{k}\quad\left[\because l_4\frac{s}{a} = -l_4\frac{a}{s}\right]$$

$$= +\frac{\nu^2}{c^2}2\pi I_0\sin 2\pi\nu t\ln\left(\frac{a}{s}\right)\hat{k}\quad\left[\because \lambda = \frac{C^2}{V^2}\right]$$

$$= \frac{1}{\lambda^2}2\pi I_0\ln\left(\frac{a}{s}\right)\sin 2\pi\nu t\hat{k}$$

$$J_d = \frac{2\pi I_0}{\lambda^2}\ln\left(\frac{a}{s}\right)\sin(2\pi\nu t)\hat{k}$$

(ii) $I_d = \int J_d s\,ds\,d\theta = \int_{s=0}^{a} J_d s\,ds\int_0^{2\pi} d\theta$

$$= \int_{s=0}^{a} J_d s\,ds \times [2\pi]$$

$$= \int_{s=0}^{a}\left[\frac{2\pi}{\lambda^2}I_0\log_e\left(\frac{a}{s}\right)s\,ds\sin(2\pi\nu t)\hat{k}\right]\times[2\pi]$$

$$= \left(\frac{2\pi}{\lambda}\right)^2\left(I_0\int_{s=0}^{a}\ln\left(\frac{a}{s}\right)s\,ds\right)\sin(2\pi\nu t)\hat{k}$$

$$\Rightarrow \left(\frac{2\pi}{\lambda}\right)^2 I_0\left[\int_{s=0}^{a}\ln\left(\frac{a}{s}\right)\frac{1}{2}d(s^2)\right]\sin(2\pi\nu t)\hat{k}$$

$$= \frac{a^2}{\lambda}\left(\frac{2\pi}{\lambda}\right)^2 I_0[\sin 2\pi\nu t\hat{k}]\left[\int_{s=0}^{a}\left[\ln\left(\frac{a}{s}\right)\cdot d\left(\frac{s}{a}\right)^2\right]ds\right]$$

$$= \frac{a^2}{2}\left(\frac{2\pi}{\lambda}\right)^2 I_0\sin 2\pi\nu t\,\hat{k}\times\left(\frac{-a}{4}\right)$$

$$\left[\because \int_{s=0}^{a}\left[\ln\left(\frac{s}{a}\right)d\left(\frac{s}{a}\right)^2\right]ds = \frac{-a}{4}\right]$$

$$\therefore\quad I_d = \frac{-a^3}{8}\left(\frac{2\pi}{\lambda}\right)^2 I_0\sin 2\pi\nu t(\hat{k})$$

$$= \frac{-a}{2} - \frac{a^2}{4}\left[\frac{2\pi}{\lambda}\right]^2 I_0\sin(2\pi\nu t)\hat{k}$$

The negative sign shows that the displacement current I_d is oposite to the conduction current I_c

So, $\frac{a}{2}\left(\frac{2\pi a}{2\lambda}\right)^4 I_0\sin 2\pi\nu t(\hat{k})$

I_d is in z-direction as I_c is in +z direction.

(iii) The displacement current.

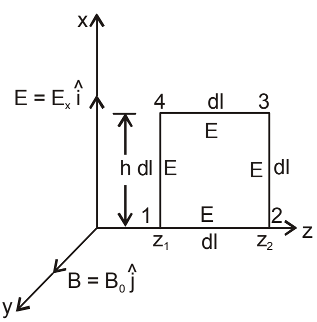
$$I_d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_0 \sin 2\pi vt = I_0^d \sin 2\pi vt$$

So, $I_0^d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_0 = \left(\frac{a\pi}{\lambda}\right)^2 I_0 \times \left(\frac{a}{2}\right)$

$$\therefore \frac{I_0^d}{I_0} = \left[\frac{\left(\frac{a\pi}{\lambda}\right)^2 I_0 \times \left(\frac{a}{2}\right)}{I_0}\right] = \frac{a}{2}\left(\frac{a\pi}{\lambda}\right)^2 = \frac{a^3\pi^2}{2\lambda^2}$$

So, the required ratio is $\frac{I_0^d}{I_0} = \frac{a^3\pi^2}{2\lambda^2}$

4. **(i)** According to the given figure



Let us consider the electromagnetic wave is propagating along z-axis, and electric field vector (E) be along x-axis and magnetic field vector B along y-axis. so, $E = E_0\hat{i}$ and $B = B_0\hat{j}$.

Line integral of E over the closed path 1234 in x-z plane, shown in above figure.

So, $\oint E.dl - \int_1^2 E.dl + \int_2^3 E.dl + \int_3^4 E.dl + \int_4^1 E.dl$

$$= \int_1^2 E.dl\cos 90 + \int_2^3 E.dl\cos 0 + \int_3^4 E.dl\cos 90 + \int_4^1 E.dl\cos 180°$$

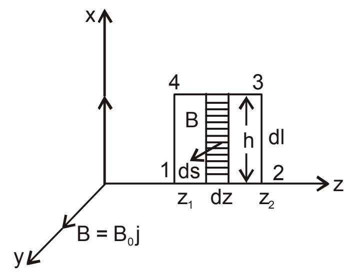
$$= E_0 h\left[\sin(kz_2 - \omega t) - \sin(kz_1 - g\omega t)\right] \quad \text{... (i)}$$

(ii) Now again let us consider the rectangle 1234 to be made of strips of area ds = hdz each and the angle between ($d\vec{S}$) and ($\vec{B}$) is zero.

So, $\int \vec{B}.\vec{ds} = \int B.ds\cos 0 = \int B.ds$

$$= \int_{Z_1}^{Z_2} B_0 \sin(kz - \omega t) hdz$$

$$= \frac{-B_0 h}{k}\left[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\right] \quad \text{... (ii)}$$



(iii) As given that,

$$\oint E.dl = \frac{-d\phi_B}{dt} = -\frac{d}{dt}\oint B.ds$$

$$\left[\because B = B_0 \sin(kz - \omega t)\right]$$

Equating Eqs. (i) and (ii), we have

$$E_0 h\left[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)\right]$$

$$= \frac{-d}{dt}\left[\frac{B_0 h}{k}\left\{\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\right\}\right]$$

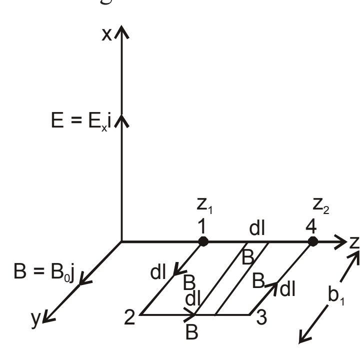
$$E_0 h\left[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)\right]$$

$$= \frac{B_0 h}{k}\omega\left[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)\right]$$

So, $E_0 = \frac{B_0\omega}{k} = B_y c \quad \left[\because \frac{\omega}{k} = c\right]$

$$\frac{E_0}{B_0} = c$$

(iv) Let us consider a loop 1234 in y-z plane as shown in figure.



$$\oint B.dl = \int_1^2 B.dl + \int_2^3 B.dl + \int_3^4 B.dl + \int_4^1 B.dl$$

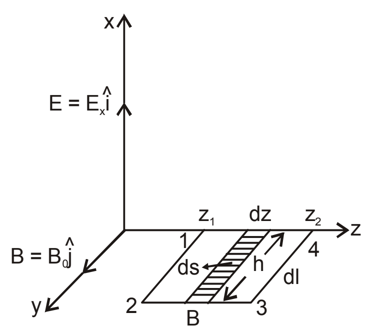
$$= \int_1^2 B.dl\cos 0 + \int_2^3 B.dl\cos 90° + \int_3^4 B.dl\cos 180°$$

$$+ \int_4^1 B.dl\cos 90°$$

$$= B_0 h\left[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)\right] \quad ...(iii)$$

$$[\because dl = h]$$

Now to calculate $\phi_e = \int E.ds$, consider the rectangle strip of loop 1, 2, 3, 4 of area (ds) each (ds = hdz).



$$\phi_E = \int E.ds = \int Eds\cos 0° = \int Eds$$

$$= \int_{z_1}^{z_2} E_0 \sin(kz_1 - \omega t) hdz$$

$$\phi_E = -\frac{E_0 h}{k}\left[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\right]$$

$$\therefore \frac{d\phi_E}{dt} = \frac{E_0 h\omega}{k}\left[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)\right]$$

...(iv)

By applying ampere's law,

$$\oint B.dl = \mu_0\left(I_c + \frac{\varepsilon_0 d\phi_E}{dt}\right)$$

where, I_c = conduction current

For vacuum, $I_c = 0$

So, $$\oint B.dl = \mu_0 \varepsilon \frac{d\phi_E}{dt}$$

From Eqs. (iii) and (iv) then we have,

$$B_0 h\left[\sin(kz_1 - \omega t)\sin(kz_2 - \omega t)\right]$$

$$= \mu_0 \varepsilon_0 \frac{E_0 h\omega}{k}\left[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)\right]$$

$$B_0 = E_0 \frac{\omega \mu_0 \varepsilon_0}{k}$$

So, $$\frac{E_0}{B_0}\frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0} \qquad \left(\because \frac{E_0}{B_0} = c \text{ and } \omega = ck\right)$$

then, $$\frac{c.ck}{k} = \frac{1}{\mu_0 \varepsilon_0} \text{ or } c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Hence, $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

5. **(i)** Due to electric field vector and magnetic field vector the electromagnetic wave carry energy. In electromagnetic wave, E and B vary with time from point to point and from moment to moment.

Consider E and B be the time averages.

So, the energy density due to electric field E is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

And the energy density due to magnetic field B is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

So, total average energy density of electromagnetic wave

$$u_{av} = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

Consider the electromagnetic wave propagating along z-direction. The electric field vector and magnetic field vector are :

$$E = E_0 \sin(kz - \omega t)$$
$$B = B_0 \sin(kz - \omega t)$$

Then the time average value of E^2 over complete cycle $= \frac{E_0^2}{2}$

Similarly the time average value of B^2 over complete cycle $= \frac{B_0^2}{2}$

So, $$u_{av} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{2} + \frac{1}{2}\mu_0\left(\frac{B_0^2}{2}\right)$$

Hence, $u_{av} = \frac{1}{4}\left[\varepsilon_0 E_0^2 + \frac{B_0^2}{4\mu_0}\right]$

(ii) As we know that the relationship between ε_0 and E_0 is $E_0 = cB_0$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

And, From (i) part, when, $u_{av} = 0$

$$\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{-1}{4}\varepsilon_0 E_0^2 = \frac{-1}{4}\frac{E_0^2}{\mu_0 c^2}$$

when (–ve) neglected.

So, $\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2/c^2}{\mu_0} = \frac{E_0^2}{4\mu_0} \times \mu_0 \varepsilon_0$

$$\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2}{\varepsilon_0}$$

or $\frac{1}{2}\frac{B_0^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E_0^2$

$\therefore$ $u_B = u_E$

Again from (i) part,

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0} \quad \left[\because c = \frac{E_0}{B_0} \Rightarrow c^2 = \left(\frac{E_0^2}{B_0^2}\right)\right]$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2$$

$$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0} \quad \left[\because E_0^2 = \frac{B_0}{\mu_0 \varepsilon_0}\right]$$

$\therefore$ $u_E = u_B$

So, time average intensity of the wave

$$I_{av} = u_{av} C = \frac{1}{2}\varepsilon_0 B_0^2 c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

Hence, $I_{av} = \frac{1}{2}\varepsilon_0 E_0^2 c$

Chapter Nine

RAY OPTICS AND OPTICAL INSTRUMENTS

SUMMARY

1. Reflection is governed by the equation $\angle i = \angle r'$ and refraction by the Snell's law, $\sin i/\sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane. Angles of incidence, reflection and refraction are i, r' and r, respectively.
2. The *critical angle of incidence* i_c for a ray incident from a denser to rarer medium, is that angle for which the angle of refraction is 90°. For $i > i_c$, total internal reflection occurs. Multiple internal reflections in diamond ($i_c \cong 24.4°$), totally reflecting prisms and mirage, are some examples of total internal reflection. Optical fibres consist of glass fibres coated with a thin layer of material of *lower* refractive index. Light incident at an angle at one end comes out at the other, after multiple internal reflections, even if the fibre is bent.
3. *Cartesian sign convention*: Distances measured in the same direction as the incident light are positive; those measured in the opposite direction are negative. All distances are measured from the pole/optic centre of the mirror/lens on the principal axis. The heights measured upwards above x-axis and normal to the principal axis of the mirror/lens are taken as positive. The heights measured downwards are taken as negative.
4. *Mirror equation*:

   $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

   where u and v are object and image distances, respectively and f is the focal length of the mirror. f is (approximately) half the radius of curvature R. f is negative for concave mirror; f is positive for a convex mirror.
5. For a prism of the angle A, of refractive index n_2 placed in a medium of refractive index n_1,

   $$n_{21} = \frac{n_2}{n_1} = \frac{\sin\left[\left(A + D_m\right)/2\right]}{\sin\left(A/2\right)}$$

   where D_m is the angle of minimum deviation.
6. *For refraction through a spherical interface* (from medium 1 to 2 of refractive index n_1 and n_2, respectively)

   $$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

   Thin lens formula

   $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Lens maker's formula

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

R_1 and R_2 are the radii of curvature of the lens surfaces. f is positive for a converging lens; f is negative for a diverging lens. The power of a lens $P = 1/f$.

The SI unit for power of a lens is dioptre (D): 1 D = 1 m^{-1}.

If several thin lenses of focal length f_1, f_2, f_3,.. are in contact, the effective focal length of their combination, is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots$$

The total power of a combination of several lenses is

$P = P_1 + P_2 + P_3 + \ldots$

7. *Dispersion* is the splitting of light into its constituent colours.
8. *The Eye*: The eye has a convex lens of focal length about 2.5 cm. This focal length can be varied somewhat so that the image is always formed on the retina. This ability of the eye is called *accommodation*. In a defective eye, if the image is focussed before the retina (myopia), a diverging corrective lens is needed; if the image is focussed beyond the retina (hypermetropia), a converging corrective lens is needed. Astigmatism is corrected by using cylindrical lenses.
9. *Magnifying power m of a simple microscope* is given by $m = 1 + (D/f)$, where D = 25 cm is the least distance of distinct vision and f is the focal length of the convex lens. If the image is at infinity, $m = D/f$. For a compound microscope, the magnifying power is given by $m = m_e \times m_0$ where $m_e = 1 + (D/f_e)$, is the magnification due to the eyepiece and m_o is the magnification produced by the objective. *Approximately,*

$$m = \frac{L}{f_o} \times \frac{D}{f_e}$$

where f_o and f_e are the focal lengths of the objective and eyepiece, respectively, and L is the distance between their focal points.

10. *Magnifying power m of a telescope* is the ratio of the angle β subtended at the eye by the image to the angle α subtended at the eye by the object.

$$m = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$$

where f_0 and f_e are the focal lengths of the objective and eyepiece, respectively.

POINTS TO PONDER

1. The laws of reflection and refraction are true for all surfaces and pairs of media at the point of the incidence.
2. The real image of an object placed between f and $2f$ from a convex lens can be seen on a screen placed at the image location. If the screen is removed, is the image still there? This question puzzles many, because it is difficult to reconcile ourselves with an image suspended in air

without a screen. But the image does exist. Rays from a given point on the object are converging to an image point in space and diverging away. The screen simply diffuses these rays, some of which reach our eye and we see the image. This can be seen by the images formed in air during a laser show.

3. Image formation needs regular reflection/refraction. In principle, all rays from a given point should reach the same image point. This is why you do not see your image by an irregular reflecting object, say the page of a book.
4. Thick lenses give coloured images due to dispersion. The variety in colour of objects we see around us is due to the constituent colours of the light incident on them. A monochromatic light may produce an entirely different perception about the colours on an object as seen in white light.
5. For a simple microscope, the angular size of the object equals the angular size of the image. Yet it offers magnification because we can keep the small object much closer to the eye than 25 cm and hence have it subtend a large angle. The image is at 25 cm which we can see. Without the microscope, you would need to keep the small object at 25 cm which would subtend a very small angle.

EXERCISES

9.1 A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

9.2 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

9.3 A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

9.4 Figures 9.34(a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. 9.34(c)].

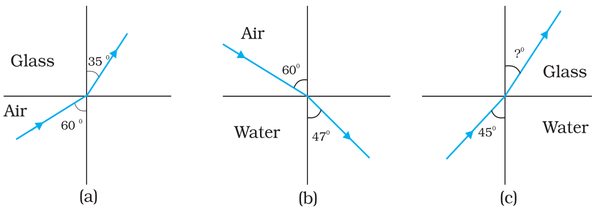


FIGURE 9.34

9.5 A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

9.6 A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

9.7 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

9.8 A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

9.9 An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

9.10 What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

9.11 A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

9.12 A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope,

9.13 A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

9.14 (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.

9.15 Use the mirror equation to deduce that:

(a) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

[*Note:* This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

9.16 A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

9.17 (a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?

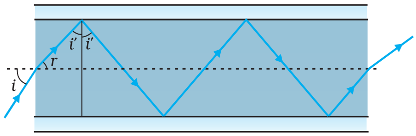


FIGURE 9.35

9.18 Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?

(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

9.19 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

9.20 A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

9.21 (a) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

9.22 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

9.23 You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion,

(b) disperse (and displace) a pencil of white light without much deviation.

9.24 For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

9.25 Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

9.26 A myopic person has been using spectacles of power –1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power + 2.0 dioptres. Explain what may have happened.

9.27 A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

9.28 A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?

(b) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope?

9.29 A card sheet divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.

(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) What is the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

9.30 (a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power?

(b) What is the magnification in this case?

(c) Is the magnification equal to the magnifying power in this case? Explain.

9.31 What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

[*Note:* Exercises 9.29 to 9.31 will help you clearly understand the difference between magnification in absolute size and the angular magnification (or magnifying power) of an instrument.]

9.32 Answer the following questions:

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

9.33 An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

9.34 A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

9.35 (a) For the telescope described in Exercise 9.34 (a), what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c) What is the height of the final image of the tower if it is formed at 25 cm?

9.36 A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

9.37 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

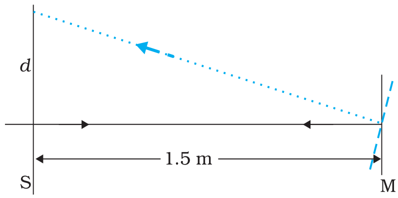


FIGURE 9.36

9.38 Figure 9.37 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?

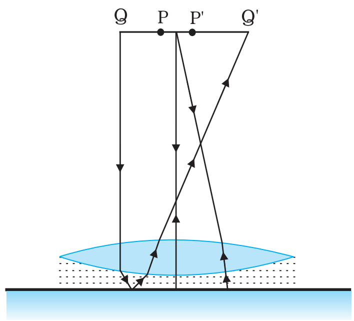


FIGURE 9.37

Chapter 9:- Ray Optics

Multiple Choice Questions (MCQs)

1. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is

(a) 7.5° (b) 5°
(c) 15° (d) 2.5°

2. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is

(a) blue (b) green
(c) violet (d) red

3. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image

(a) moves away from the lens with an uniform speed 5 m/s
(b) moves away from the lens with an uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

4. A passenger in an aeroplane shall

(a) never see a rainbow
(b) may see a primary and a secondary rainbow as concentric circles
(c) may see a primary and a secondary rainbow as concentric arcs
(d) shall never see a secondary rainbow

5. You are given four sources of light each one providing a light of a single colour - red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence?

(a) The beam of red light would undergo total internal reflection
(b) The beam of red light would bend towards normal while it gets refracted through the second medium
(c) The beam of blue light would undergo total internal reflection
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium

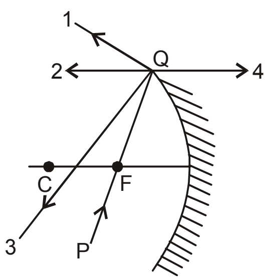
6. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will

(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

7. The phenomena involved in the reflection of radiowaves by ionosphere is similar to

(a) reflection of light by a plane mirror
(b) total internal reflection of light in air during a mirage
(c) dispersion of light by water molecules during the formation of a rainbow
(d) scattering of light by the particles of air

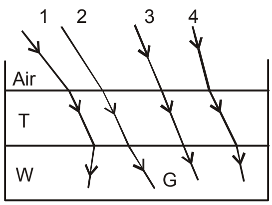
8. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray?



(a) 1 (b) 2 (c) 3 (d) 4

9. The optical density of turpentine is higher than tnat of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?

(a) 1 (b) 2 (c) 3 (d) 4



10. A car is moving with at a constant speed of 60 km h^{-1} on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 100 m and is approaching with a speed of 5 kmh^{-1}.

In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement (s) is/are correct?

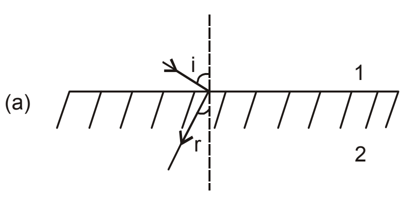
(a) The speed of the car in the rear is 65 km h^{-1}

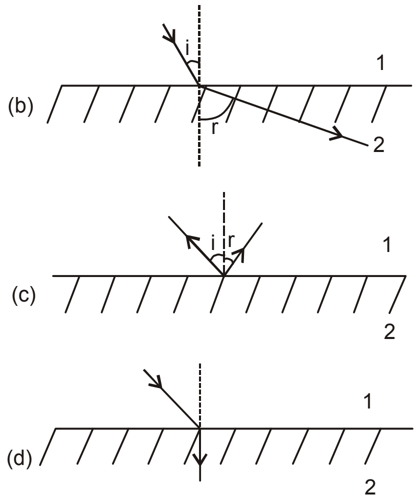
(b) In the side mirror, the car in the rear would appear to approach with a speed of 5 kmh^{-1} to the driver of the leading car

(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases

(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases

11. There are certain material developed in laboratories which have a negative refractive index figure. A ray incident from air (Medium 1) into such a medium (Medium 2) shall follow a path given by





Multiple Choice Questions (MCQs) (More than one option correct)

1. Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because

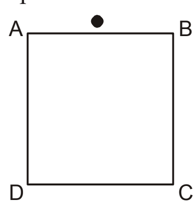
(a) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge

(b) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air

(c) some of the points of the object far away from the edge may not be visible because of total internal reflection

(d) water in a trough acts as a lens and magnifies the object

2. A rectangular block of glass ABCD has a refractive index 1.6. A pin is placed midway on the face AB figure. When observed from the face AD, the pin shall



(a) appear to be near A

(b) appear to be near D

(c) appear to be at the centre of AD

(d) not be seen at all

3. Between the primary and secondary rainbows, there is a dark band known as Alexandar's dark band. This is because
(a) light scattered into this region interfere destructively
(b) there is no light scattered into this region
(c) light is absorbed in this region
(d) angle made at the eye by the scattered rays with respect to the incident light of the sun lies between approximately 42° and 50°

4. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in
(a) a larger angle to be subtended by the object at the eye and hence, viewed in greater detail
(b) the formation of a virtual erect image
(c) increase in the field of view
(d) infinite magnification at the near point

5. An astronomical refractive telescope has an objective of focal length 20m and an eyepiece of focal length 2 cm.
(a) The length of the telescope tube is 20.02 m
(b) The magnification is 1000
(c) The image formed is inverted
(d) An objective of a larger aperture will increase the brightness and reduce chromatic aberration of the image

Very Short Answer Questions

1. Will the focal length of a lens for red light be more, same or less than that for blue light?

2. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope ?

3. An unsymmetrical double convex thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed?

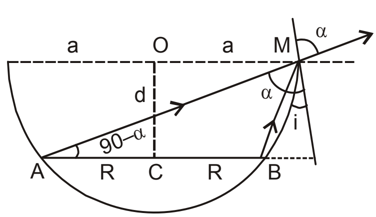
4. Three immiscible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $\frac{h}{3}$. A dot is made at the bottom of the beaker. For near normal vision, find the apparent depth of the dot.

5. For a glass prism $(\mu = \sqrt{3})$, the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.

Short Answer Questions

1. A short object of length L is placed aLong the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L << |v - f|$.

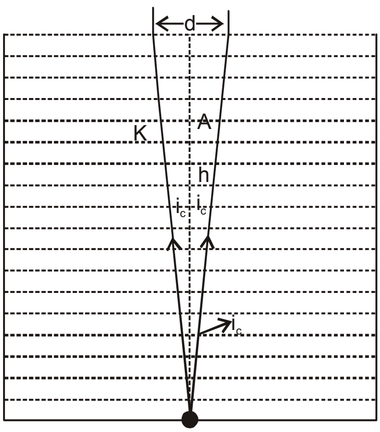
2. A circular disc of radius R is placed co-axially and horizontally inside an opaque hemispherical bowl of radius a figure. The far edge of the disc is just visible when viewed from the edge of the bowl. The bowl is filled with transparent liquid of refractive index μ and the near edge of the disc becomes just visible. How far below the top of the bowl is the disc placed?



3. A thin convex lens of focal length 25 cm is cut into two pieces 0.5cm above the principal axis. The top part is placed at (0, 0) and an object placed at (–50 cm, 0). Find the coordinates of the image.

4. In many experimental set-ups, the source and screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of the image sizes for these two points.

5. A jar of height h is filled with a transparent liquid of refractive index μ figure. At the centre of the jar on the bottom surface is a dot. Find the minimum diameter of a disc, such that when placed on the top surface



Long Answer Questions

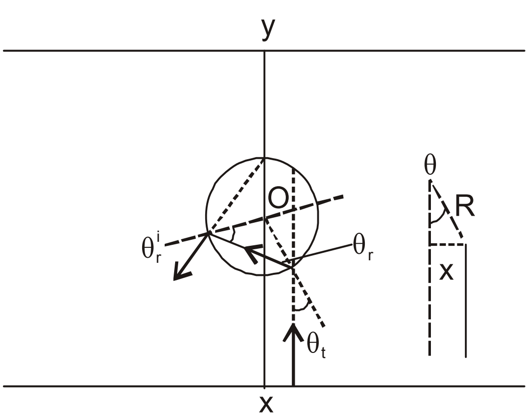
1. The mixture a pure liquid and a solution in a long vertical column (i.e., horizontal dimensions << vertical dimensions) produces diffusion of solute particles and hence a refractive index gradient along the vertical dimension. A ray of light entering the column at right angles to the vertical is deviated from its original path. Find the deviation in travelling a horizontal distance d << h, the height of the column.

2. If light passes near a massive object, the gravitational interaction causes a bending of the ray. This can be thought of as happening due to a change in the effective refractive index of the medium given by

$$n(r) = 1 + 2GM/rc^2$$

where r is the distance of the point of consideration from the centre of the mass of the massive body, G is the universal gravitational constant, M the mass of the body and c the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.

3. An infinitely long cylinder ot radius R is made of an unusual exotic material with refractive index-1 (figure). The cylinder is placed between two planes whose normals are along the y-direction. The centre of the cylinder O lies along the y-axis. A narrow laser beam is directed along the y-direction from the lower plate. The Laser source is at a horizontal distance x from the diameter in the y direction. Find the range of x such that light emitted from the lower plane does not reach the upper plane.



NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(a)** As we know that the deviation

$$\delta = (\mu - 1) A \quad \text{..... (i)}$$

By geometry, the angle of refraction by first surface is 5° and given $\mu = 1.5$

So, $\delta = (1.5 - 1) \times 5°$

$= 2.5°$

also, $\delta = \theta - r$, ... (ii)

By putting the value of δ and r in equation (ii)

$$2.5° = \theta - 5°$$

So, $\theta = 5 + 2.5 = 7.5°$

2. **(d)** As we know that when light ray goes from one medium to other medium, the frequency of light remains unchanged.

And, $c = \nu\lambda$

So, $c \propto \lambda$ the light of red colour is of highest wavelength and therefore of highest speed. Thus, after travelling through the slab, the red colour emerge first,

3. **(c)** According to the question, when object is at different position, and if an object approaches towards a convergent lens from the left of the lens with a uniform speed of 5 m/s, the image move away from the lens to infinity with a non-uniform acceleration.

4. **(b)** When a passenger in an aeroplane then he may see primary and secondary rainbow such as concentric circles.

5. **(c)** Among all given sources of light, the bule light have smallest wavelength.According to Cauchy relationship, smaller the wavelength higher the refractive index and consequently smaller the critical angle as $\mu = \frac{1}{\sin c}$.

Hence, corresponding to blue colour, the critical angle is least which facilitates total internal reflection for the beam of blue light and the beam of green light would also undergo total internal reflection.

6. **(c)** Using lens maker's formula for plano-convex lens, so focal length is

$$\frac{1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If object on curved suface

so, $R_2 = \infty$

then, $f = \frac{R_1}{(\mu_2 - \mu_1)}$

Lens placed in air, $\mu_1 = 1$.

(As given that, R = 20cm, $\mu_2 = 1.5$, on substituting the values in)

$$f = \frac{R_1}{\mu - 1} = \frac{20}{1.5 - 1} = 40 \text{ cm}$$

So, f is converging nature, as f > 0. Hence, lens will always act as a convex lens irrespective of the side on which the object lies.

7. **(b)** The reflection of radiowaves by ionosphere is similar to total internal reflection of light in air during a mirage because angle of incidence is greater than critical angle so that internal reflection of radio wave, take place.

8. **(b)** The incident PQ ray of light passes through focus F on the concave mirror, after reflection should become parallel to the principal axis, i.e., ray-2.

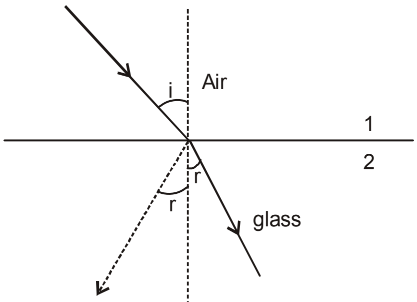
9. **(b)** As we know, when the ray goes from rarer medium air to optically denser turpentine, then it bends towards the normal i.e., i > r whereas when it goes from optically denser medium turpentine to rarer medium water, then it bends away from normal i.e., i < r.

So, the path of ray 2 is correct.

10. **(d)** As we know that, the image formed by convex mirror does not depend on the relative position of object w.r.t. mirror.

So, when the car approaches in the rear side, initially it appear at rest as images is formed at focus. Hence the speed of the image of the car would appear to increase as the distance between the cars decreases.

11. **(a)** When the negative refractive index materials are those in which incident ray from air (Medium 1) to them refract or bend differently to that of positive refractive index medium.



Multiple Choice Questions (More Than One Option)

1. **(a, b, c)**

When immersed object is seen from close to the edge of the trough the object looks distorted because the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge.

The angle subtended by an object is larger than the angle subtended by its image in water.

Some of the points of the object far away from the edge may not be visible because of the incidence angles for rays is more than critical angle can cause total internal reflection.

2. **(d)** As we know that

$$\therefore \quad \sin c = \frac{1}{\mu}$$

For $\mu = 1.6$, the critical angle, $\sin c = \frac{1}{1.6}$, we get $C = 38.7°$, when viewed from AD, as long as angle of incidence on AD of the ray emanating from pin is greater than the critical angle , the light suffers total internal reflection so object cannot be seen through AD or cannot be observed.

3. **(a, d)** Between the primary and secondary rainbows, there is a dark band known as Alexandar's dark, this forms due to light scattered into this region interfere destructively.
Since, primary and secondary rainbows subtends an angle nearly 41° to 42° and 51° to 54° respectively at observer's eye w.r.t. incident light ray. Then, the scattered rays with respect to the incident ray of the sun lies between approximately 42° and 50°.

4. **(a, b)**
In magnifying glass the object to be viewed can be brought closer to the eye than the normal near point because the results in a larger angle to be subtended by the object at the eye and so, viewed is greater. Morever, the formation of a virtual erect image enlarged and magnified image, takes place.

5. **(a, b, c)** As we know that,
the magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α subtended by object.

$$m = \frac{\beta}{\alpha} = \frac{h}{f_e}\frac{f_0}{h} \quad \text{(in normal adjustment)}$$

or $$m = \frac{f_0}{f_e}$$

In this case, the length of the telescope tube is $f_0 + f_e$,

So, $L = f_0 + f_e$

As given that, $f_0 = 20\,m$,

$f_e = 0.02\,m$

So, the length of the telescope tube is

$$(L) = f_0 + f_e = 20 + (0.02) = 20.02\,m$$

So, $$m = \frac{f_0}{f_e},$$

$$m = 20/0.02 = 1000$$

So, the final image formed is inverted and real.

Very Short Answer Questions

1. As we know that, the refractive index for red is less than that for blue, parallel beams of light incident on a lens will be bent more towards the axis for blue light compared to red.

So, $(\mu_b > \mu_r)$, so, $(\frac{1}{f})$ will be large for blue light and smaller for red coloured light.
By using lens maker's formula.

$$\frac{1}{f} = (\mu_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So, $f_b < f_r$

Hence, the focal length will be larger for red coloured light.

2. The least distance of distinct vision of an average person,
(i.e., D) is 25 cm
In order to view an object with magnification 10.
As given that, $v = D = 25$ cm
and $u = -f$,
Now, the magnification (m) is

$$m = \frac{v}{u}$$

$$m = \frac{-25}{-f} = \frac{25}{f}$$

So, $$f = \frac{D}{m} = \frac{25}{m} = \frac{25}{10} = 2.5 = 0.025\text{ m.}$$

Hence, power of lens $P = \frac{1}{0.025} = 40\,D$

3. Using lens Maker's formula,

$$\boxed{\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

Due to reversibility of the lens position of the image will not change

4. Consider the layer l_1. Let object is at its bottom at P then distance (X_1) of image of P by liquid of refractive index μ_1 from layer l_1 and apparent depth or X_1 with the medium μ_2 is :

$$X_1 = \frac{(-h/3)}{{}_2\mu_1} \quad \left(\because {}_2\mu_1 = \frac{\mu_1}{\mu_2}\right)$$

$$X_1 = \frac{\mu_2}{\mu_1}\frac{h}{3}$$

or $$X_1 = \frac{h\mu_2}{3\mu_1}$$

(Since, apparent depth = real depth/refractive index μ)

Since, the image formed by Medium μ_1 then X_2 act as an object for Medium μ_2.

If seen from μ_3, the apparent depth is X_2

Similarly, the image formed by Medium μ_2 then X_2 act as an objects Medium μ_3.

$$\text{Real depth} = \left(\frac{h}{3} + X_1\right)$$

$$X_2 = \frac{\text{Real depth}}{3\mu_2}$$

$$= \frac{\text{Real depth}}{(\mu_2/\mu_3)}$$

So, $$X_2 = \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + X_1\right)$$

$$= \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + \frac{\mu_2 h}{\mu_1 3}\right)$$

$$= \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_2}{\mu_1}\right)$$

When we seen from outside, the apparent height is

$$X_3 = \frac{\mu_0}{\mu_3}\left(\frac{h}{3} + X_2\right)$$

$$= \frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right]$$

So, $$X_3 = \frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$$

This is the required apparent depth of dot placed.

5. As we know that, the relationship between refractive index, prism angle A and angle of minimum deviation is

$$\mu = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

As given that,

$$\delta_m = A$$

By putting the value of (δ_m) in above expression,

$$\mu = \frac{\sin A}{\sin \frac{A}{2}}$$

or, $$\mu = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

For the given value of refractive index

$$2\cos\frac{A}{2} = \sqrt{3}$$

or, $$\cos\frac{A}{2} = \frac{\sqrt{3}}{2} = \cos 30°$$

So, $$A = 60°$$

Hence, the angle of the prism is 60°.

Short Answer Questions

1. As we know that, the length of image is the difference between the images formed by mirror of the extremities of object.

As the object distance is u, consider the two ends of the object, be at distance $u_1 = (u - L/2)$ and $u_2 = (u + L/2)$ respectively so that $|u_1 - u_2| = L$

Consider the image of the two ends be formed at v_1 and v_2, respectively so that the image length on principal axis would be

$$L' = |v_1 - v_2| \quad \text{... (i)}$$

By using mirror formula, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or, $$v = \frac{fu}{u - f} \quad \text{... (ii)}$$

So, $$v_1 = \left(\frac{fu_1}{u_1 - f}\right) \quad \text{... (iii)}$$

and $$v_2 = \left(\frac{fu_2}{u_2 - f}\right) \quad \text{... (iv)}$$

To find the value of v_1 and v_2, put the value of u_1 and u_2 in (iii) and (iv) equations respectively. By solving, the positions of two images are

$$v_1 = \frac{f(u - L/2)}{u - f - L/2},$$

$$v_2 = \frac{f(u + L/2)}{u - f + L/2},$$

For image length, substituting the value of v_1 and v_2 in equation (i), we get,

$$L' = |v_1 - v_2|$$

$$\left[L' = \frac{f^2 L}{\left((u-f)^2 - L^2/4\right)}\right].$$

Since, the object is short and kept away from focus, we get

$\therefore$ $L << (u - f)$ or $(L^2/4) << (u - f)^2$

Thus, neglecting the term $(L^2/4)$

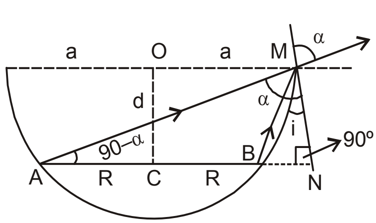
So, $$\boxed{L' = \frac{f^2}{(u-f)^2} L}$$

This is the required expression of length of image.

2. According to the given figure, AM is the direction of incidence ray before liquid is filled. After liquid is filled in, BM is the direction of the incident ray. Refracted ray in both cases is same as that along AM. MN is tangent at M, so $MN \perp AB$

i.e., $\angle N = 90°$

Consider the disc is separated by O at a distance d as shown in figure given below,



Now let us consider an angle at

$$(N) = 90°,$$

$$OM = a$$

$$NB = a - R,$$

$$MB = \sqrt{d^2 + (a - R)^2}$$

From, above figure,

$$\sin i = \left[\frac{NB}{MB}\right] = \frac{a - R}{\sqrt{d^2 + (a - R)^2}}$$

$$AN = AB + BN = 2R + a - R = (a + R)$$

$$AM = \sqrt{d^2 + (a + R)^2}$$

And, $$\sin r = \cos(90 - \alpha)$$

$$= \left[\frac{AN}{AM}\right] = \frac{a + R}{\sqrt{d^2 + (a + R)^2}}$$

Now by applying Snell's law,

So, $${}_1\mu_0 = \frac{\mu_0}{\mu_1} = \frac{\sin i}{\sin r}$$

(μ_0 for air = 1) & (μ_1 for liquid)

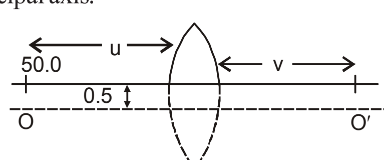
So, $$\frac{1}{\mu} = \frac{\sin i}{\sin r}$$

By putting the values of sin i & sin r we get the distance (d) is

$$d = \frac{\mu(a^2 - b^2)}{\sqrt{(a + r)^2 - \mu(a - r)^2}}$$

This is the required expression.

3. As we know that, there is no effect on the focal length of the lens if it is cut by plane parallel to principal axis.



When there was no cut, then the object would has been at a height of 0.5 cm from the principal axis OO'.

As given that, $u = -50$ cm,

$f = 25$ cm

$V = ?$

By using lens formula, we get

$$\frac{1}{f} = \left(\frac{1}{v} - \frac{1}{u}\right)$$

or, $$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{-50} + \frac{1}{25} = \frac{1}{50}$$

$\therefore$ Mangnification is $m = \frac{v}{u} = -\frac{50}{50} = -1$

So, the size of image is equal to the object, m is negative so image is inverted. Therefore, the image would have been formed at 50 cm from the pole and 0.5 cm below the principal axis. Hence, with respect to the X-axis passing through the edge of the cut lens, the coordinates of the image are (50 cm, –1 cm).

4. The Principal of reversibility states that the position of object and image are interchangeable. Then, by the versibility of u and v, as seen from the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \text{..... (i)}$$

It shows that there are two positions for which there shall be an image.

Consider the first position be when the lens is at O.

As given that,

$$-u + v = D$$

$$\Rightarrow \quad u = -(D - v) \qquad \text{..... (ii)}$$

By putting the value of u in equation (i),

so, $$\frac{1}{(D-v)} + \frac{1}{v} = \frac{1}{f}$$

or $$\frac{v + D - v}{(D-v)v} = \frac{1}{f}$$

$$v^2 - Dv + Df = 0$$

To find the coordinate of this equation then using

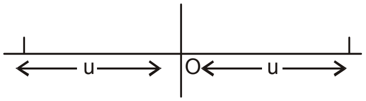
$$x = \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$v = \frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}$$

By putting the value of v in equation (ii) then,

$$u = -\left[D - \left[\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right]\right]$$

So, $$u = -\left(\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right) \qquad \text{... (iii)}$$



From (iii)

When, the position of object distance is

$$u_2 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$$

The image forms at

$$v_2 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$$

Similarly, when the position of the object distance is

$$u_1 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$$

The image forms at

$$v_1 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$$

The distance between the poles for these two positions of lens (d) = $v_1 - v_2$

$$d = \left[\frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}\right] . \left[\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}\right]$$

So, $$d = \sqrt{D^2 - 4Df}$$

In first case :

If $u_2 = \left[\frac{D}{2} + \frac{d}{2}\right]$, then the image is at

$$v_2 = \left[\frac{D}{2} - \frac{d}{2}\right]$$

So, the magnification $m_2 = \frac{v_2}{u_2} = \frac{\frac{D}{2} - \frac{d}{2}}{\frac{D}{2} + \frac{d}{2}}$

$$m_2 = \frac{D - d}{D + d}$$

In second case :

If $\left[u_1 = \frac{D - d}{2}\right]$, then the image is

$$v_1 = \left[\frac{D+d}{2}\right]$$

∴ The magnification (m_1) is

$$m_1 = \left|\frac{v_1}{u_1}\right| = \frac{(D+d)\times 2}{2\times(D-d)}$$

$$m_1 = \frac{D+d}{D-d}$$

Hence, $$\frac{m_1}{m_2} = \left(\frac{D+d}{D-d}\right)^2$$

5. According to the given figure, consider d be the diameter of the disc. The spot shall be invisible if the incident rays from the dot at O to the surface at d/2 at the critical angle.

As we know that, the relationship between refractive index and critical angle then,

$$\sin i_c = \frac{1}{\mu} = \frac{L}{K}$$

So, $L = 1, K = \mu$ and $A = \sqrt{\mu^2 - 1}$

By using geometry and trigonometry, for incidence angle,

Now, $$\tan i_c = \left[\frac{\left(\frac{d}{2}\right)}{h}\right]$$

or, $$\frac{d/2}{h} = \tan i_c \qquad \left[\tan i_c = \frac{L}{A}\right]$$

So, $$\tan i_c = \frac{1}{\sqrt{\mu^2-1}}$$

$$\frac{d}{2} = h \tan i_c$$

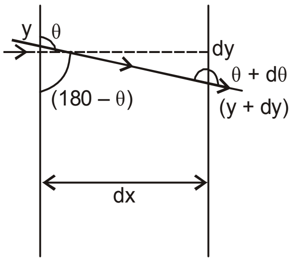
$$= h\left[\sqrt{\mu^2-1}\right]^{-1}$$

So, $$d = \frac{2h}{\sqrt{\mu^2-1}}$$

This is the minimum diameter.

Long Answer Questions

1. Consider a portion of a ray between x and x + dx inside a long vertical column of transparent liquid and the angle of incidence at x be θ and let it enter the thin column at height y. Because of the bending it shall emerge at (x + dx) with an angle (θ + dθ) and at a height (y + dy).



From Snell's law,

$$\mu(y)\sin\theta = \mu(y+dy)\sin(\theta+d\theta)$$

or, $$\mu(y)\sin\theta = \left(\mu(y) + \frac{d\mu}{dy}dy\right) \times (\sin\theta\cos d\theta + \cos\theta\sin d\theta)$$

As dμ, dθ are very small

So, $$\sin d\theta = d\theta$$

$$\mu(y\sin\theta) = \mu(y)\sin\theta + \mu(y)\cos\theta d\theta + \frac{d\mu}{dy}dy\sin\theta$$

$$\mu(y)\cos\theta d\theta = \frac{-d\mu}{dy}dy\sin\theta$$

$$d\theta = \frac{-d\mu}{\mu dy}dy\tan\theta$$

From figure : $$\tan\theta = \frac{dx}{dy}$$

By solving, we get,

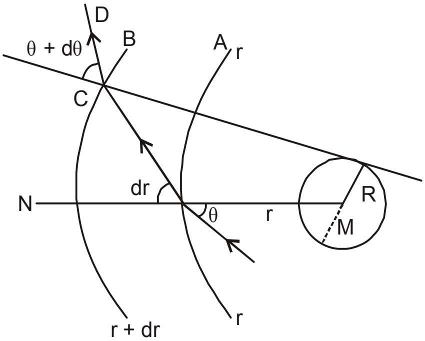
$$d\theta = \frac{-1d\mu}{\mu dy}dx$$

Integration on both sides

$$\int_0^\theta \theta = \frac{-1d\mu}{\mu dy}\int_0^d dx = \frac{-1d\mu}{\mu dy}d$$

∴ $$\boxed{\theta = \frac{-1}{\mu}\cdot\frac{d\mu}{dy}d}$$

2. Consider two planes A and B at r and (r + dr) distance from the center of massive object of mass M and let the ray be incident at an angle θ at the plane at r and leave r + dr at an angle $\theta + d\theta$.



By Snell's law,

$n(r)\sin\theta \cong n(r+dr)\sin(\theta+d\theta)$

$$n(r)\sin\theta \cong \left(n(r)+\frac{dn}{dr}dr\right)\times(\sin\theta\cos d\theta + \cos\theta\sin d\theta)$$

$$n(r)\sin\theta \cong \left(n(r)+\frac{dn}{dr}dr\right)(\sin\theta+\cos\theta d\theta)$$

$[\because d\theta \to 0$ then $\cos d\theta = 1$ and $(\sin d\theta = d\theta)]$

Ignoring the product of differentials

$$n(r)\sin\theta = n(r)\sin\theta + r\sin\theta\left(\frac{dn}{dr}\right) + n(r)\cos\theta\left(\frac{d\theta}{dr}\right)+\left(\frac{dn}{dr}\right)\cos\theta(d\theta)$$

or, $$-\frac{dn}{dr}\tan\theta = n(r)\frac{d\theta}{dr}$$

$$\left[\text{given } n(r) = 1+\frac{2GM}{rc^2}\right]$$

$$\frac{2GM}{r^2c^2}\tan\theta = \left(1+\frac{2GM}{rc^2}\right)\frac{d\theta}{dr} \approx \frac{d\theta}{dr}$$

As the G is very small and c^2 is very large

So, $$\frac{2GM}{rc^2} \to 0$$

$$\frac{2GM}{r^2c^2}\tan\theta = \frac{d\theta}{dr}$$

intigrating both sides,

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{\tan\theta dr}{r^2} \quad \text{... (i)}$$

Now substitution for integrals, we get

$$r^2 = x^2 + R^2 \text{ and } \tan\theta = \frac{R}{x}$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{1}{r^2}\frac{R}{r}dx = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{Rx}{xr^3}dx$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{R}{x}\frac{xdx}{(x^2+R^2)^{3/2}}$$

$x = R\tan\phi$ (putting)

$dx = R\sec^2\phi d\phi$

$$\theta_0 = \frac{2GMR}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R\sec^2\phi d\phi}{R^3\sec^3\phi}$$

$$= \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\phi d\phi = \frac{4Gm}{Rc^2}$$

or, $$\boxed{\theta_0 = \frac{4GM}{Rc^2}}$$

3. As, the cylinder is made of material of refractive index (–1) and is placed in air of ($\mu = 1$), so, when ray AB is incident at B to cylinder, θ_r, is negative and θ'_t positive.

Now, $|\theta_t| = |\theta_r| = |\theta'_r|$

The total deviation of the outcoming ray from the incoming ray is $4\theta_t$. Rays shall not reach the recieving plane if

$$\frac{\pi}{2} \le 4\theta_t \le \frac{3\pi}{2}$$

So, angles measured clockwise from the y-axis. On dividing by 4 to all sides

By solving, $$\frac{\pi}{8} \le \theta_t \le \frac{3\pi}{8}$$

Now, $$\sin\theta_t = \frac{x}{R}$$

$$\frac{\pi}{8} \le \sin^{-1}\frac{x}{R} \le \frac{3\pi}{8}$$

or, $$\frac{\pi}{8} \le \frac{x}{R} \le \frac{3\pi}{8}$$

Hence, for light emitted from the source shall not reach the receiving plane. If $\frac{R\pi}{8} \le x \le \frac{R3\pi}{8}$.

Chapter Ten

WAVE OPTICS

SUMMARY

3. The principle of superposition of waves applies whenever two or more sources of light illuminate the same point. When we consider the intensity of light due to these sources at the given point, there is an interference term in addition to the sum of the individual intensities. But this term is important only if it has a non-zero average, which occurs only if the sources have the same frequency and a stable phase difference.
4. Young's double slit of separation d gives equally spaced fringes of angular separation λ/d. The source, mid-point of the slits, and central bright fringe lie in a straight line. An extended source will destroy the fringes if it subtends angle more than λ/d at the slits.
5. A single slit of width a gives a diffraction pattern with a central maximum. The intensity falls to zero at angles of $\pm\frac{\lambda}{a}, \pm\frac{2\lambda}{a}$, etc., with successively weaker secondary maxima in between. Diffraction limits the angular resolution of a telescope to λ/D where D is the diameter. Two stars closer than this give strongly overlapping images. Similarly, a microscope objective subtending angle 2β at the focus, in a medium of refractive index n, will just separate two objects spaced at a distance $\lambda/(2n \sin \beta)$, which is the resolution limit of a microscope. Diffraction determines the limitations of the concept of light rays. A beam of width a travels a distance a^2/λ, called the Fresnel distance, before it starts to spread out due to diffraction.
6. Natural light, e.g., from the sun is unpolarised. This means the electric vector takes all possible directions in the transverse plane, rapidly and randomly, during a measurement. A polaroid transmits only one component (parallel to a special axis). The resulting light is called linearly polarised or plane polarised. When this kind of light is viewed through a second polaroid whose axis turns through 2π, two maxima and minima of intensity are seen. Polarised light can also be produced by reflection at a special angle (called the Brewster angle) and by scattering through $\pi/2$ in the earth's atmosphere.

POINTS TO PONDER

1. Waves from a point source spread out in all directions, while light was seen to travel along narrow rays. It required the insight and experiment of Huygens, Young and Fresnel to understand how a wave theory could explain all aspects of the behaviour of light.
2. The crucial new feature of waves is interference of amplitudes from different sources which can be both constructive and destructive, as shown in Young's experiment.
3. Even a wave falling on single slit should be regarded as a large number of sources which interefere constructively in the forward direction ($\theta = 0$), and destructively in other directions.
4. Diffraction phenomena define the limits of ray optics. The limit of the ability of microscopes and telescopes to distinguish very close objects is set by the wavelength of light.
5. Most interference and diffraction effects exist even for longitudinal waves like sound in air. But polarisation phenomena are special to transverse waves like light waves.

EXERCISES

10.1 Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

10.2 What is the shape of the wavefront in each of the following cases:

(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

10.3 (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×10^8 m s^{-1})

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

10.4 In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

10.5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\lambda/3$?

10.6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

10.7 In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

10.8 What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

10.9 Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

10.10 Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm.

ADDITIONAL EXERCISES

10.11 The 6563 Å Hα line emitted by hydrogen in a star is found to be red-shifted by 15 Å. Estimate the speed with which the star is receding from the Earth.

10.12 Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

10.13 You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

10.14 Let us list some of the factors, which could possibly influence the speed of wave propagation:

(i) nature of the source.

(ii) direction of propagation.

(iii) motion of the source and/or observer.

(iv) wavelength.

(v) intensity of the wave.

On which of these factors, if any, does

(a) the speed of light in vacuum,

(b) the speed of light in a medium (say, glass or water),

depend?

10.15 For sound waves, the Doppler formula for frequency shift differs slightly between the two situations: (i) source at rest; observer moving, and (ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

10.16 In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

10.17 Answer the following questions:

(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

(d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.

(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

10.18 Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

10.19 A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.

10.20 Answer the following questions:

(a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

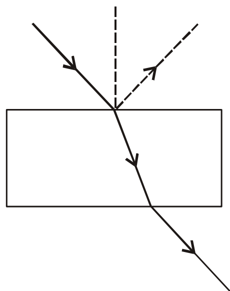
10.21 In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $n\lambda/a$. Justify this by suitably dividing the slit to bring out the cancellation.

Chapter 10:- Wave Optics

Multiple Choice Questions (MCQs)

1. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure.

A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.



(a) For a particular orientation, there shall be darkness as observed through the polaroid

(b) The intensity of light as seen through the polaroid shall be independent of the rotation

(c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid

(d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid

2. Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall

(a) be a fine sharp slit white in colour at the centre

(b) a bright slit white at the centre diffusing to zero intensities at the edges

(c) a bright slit white at the centre diffusing to regions of different colours

(d) only be a diffused slit white in colour

3. Consider a ray of light incident from air onto a slab of glass (refractive index n) of width d, at an angle θ. The phase difference between the ray reflected bv the top surface of the glass and the bottom surface is

(a) $\frac{2\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}+\pi$

(b) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}$

(c) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}+\frac{\pi}{2}$

(d) $\frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{1/2}+2\pi$

4. In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case,

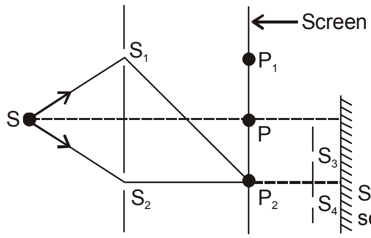
(a) there shall be alternate interference patterns of red and blue

(b) there shall be an interference pattern for red distinct from that for blue

(c) there shall be no interference fringes

(d) there shall be an interference pattern for red mixing with one for blue

5. Figure shows a standard two slit arrangement witn slits S_1, S_2, P_1, P_2 are the two minima points on either side of P (figure).



At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them.

(a) There would be no interference pattern on the second screen but it would be lighted

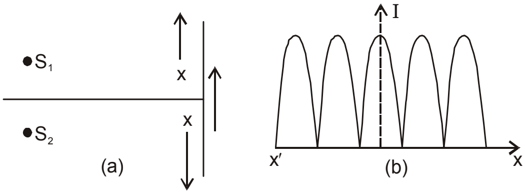
(b) The second screen would be totally dark

(c) There would be a single bright point on the second screen

(d) There would be a regular two slit pattern on the second screen

Multiple Choice Questions (MCQs) (More Than One Option Correct)

1. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [Fig. (a)]. The pattern of intensity distribution seen in the central portion is given by Fig. (b).



In this case, which of the following statements are true?
   (a) S_1 and S_2 have the same intensities
   (b) S_1 and S_2 have a constant phase difference
   (c) S_1 and S_2 have the same phase
   (d) S_1 and S_2 have the same wavelength

2. Consider sunlight incident on a pinhole of width 10^3 Å. The image of the pinhole seen on a screen shall be
   (a) a sharp white ring
   (b) different from a geometrical image
   (c) a diffused central spot, white in colour
   (d) diffused coloured region around a sharp central white spot

3. Consider the diffraction pattern for a small pinhole. As the size of the hole is increased
   (a) the size decreases
   (b) the intensity increases
   (c) the size increases
   (d) the intensity decreases

4. For light diverging from a point source,
   (a) the wavefront is spherical
   (b) the intensity decreases in proportion to the distance squared
   (c) the wavefront is parabolic
   (d) the intensity at the wavefront does not depend on the distance

Very Short Answer Questions

1. Is Huygen's principle valid for longitudinal sound waves?

2. Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image?

3. What is the shape of the wavefront on earth for sunlight?

4. Why is the diffraction of sound waves more evident in daily experience than that of light wave?

5. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ rad and a typical photoprinter prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance z should a printed page be held so that one does not see the individual dots.

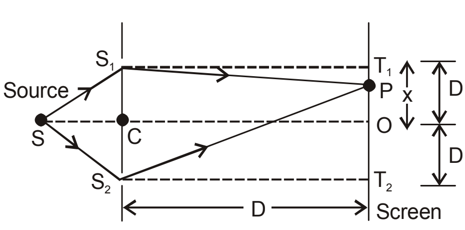
6. A polaroid (I) is placed infront of a monochromatic source. Another polariod (II) is placed in front of this polaroid (I) and rotated till no light passes. A third polaroid (III) is now placed in between (I) and (II). In this case, will light emerge from (II). Explain.

Short Answer Questions

1. Can reflection result in plane polarised light if the light is incident on the interface from the side with higher refractive index?

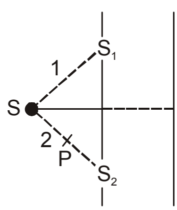
2. For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of 5000 Å and electrons accelerated through 100 V used as the illuminating substance.

3. Consider a two slit interference arrangements (figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minima on the screen falls at a distance D from the centre O.

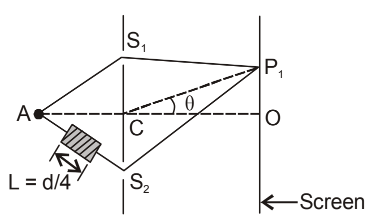


Long Answer Questions

1. Figure shown a two slit arrangement with a source which emits unpolarised light. P is a polariser with axis whose direction is not given. If I_0 is the intensity of the principal maxima when no polariser is present, calculate in the present case, the intensity of the principal maxima as well as of the first minima.



2.



$AC = CO = D, S_1C = S_2C = d << D$

A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (figure). What will be the distance from O of the principal maxima and of the first minima on either side of the principal maxima obtained in the absence of the glass slab?

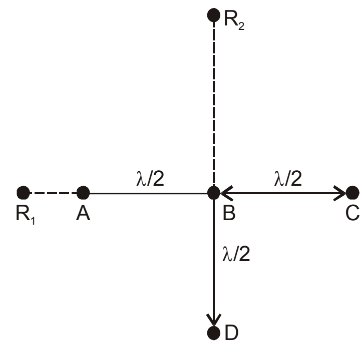
3. Four identical monochromatic sources A, B, C, D as shown in the (figure) produce waves of the same wavelength λ and are coherent. Two receiver R_1 and R_2 are at great but equal distances from B.

(i) Which of the two receivers picks up the larger signal?

(ii) Which of the two receivers picks up the larger signal when B is turned off?

(iii) Which of the two receivers picks up the larger signal when D is turned off?

(iv) Which of the two receivers can distinguish which of the sources B or D has been turned off?



$R_1B = d = R_2B$

$AB = BC = BD = \lambda/2$

4. The optical properties of a medium are governed by the relative permittivity (ε_r) and relative permeability (μ_r). The refractive index is defined as $\sqrt{\mu_r \varepsilon_r} = \mu$. For ordinary material, $\varepsilon_r > 0$ and $\mu_r > 0$ and the positive sign is taken for the square root.

In 1964, a Russian scientist V. Veselago postulated the existence of material with $\varepsilon_r < 0$ and $\mu_r < 0$. Since, then such metamaterials have been produced in the laboratories and their optical properties studied. For such materials $\mu = -\sqrt{\mu_r \varepsilon_r}$. As light enters a medium of such refractive index the phases travel away from the direction of propagation.

(i) According to the description above show that if rays of light enter such a medium from air (refractive index = 1) at an angle θ in 2nd quadrant, then the refracted beam is in the 3rd quadrant.

(ii) Prove that Snell's law holds for such a medium.

5. To ensure almost 100% transmittivity, photographic lenses are often coated with a thin layer of dielectric material. The refractive index of this material is intermediated between that of air and glass (which makes the optical element of the lens). A typically used dielectric film is MgF_2 (n = 1.38). What should the thickness of the film be so that at the centre of the visible spectrum (5500 Å) there is maximum transmission.

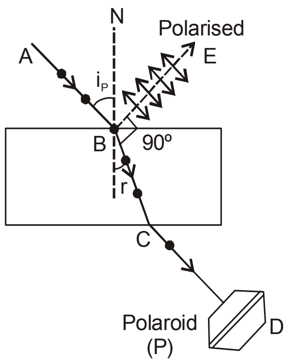
NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(c)** Let us consider the diagram shown below the light beam incident from air to the glass slab at Brewster's angle (i_p). The angle between reflected ray BE and BC is 90°.

Then only reflected ray is plane polarised represented by arrows.

As the emergent and incident ray are unpolarised, so, polaroid rotated in the way of CD then the intensity cannot be zero but varies in one complete rotation.



2. **(a)** As given that the width of the slit

$=10^4$ Å $= 10000$ Å

$= 10^4 \times 10^{-10}\,\text{m} = 10^{-6}\,\text{m} = 1\,\mu\text{m}$

Wavelength of visible sunlight varies from 4000 Å to 8000 Å.

Thus the width of slit is 10000 Å comparable to that of wavelength visible light i.e., 8000 Å. So diffraction occurs with maxima at centre. Hence at the centre all colours appear i.e., mixing of colours form white patch at the centre.

3. **(a)** Let, us consider the diagram, the ray (P) is incident at an angle θ and gets reflected in the direction P' and refracted in the direction P' through O'. Due to reflection from the glass medium there is a phase change of π. The time difference between two refracted ray OP' and O'P'' is equal to the time taken by ray to travel along OO'.

$$\Delta t = \frac{OO'}{V_g} = \frac{d/\cos r}{c/n} = \frac{nd}{c\cos r}$$

From Snell's law, $n = \dfrac{\sin\theta}{\sin r}$

$$\sin r = \frac{\sin\theta}{n}$$

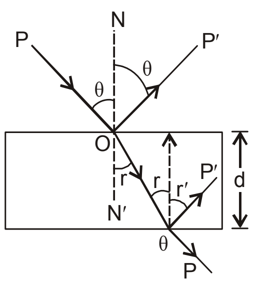
As we know that,

$$\cos r = \sqrt{1-\sin^2 r},$$

so by putting sin r value in that relation.

So, $\cos r = \sqrt{1-\sin^2 r}$

$$\cos r = \sqrt{1-\frac{\sin^2\theta}{n^2}}$$



$$\therefore \quad \Delta t = \frac{nd}{c\left(1-\dfrac{\sin^2\theta}{n^2}\right)^{1/2}}$$

$$= \frac{nd}{c}\left(1-\frac{\sin^2\theta}{n^2}\right)^{-1/2}$$

$$\text{Phase difference} = \Delta\phi = \frac{2\pi}{T}\times\Delta t$$

$$= \frac{2\pi d}{\dfrac{1}{\nu}\cdot\nu\lambda}\left(1-\frac{\sin^2\theta}{n^2}\right)^{-1/2}$$

$$\Delta\phi = \frac{2\pi d}{\lambda}\left[1-\frac{\sin^2\theta}{n^2}\right]^{-1/2}$$

∴ Hence the net phase difference $= \Delta\phi + \pi$

$$= \frac{2\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{-1/2} + \pi$$

4. **(c)** For sustained interference pattern to be formed on the screen, the sources must be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present which has different frequency. In this monochromatic light is used for the formation of fringes on the screen. So, in that case there shall be no interference fringes.

5. **(d)** Consider the given figure there is a hole at point P_2. By Huygen's principle, wave will propagates from the sources S_1 and S_2. Each point on the screen will acts as sources of secondary wavelets.

Wavefront starting from P_2 reaches at S_3 and S_4 which will again act as two monochromatic or coherent sources.

Hence, there will be always a regular two slit pattern on the second screen.

Multiple Choice Questions (More Than One Options)

1. **(a, b, d)**

Consider the pattern of the intensity shown in the figure

(i) As the intensities of dark fringe is zero, hence we can say that two sources S_1 and S_2 are having same intensities.

(ii) As width of the successive maxima and minima are symmetric, so the wave from S_1 and S_2 are have same or constant phase difference, so, maxima (puises) increases in continuous manner, we can say that the path difference (x) or phase difference varies in continuous manner.

(iii) As we are using monochromatic light in Young's double slit experiment to avoid overlapping and we have very clear pattern on the screen.

2. **(b, d)** As given that,

The width of pinhole $= 10^3$ Å $= 1000$ Å

As we know that,

Wavelength of visible light in sunlight ranges varies from 4000 Å to 8000 Å

So, clearly, wavelength $\lambda <$ width of the slit

So, light from pinhole will diffracted from the hole. Due to diffraction pattern from the slight or fringes the image formed on the screen will be different from the geometrical image.

3. **(a, b)** We know that width (B_0) of central maxima

$$\beta_0 = \frac{D\lambda}{d}$$

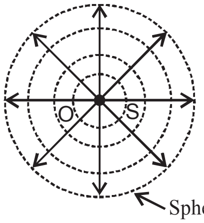
where D is the distance between slit and screen λ is the wave length of source does not change.

(a) When increases the width of the hole 'd' increase so, size decrease.

(b) When, size of hole is decrease, then the light energy is distributed over a small area and intensity $\propto \frac{1}{\text{area}}$ as area is decreasing so intensity increases.

4. **(a, b)**

Let us consider the diagram in which light diverges from a point source (O).



Due to the light from point source propagates in all around the source with same speed hence, wavefront will be spherical surface of wavefront.

If power of the source is P, then intensity of the source will be

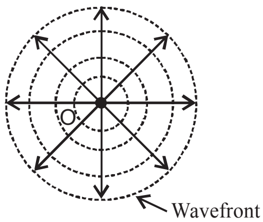
$$I = \frac{P}{4\pi r^2}$$

where, r is radius of the spherical wavefront at any time

So, the intensity (I) always decrease as reciprocal of square of distance.

Very Short Answer Questions

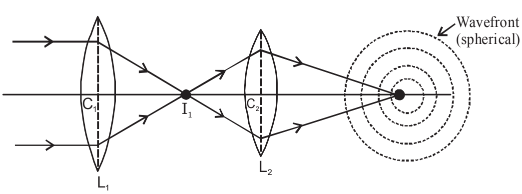
1. Considering a point source of sound wave and rarefactions forward in all direction with same velocity. So, the logitudinal waves due to the source propagates in spherical symmetry that is in all direction. The formation of wavefront is in accordance with Huygen's principle.



On a surface of sphere there will be either compression or rarefaction and that part can also behave like a source of sound but with low intensity.

Hence, Huygen's principle is valid for longitudinal sound waves also.

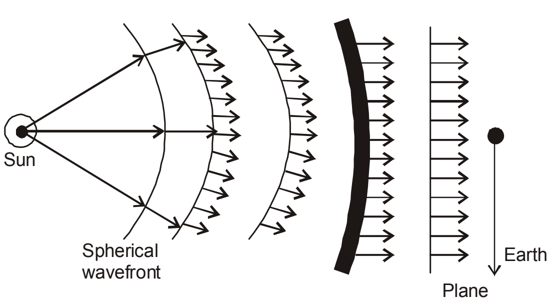
2. Let us consider the figure shown below



Due to the converging lense L_1 is at the focal point the point image formed I_1 and due to the converging lense L_2, the final image formed is I which is point image, so it behave like a point source of rays hence the wavefront for this image will be spherical symmetry.

3. As we know that, the sun is very far from the earth. Considering sun as spherical, it can be considered as point source situated at infinity.

Due to the very large distance the radius of wavefront can be considered as large infinity and so, wavefront is plane.



4. We know that

The frequencies of sound waves lie between 20 Hz to 20 kHz so its corresponding wavelength ranges varies between 15 m to 15 mm respectively. The diffraction occur if the wavelength of waves is nearly equal to slit width. But, the wavelength of visible light waves varies from is 7000×10^{-10} m to 4000×10^{-10} m which is very small and the slit width is very near to the wavelength of sound waves as compared to light waves. Hence, the diffraction of sound waves is more evident in daily life than that of light waves.

5. As given that, the angular resolution of human eye,

$$\phi = 5.8 \times 10^{-4} \text{ radian.}$$

Photo printer prints = 300 dots per inch

So, the average linear distance between any two dots is

$$l = \frac{2.54}{300} \text{ cm}$$

$$= 0.85 \times 10^{-2} \text{ cm.}$$

Now, at a distance of z cm, subtend angle,

$$\phi = \frac{l(\text{Arc})}{z(\text{radian})}$$

$$= \frac{0.85 \times 10^{-2}}{z}$$

So, maximum distance up to which human eye cannot see 2 dots distinctly.

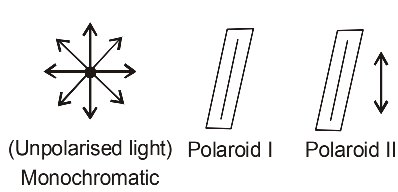
$$z = \frac{l}{\phi}$$

$$= \frac{0.85 \times 10^{-2} \text{ cm}}{5.8 \times 10^{-4}}$$

$$= 14.5 \text{ cm}$$

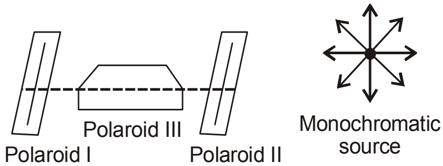
Which is less than distance of distinct vision so a normal person can not see the dots.

6. When a monochromatic light is placed infront of polaroid (I) as shown in diagram



According to question, monochromatic light emerging from polaroid (I) is plane polarised.

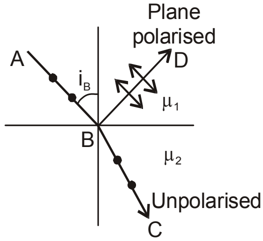
When polaroid (II) is placed infront of this polaroid (I), and rotated till no light passes through polaroid (ii), then (i) and (ii) are set in crossed positions, i.e., pass axes of I and II are at 90°.



From the above figure, where a third polaroid (III) is placed between polaroid (I) and polaroid II. When a third polaroid (III) is placed in between (I) and (II), no light will emerge from (II), if pass axis of (III) is parallel to pass axis of (I) or (II) (when $\theta = 0°$). In all other cases, light will emerge from (I). as pass axis of (II) will no longer be at 90° to the pass axis of (III).

Short Answer Questions

1. When a ray of light passes from a medium (air) of refractive index μ_1 to another medium of refractive index μ_2 less then the μ_1 with angle of incidence is equal to Brewster's angle, the transmitted light is unpolarised and reflected light is plane polarised.



Consider the diagram in which unpolarised light is represented by dot and plane polarised light is represented by arrows.

Polarisation by reflection occurs when the angle of incidence is the Brewster's angle

So, $$\tan i_B = {}^1\mu_2 = \frac{\mu_2}{\mu_1}$$

where, $\mu_2 < \mu_1$

So, when the light rays travels in such a medium, the critical angle is

$$\sin i_C = \frac{\mu_2}{\mu_1}$$

Where, $\mu_2 < \mu_1$

As $|\tan i_B| > |\sin i_c|$ for large angle $i_B < i_C$.

Hence, the polarisation by reflection occurs definitely.

2. As we know that,

The resolving power of microscope is

$$(P) = \frac{1}{d} = \frac{2\sin\beta}{1.22\lambda}$$

Where, λ is the wavelength of light and β is the angle subtended by the objective at the object.

So, $$\frac{1}{d_{min}} = \frac{2\sin\beta}{1.22\lambda}$$

$$\Rightarrow \quad d_{min} = \frac{1.22\lambda}{2\sin\beta}$$

As we know,

For the light of wavelength 5500 Å

$$\lambda = 5000 \times 10^{-10}\,m$$

So, $$d_{min} = \frac{1.22 \times 5000 \times 10^{-10}}{2\sin\beta} \quad ...(i)$$

For electrons accelerated through 100 V, the de-Broglie wavelength (λ_d) of illuminated light is

$$\lambda_d = \frac{12.2}{\sqrt{V}}\,nm = \frac{12.2}{\sqrt{100}}\,nm = 0.12 \times 10^{-9}\,m$$

The limit of resolution by 100 V light d'_{min}

$$d'_{min} = \frac{1.22\,\lambda_d}{2\sin\beta}$$

$$d'_{min} = \frac{1.22 \times 0.12 \times 10^{-9}}{2\sin\beta}$$

The required ratio of the least separation

So, $$\frac{d'_{min}}{d_{min}} = \frac{1.22 \times 0.12 \times 10^{-10}}{1.22 \times 5000 \times 10^{-10}}$$

$$\frac{d'_{min}}{d_{min}} = \frac{0.12 \times 10^{-9}}{5000 \times 10^{-10}}$$

$$= 0.2 \times 10^{-3}$$

3. As the given figure of two slit interference arrangements, we can write

$$T_2P = T_2O + OP = D + x \quad (\because OP = x)$$

and $$T_1P = T_1O - OP = D - x$$

$$S_1P = \sqrt{(S_1T_1)^2 + (PT_1)^2}$$

$$= \sqrt{D^2 + (D-x)^2}$$

and $$S_2P = \sqrt{(S_2T_2)^2 + (T_2P)^2}$$

$$= \sqrt{D^2 + (D+x)^2}$$

So, path difference = $(S_2P - S_1P)$

The minima will occur when

$$S_2P - S_1P = (2n-1)\frac{\lambda}{2}$$

For first minima $P = \frac{\lambda}{2}$ at $n = 1$

So, $\left[D^2 + (D+x)^2\right]^{1/2} - \left[D^2 + (D-x)^2\right]^{1/2} = \frac{\lambda}{2}$

If $(x = D)$ then,

We can write $[D^2 + 4D^2]^{1/2} - [D^2 + 0]^{1/2} = \frac{\lambda}{2}$

$$[5D^2]^{1/2} - [D^2]^{1/2} = \frac{\lambda}{2}$$

$$\sqrt{5}D - D = \frac{\lambda}{2}$$

$$D(\sqrt{5} - 1) = \frac{\lambda}{2}$$

or $$D = \frac{\lambda}{2(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{3.236\lambda}{2 \times 4}$$

$$D = \frac{3.236}{8}\lambda = 0.404\lambda$$

Long Answer Questions

1. Let the amplitude of ray 1 and 2 are A_1 and A_2 respectively and resultant amplitude is A, so resultant amplitude is :

A = A parallel $(A_\parallel)$ + A perpendicular $(A_\perp)$

$$A_1 = A_\perp^{\,1} + A_\parallel^{\,1}$$

and $$A_2 = A_\perp^{\,2} + A_\parallel^{\,2}$$

Without P,

So, $$A_\perp^{\,0} = A'_\perp + A_\perp^2$$

$$= A_\perp^0 \sin(kx - \omega t) + A_\perp^0 \sin(kx - \omega t + \phi)$$

$$A_\parallel^0 = A_\parallel^{(1)} + A_\parallel^{(2)}$$

$$A_\parallel^0 = A^0\|[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

where $A_\perp^0, A_\parallel^0$ are the amplitudes of either of the beam in perpendicular and parallel polarisations.

Consider when their is no polaroid then net amplitude

$$A = A_\perp^0 + A_\parallel^0$$

$\therefore$ Intensity $= \{|A_\perp^2| + |A_\parallel^2|\}\ [\sin^2(kx - \omega t)(1 + \cos^2\phi + 2\sin\phi) + \sin^2(kx - \omega t)\sin^2\phi]$

$$= \{|A_\perp^0|^2 + |A_\parallel^0|^2\}\left(\frac{1}{2}\right)2(1 + \cos\phi)$$

$= 2|A_\perp^0|^2 (1 + \cos\phi)$, since, $|A_\perp^{\,0}|_{av} = |A_\parallel^{\,0}|_{av}$

With P

Consider $A_\perp^{\,2}$ is blocked or zero,

So, Intensity $= [A_\parallel^{\,0}]^2 + [A_\perp^{\,0}]^2$

$$\left[A_\perp^0 = \left(A_\perp^1 + A_\perp^2\right) = A_\perp^1,\ A_\parallel^0 = \left(A_\parallel^1 + A_\parallel^2\right) = A_\parallel^1\right]$$

So, Intensity $= (A_\parallel^{\,1} + A_\parallel^{\,2})^2 + (A_\perp^{\,1})^2$

$$= |A_\perp^{\,0}|^2 (1 + \cos\phi) + |A_\perp^{\,0}|^2 . \frac{1}{2}$$

$$|A_1^0|^2 [1 + \cos\phi + \frac{1}{2}] = |A_1^{\,0}|^2 [\frac{3}{2} + \cos\phi]$$

$$= |A_1^0|^2 \times \left[\frac{5}{2}\right]$$

$[\because \cos\phi = 1$ for maxima$]$

As given that, $I_0 = 4|A_\perp^{\,0}|^2$ = Intensity without polariser at principal maxima.

So, $|A_1^0|^2 = (I_0/4)$

Intensity at principal maxima with polariser

$$= |A_\perp^{\,0}|^2 \left(\frac{5}{2}\right) = \frac{5}{8} I_0$$

Intensity at first minima with polariser

$$= |A_\perp^0|^2 (1 - 1) + \frac{|A_\perp^0|^2}{2} = \frac{I_0}{8}$$

2. When, ray pass through a transparent glass slab of thickness L of refractive index μ so the path difference caused by slab is (μ – 1)L and path difference caused by YDSE is 2dsinA. then the total path difference will be calculated as

$$\Delta x = 2d\sin\theta + (\mu - 1)L.$$

In case of transparent glass slabe of refractive index μ,

the path difference = $2d\sin\theta + (\mu - 1)L$

For the principal maxima, (path difference is zero),

So, $\Delta x = 0$

$$2d\sin\theta_0 + (\mu - 1)L = 0$$

$$\sin\theta_0 = -\frac{L(\mu-1)}{2d} = \frac{-L(0.5)}{2d}$$

$$= \frac{-L\times 0.5}{2\times 4L} \quad \left[\because L = \frac{d}{4};\ d = 4L\right]$$

$$\sin\theta_0 = \frac{-1}{16}$$

∴ For central maxima

$$OP = D\tan\theta_0 \approx D\sin\theta_0 = \frac{-D}{16}$$

For small $\angle\theta_0$, $\sin\theta_0 = \theta_0$ and $\tan\theta_0 = \theta_0$

And for the first minima, the path difference is $\pm\frac{\lambda}{2}$

$$\text{So, } 2d\sin\theta_1 + (\mu - 1) = \pm\frac{\lambda}{2}$$

(∵ for both upper and lower side from 0)

$$\therefore 2d\sin\theta_1 + 0.5L = \pm\frac{\lambda}{2}$$

$$\text{or} \quad \sin\theta_1 = \frac{\pm\frac{\lambda}{2} - 0.5L}{2d} = \frac{\pm\frac{\lambda}{2} - \frac{d}{8}}{2d} \quad \left[\because L = \frac{d}{4}\right]$$

$$= \frac{\pm\frac{\lambda}{2} - \frac{\lambda}{8}}{2\lambda} = \pm\frac{1}{4} - \frac{1}{16}$$

So, the diffraction occurs if the wavelength of waves is nearly equal to the side width (d)

On the positive side $\sin\theta'^{+}_{1} = +\frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

On the negative side $\sin\theta'^{-}_{1} = -\frac{1}{4} - \frac{1}{16} = -\frac{5}{16}$

The first principal maxima on the positive side is at distance

$$D\tan\theta'^{+}_{1} = D\frac{\sin\theta'^{+}_{1}}{\sqrt{1-\sin^2\theta^{+}_{1}}} = D\frac{3}{\sqrt{16^2 - 3^2}}$$

$$= \frac{3D}{\sqrt{247}} \text{ above point O}$$

The distance of first principal minima on the negative side is

$$D\tan\theta'_1 = \frac{D\sin\theta''^{-}_{1}}{\sqrt{1-\sin^2\theta''^{-}_{1}}} = \frac{\frac{-5D}{16}}{\sqrt{\frac{16^2-5^2}{16}}}$$

$$= \frac{-5D}{\sqrt{16^2 - 5^2}} = \frac{-5D}{\sqrt{231}} \text{ below point O.}$$

3. The resultant disturbance at a point will be calculated by some of disturbances due to individual sources.

Consider all the disturbances at the receiver R_1 which is at a distance d from B.

Let the wave at R_1 has zero path difference from source A, $Y_A = a\cos\omega t$.

The path difference of the signal from A with that from B is λ/2 and so, the phase difference is π.

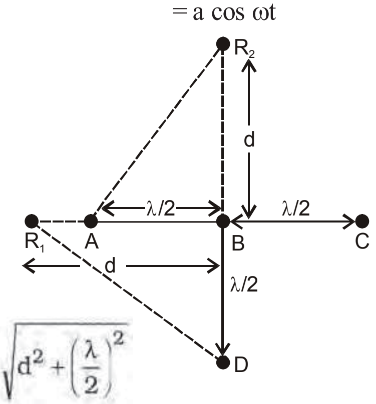
Then, the wave at R_1 because of B is

$$y_B = a\cos(\omega t - \pi) = -a\cos\omega t.$$

The path difference of the signal from C with that from A is λ and hence the phase difference is 2π.

So, the wave at R_1 because of C is

$$Y_c = a\cos(\omega t - 2\pi)$$
$$= a\cos\omega t$$



Path difference between the signal from D with that of A is $= R_1D - AR_1$

$$= \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - \left(d - \frac{\lambda}{2}\right)$$

$$= d\left(1 + \frac{\lambda}{4d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left(1 + \frac{\lambda^2}{8d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

Neglecting the term $\frac{\lambda^2}{8d^2}$, So path difference

$$= d + 0 - d + \frac{\lambda}{2} = \frac{\lambda}{2}$$

Therefore, phase difference is π. i.e., $\frac{\lambda}{2} = \pi$

$Y_D = a\cos(\omega t - \pi) = -a\cos\omega t$

So, the signal picked up at R_1 from all the four sources is

$$Y_{R_1} = y_A + y_B + y_C + y_D$$

$= a\cos\omega t - a\cos\omega t + a\cos\omega t - a\cos\omega t = 0$

($Y_{R_1} = 0$, so $IR_1 = 0$)

So, net signal = 0

(i) Consider the signal picked up at R_2 from B be $y_B = a_1\cos\omega t$.
The path difference between signal at D and that at B is

$$D = \frac{\lambda}{2} \text{ or } \pi$$

The signal picked up by R_2 from

$$D = \cos(\omega t - \pi)$$

$$y_D = -a_1\cos\omega t$$

The path difference between signal at A and D at R_2 is $= AR_2 - R_2B$

$$\Delta x = \left[\sqrt{(d)^2 + \left(\frac{\lambda}{2}\right)^2} - d\right] = \left[d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d\right]$$

$$= \frac{1\lambda^2}{8d^2}$$

As $d >> \lambda$, so this path difference is tends to 0

$$\text{Phase difference} = \frac{2\pi}{\lambda}\Delta x$$

By putting the value of Δx, so

$$\text{phase difference} = \frac{2\pi}{\lambda}\left(\frac{1}{8}\frac{\lambda^2}{d^2}\right) = \frac{2\pi\lambda}{8d} \to \phi$$

So, signal received by R_2 from A & C are :

Hence, $Y_A = a_1\cos(\omega t - \phi)$

Similarly, $Y_C = a_1\cos(\omega t - \phi)$

$\therefore$ Signal picked up by R_2 from A, B, C and D.

$$Y_{R_2} = y_A + y_B + y_C + y_D$$

$$Y_{R_2} = 2a_1\cos(\omega t - \phi)$$

$\therefore$ $|y|^2 = 4a_1^2\cos^2(\omega t - \phi)$

$\therefore$ $\langle I \rangle = 2a_1^2$

$$\because I = A^2$$

Hence, R_2 picks up the larger signal then R_1.

(ii) If B is switched off
Signal picked by

$$R_1 = Y_A + Y_C + Y_D$$

$= a\cos\omega t + a\cos\omega t - a\cos\omega t$

R_1 picks up $Y_{R_1} = a\cos\omega t$

$$(I_{R_1}) = \frac{1}{2}a^2$$

when signal picked by $(R_2) = Y_A + Y_C + Y_D$

$= a\cos(\omega t - \phi) + a\cos(\omega t - \phi) - a\cos\omega t$

R_2 picks up, $y = a\cos\omega t$

$$(I_{R_2}) = a^2 \langle \cos^2\omega t \rangle = \frac{a^2}{2}$$

So, $(I_{R_1}) = (I_{R_2})$

Hence R_1 and R_2 pick up the same signal at same intensity.

(iii) If D is switched off
Signal picked up by reciver R_1

$$R_1 = Y_B + Y_C + Y_A$$

$= -a\cos\omega t + a\cos\omega t + a\cos\omega t$

R_1 picks up $Y_{R_1} = a\cos\omega t$

$$\therefore \quad (I_{R_1}) = \frac{1}{2}a^2$$

signal picked up by receiver R_2

$$R_2 = Y_A + Y_B + Y_C$$

$= a\cos(\omega t - \phi) + a\cos\omega t + a\cos(\omega t - \phi)$

$= 2a\cos(\omega t - \phi) + a\cos\omega t$

$$\therefore \quad \frac{2\pi\lambda}{a} = \phi$$

is very small so neglected it.

R_2 picks up $\quad R_2 = 2a \cos \omega t + a \cos \omega t$

$$y = 3a \cos \omega t$$

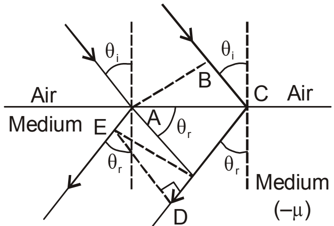
$$\therefore \quad (I_{R_2}) = 9a^2 < \cos^2 \omega t > = \frac{9a^2}{2}$$

$$(I_{R_1}) < (I_{R_2})$$

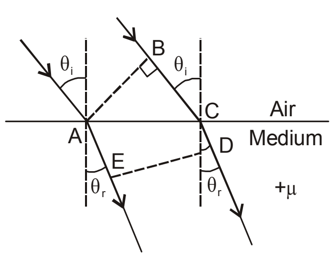
So, R_2 picks up larger signal compared to R_1.

(iv) A signal at R_1 indicates B has been switched off and A signal at R_2 indicates D has been switched off.

4. If let, the given postulate $-\sqrt{\mu_r \varepsilon_r} = \mu$ and $+\sqrt{\mu_r \varepsilon_r} = \mu$ are true, then two parallel rays would proceed as shown in the figure below



(i)



(ii)

(i) Let two parallel rays at incidence angle θ_i from air would proceed in medium shown in figure so, AB represent the incident wavefront and DE represent the refracted wavefront.

All points on a wavefront must be in same phase and in turn, must have the same optical path length.

So, $-\sqrt{\varepsilon_r \mu_r} AE = BC - \sqrt{\varepsilon_r \mu_r} CD$

or $\quad BC = \sqrt{\varepsilon_r \mu_r}(CD - AE)$

$$BC > 0, CD > AE$$

This showing that the postulate is reasonable. If however, the light proceeded in the sense it does for ordinaiy material refracted rays are in the fourth quadrant, shown in figure second :

Then, $-\sqrt{\varepsilon_r \mu_r} AE = BC - \sqrt{\varepsilon_r \mu_r} CD$

or $\quad BC = \sqrt{\varepsilon_r \mu_r}(CD - AE)$

If $BC > 0$, then $CD > AE$

which is obvious from Fig (i).

Hence, the postulate reasonable.

However, if the light proceeded in the sense it does for ordinary material, (going from 2nd quadrant to 4th quadrant) as shown in Fig. (i)., then proceeding as above,

$$-\sqrt{\varepsilon_r \mu_r} AE = BC - \sqrt{\varepsilon_r \mu_r} CD$$

or $\quad BC = \sqrt{\varepsilon_r \mu_r}(CD - AE)$

As figure shows, $AE > CD$ So, $BC < 0$ this is not possible. Hence, the given postulate is correct.

(ii) According to the Fig. (i)

$$BC = AC \sin \theta_i$$

So, $\quad CD - AE = AC \sin \theta_r$

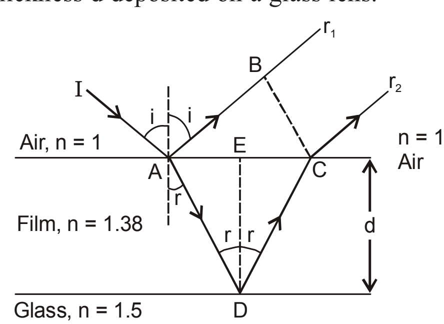
$$BC = \sqrt{\mu_r \varepsilon_r} \; [\because CD - AE = BC]$$

$\therefore \quad$ then, $AC \sin\theta_i = \sqrt{\mu_r \varepsilon_r} \; AC \sin \theta_r$

or $\quad \dfrac{\sin \theta_i}{\sin \theta_r} = \sqrt{\varepsilon_r \mu_r} = n$

Which proves Snell's law.

5. In the figure below shown a dielectric film of thickness d deposited on a glass lens.



As given that, the refractive index of film =1.38 and retractive index of glass = 1.5.

and, $\quad \lambda = 5500Å$.

Let a ray incident at an angle i. A part of this ray is reflected from the air-film interface and a part refracted inside.

That is partly reflected at the film-glass interface and a part transmitted A part of the reflected ray is reflected at the film-air interface and a part transmitted as r_2 parallel to r_1.

So, the amplitude of wave during refraction and reflection decreases.

So, rays r_1 and r_2 shall dominate the behaviour. If incident light is to be transmitted through the lens, r_1 and r_2 should interfere destructively.

Both the reflections at A and D are from tower to higher refractive index and so, there is no phase change on reflection. Then the optical path difference between r_2 and r_1 is

$$n(AD + CD) - AB \quad \text{...(i)}$$

If d is the thickness of the film, then

$$AD = CD = \frac{d}{\cos r}$$

$$AB = AC \sin i$$

$$\frac{AC}{2} = d \tan r$$

$$\therefore \quad AC = 2d \tan r$$

So, $AB = 2d \tan r \sin i.$

or, $\quad AB = 2d \tan r (\sin i)$

Thus, the optical path difference form (i)

$$= n(AD + CD) - AB = 2n(AD) - AB$$

$$(\because AD = CD)$$

$$= \frac{2nd}{\cos r} - 2d \tan r \sin i$$

$$= 2 \cdot \frac{\sin id}{\sin r \cos r} - 2d \frac{\sin r}{\cos r} \sin i$$

$$= \frac{2d \sin i}{\cos r}\left[\frac{1}{\sin r} - \frac{\sin r}{1}\right] = 2d \sin i\left[\frac{1 - \sin^2 r}{\sin r \cos r}\right]$$

$$= \frac{2d \sin i \cos^2 r}{\sin r \cos r} = 2nd \cos r \qquad \left[\because n = \frac{\sin i}{\sin r}\right]$$

For two rays to interfere destructively path difference $= \frac{\lambda}{2}$

So, $\quad 2nd \cos r = \frac{\lambda}{2}$

$$nd \cos r = \frac{\lambda}{4} \quad \text{...(i)}$$

For photographic lenses, the sources are normally in vertical plane, i.e., rays incident at very small angle so,

$$i = r = 0°$$

From Eq. (i),

$$nd \cos 0° = \frac{\lambda}{4} \qquad (\because \cos 0° = 1)$$

$$d = \frac{\lambda}{4n} \qquad (\because n = 1.38)$$

$$\boxed{d = \frac{5500 Å}{4 \times 1.38} \approx 1000 Å}$$

DUAL NATURE OF RADIATION AND MATTER

SUMMARY

1. The minimum energy needed by an electron to come out from a metal surface is called the work function of the metal. Energy (greater than the work function (ϕ_o) required for electron emission from the metal surface can be supplied by suitably heating or applying strong electric field or irradiating it by light of suitable frequency.
2. Photoelectric effect is the phenomenon of emission of electrons by metals when illuminated by light of suitable frequency. Certain metals respond to ultraviolet light while others are sensitive even to the visible light. Photoelectric effect involves conversion of light energy into electrical energy. It follows the law of conservation of energy. The photoelectric emission is an instantaneous process and possesses certain special features.
3. Photoelectric current depends on (i) the intensity of incident light, (ii) the potential difference applied between the two electrodes, and (iii) the nature of the emitter material.
4. The stopping potential (V_o) depends on (i) the frequency of incident light, and (ii) the nature of the emitter material. For a given frequency of incident light, it is independent of its intensity. The stopping potential is directly related to the maximum kinetic energy of electrons emitted: $e\,V_0 = (1/2)\,m\,v^2_{max} = K_{max}$.
5. Below a certain frequency (threshold frequency) ν_0, characteristic of the metal, no photoelectric emission takes place, no matter how large the intensity may be.
6. The classical wave theory could not explain the main features of photoelectric effect. Its picture of continuous absorption of energy from radiation could not explain the independence of K_{max} on intensity, the existence of ν_o and the instantaneous nature of the process. Einstein explained these features on the basis of photon picture of light. According to this, light is composed of discrete packets of energy called quanta or photons. Each photon carries an energy E (= $h\,\nu$) and momentum p (= h/λ), which depend on the frequency (ν) of incident light and not on its intensity. Photoelectric emission from the metal surface occurs due to absorption of a photon by an electron.
7. Einstein's photoelectric equation is in accordance with the energy conservation law as applied to the photon absorption by an electron in the metal. The maximum kinetic energy $(1/2)m\,v^2_{max}$ is equal to the photon energy ($h\nu$) minus the work function ϕ_0 (= $h\nu_0$) of the target metal:

$$\frac{1}{2}\,m\,v^2_{max} = V_0\,e = h\nu - \phi_0 = h\,(\nu - \nu_0)$$

This photoelectric equation explains all the features of the photoelectric effect. Millikan's first precise measurements confirmed the Einstein's photoelectric equation and obtained an accurate value of Planck's constant h. This led to the acceptance of particle or photon description (nature) of electromagnetic radiation, introduced by Einstein.
8. Radiation has dual nature: wave and particle. The nature of experiment determines whether a wave or particle description is best suited for understanding the experimental result. Reasoning that radiation and matter should be symmetrical in nature, Louis Victor de Broglie

attributed a wave-like character to matter (material particles). The waves associated with the moving material particles are called matter waves or de Broglie waves.

9. The de Broglie wavelength (λ) associated with a moving particle is related to its momentum p as: $\lambda = h/p$. The dualism of matter is inherent in the de Broglie relation which contains a wave concept (λ) and a particle concept (p). The de Broglie wavelength is independent of the charge and nature of the material particle. It is significantly measurable (of the order of the atomic-planes spacing in crystals) only in case of sub-atomic particles like electrons, protons, etc. (due to smallness of their masses and hence, momenta). However, it is indeed very small, quite beyond measurement, in case of macroscopic objects, commonly encountered in everyday life.
10. Electron diffraction experiments by Davisson and Germer, and by G. P. Thomson, as well as many later experiments, have verified and confirmed the wave-nature of electrons. The de Broglie hypothesis of matter waves supports the Bohr's concept of stationary orbits.

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Planck's constant	h	$[ML^2T^{-1}]$	J s	$E = h\nu$
Stopping potential	V_0	$[ML^2T^{-3}A^{-1}]$	V	$e\,V_0 = K_{max}$
Work function	ϕ_0	$[ML^2T^{-2}]$	J; eV	$K_{max} = E - \phi_0$
Threshold frequency	ν_0	$[T^{-1}]$	Hz	$\nu_0 = \phi_0/h$
de Broglie wavelength	λ	[L]	m	$\lambda = h/p$

POINTS TO PONDER

1. Free electrons in a metal are free in the sense that they move inside the metal in a constant potential (This is only an approximation). They are not free to move out of the metal. They need additional energy to get out of the metal.
2. Free electrons in a metal do not all have the same energy. Like molecules in a gas jar, the electrons have a certain energy distribution at a given temperature. This distribution is different from the usual Maxwell's distribution that you have learnt in the study of kinetic theory of gases. You will learn about it in later courses, but the difference has to do with the fact that electrons obey Pauli's exclusion principle.
3. Because of the energy distribution of free electrons in a metal, the energy required by an electron to come out of the metal is different for different electrons. Electrons with higher energy require less additional energy to come out of the metal than those with lower energies. Work function is the least energy required by an electron to come out of the metal.

4. Observations on photoelectric effect imply that in the event of matter-light interaction, *absorption of energy takes place in discrete units of hν*. This is not quite the same as saying that light consists of particles, each of energy *hν*.
5. Observations on the stopping potential (its independence of intensity and dependence on frequency) are the crucial discriminator between the wave-picture and photon-picture of photoelectric effect.
6. The wavelength of a matter wave given by $\lambda = \dfrac{h}{p}$ has physical significance; its phase velocity v_p has no physical significance. However, the group velocity of the matter wave is physically meaningful and equals the velocity of the particle.

EXERCISES

11.1 Find the
(a) maximum frequency, and
(b) minimum wavelength of X-rays produced by 30 kV electrons.

11.2 The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14}Hz is incident on the metal surface, photoemission of electrons occurs. What is the
(a) maximum kinetic energy of the emitted electrons,
(b) Stopping potential, and
(c) maximum speed of the emitted photoelectrons?

11.3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

11.4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.
(a) Find the energy and momentum of each photon in the light beam,
(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and
(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

11.5 The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

11.6 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

11.7 A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

11.8 The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

11.9 The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

11.10 Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

11.11 Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

11.12 Calculate the
(a) momentum, and
(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

11.13 What is the
(a) momentum,
(b) speed, and
(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

11.14 The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which
(a) an electron, and
(b) a neutron, would have the same de Broglie wavelength.

11.15 What is the de Broglie wavelength of
(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,
(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

11.16 An electron and a photon each have a wavelength of 1.00 nm. Find
(a) their momenta,
(b) the energy of the photon, and
(c) the kinetic energy of electron.

11.17 (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?
(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $(3/2)\ kT$ at 300 K.

11.18 Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

11.19 What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

ADDITIONAL EXERCISES

11.20 (a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The *specific charge* of the electron, i.e., its e/m is given to be 1.76×10^{11} C kg^{-1}.

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong ? In what way is the formula to be modified?

11.21 (a) A monoenergetic electron beam with electron speed of 5.20×10^{6} m s^{-1} is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76×10^{11}C kg^{-1}.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

[**Note:** Exercises 11.20(b) and 11.21(b) take you to relativistic mechanics which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

11.22 An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (~10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.

11.23 (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

11.24 In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^{9} eV)

11.25 Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radiowaves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive (~10^{-10} W m^{-2}). Take the area of the pupil to be about 0.4 cm^2, and the average frequency of white light to be about 6 × 10^{14} Hz.

11.26 Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is –1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity (~10^5 W m^{-2}) red light of wavelength 6328 Å produced by a He-Ne laser?

11.27 Monochromatic radiation of wavelength 640.2 nm (1nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

11.28 A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

λ_1 = 3650 Å, λ_2= 4047 Å, λ_3= 4358 Å, λ_4= 5461 Å, λ_5= 6907 Å,

The stopping voltages, respectively, were measured to be:

V_{01} = 1.28 V, V_{02} = 0.95 V, V_{03} = 0.74 V, V_{04} = 0.16 V, V_{05} = 0 V

Determine the value of Planck's constant *h*, the threshold frequency and work function for the material.

[*Note:* You will notice that to get *h* from the data, you will need to know *e* (which you can take to be 1.6 × 10^{-19} C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of *e* (from the oil-drop experiment) confirmed Einstein's photoelectric equation and at the same time gave an independent estimate of the value of *h*.]

11.29 The work function for the following metals is given:

Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

11.30 Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

11.31 Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of inter-atomic spacing in the lattice) (m_e=9.11 × 10^{-31} kg).

11.32 (a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. (m_n = 1.675 × 10^{-27} kg)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

11.33 An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

11.34 The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

11.35 Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

11.36 Compute the typical de Broglie wavelength of an electron in a metal at 27 °C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

[*Note:* Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distintguished apart from one another. This indistinguishibility has many fundamental implications which you will explore in more advanced Physics courses.]

11.37 Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [$(+2/3)e$; $(-1/3)e$]. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:

$$E = h\nu, \; p = \frac{h}{\lambda}$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

APPENDIX

11.1 The history of wave-particle flip-flop

What is light? This question has haunted mankind for a long time. But systematic experiments were done by scientists since the dawn of the scientific and industrial era, about four centuries ago. Around the same time, theoretical models about what light is made of were developed. While building a model in any branch of science, it is essential to see that it is able to explain all the experimental observations existing at that time. It is therefore appropriate to summarize some observations about light that were known in the seventeenth century.

The properties of light known at that time included (a) rectilinear propagation of light, (b) reflection from plane and curved surfaces, (c) refraction at the boundary of two media, (d) dispersion into various colours, (e) high speed. Appropriate laws were formulated for the first four phenomena. For example, Snell formulated his laws of refraction in 1621. Several scientists right from the days of Galileo had tried to measure the speed of light. But they had not been able to do so. They had only concluded that it was higher than the limit of their measurement.

Two models of light were also proposed in the seventeenth century. Descartes, in early decades of seventeenth century, proposed that light consists of particles, while Huygens, around 1650-60, proposed that light consists of waves. Descartes′ proposal was merely a philosophical model, devoid of any experiments or scientific arguments. Newton soon after, around 1660-70, extended Descartes′ particle model, known as *corpuscular theory*, built it up as a scientific theory, and explained various known properties with it. These models, light as waves and as particles, in a sense, are quite opposite of each other. But both models could explain all the known properties of light. There was nothing to choose between them.

The history of the development of these models over the next few centuries is interesting. Bartholinus, in 1669, discovered double refraction of light in some crystals, and Huygens, in 1678, was quick to explain it on the basis of his wave theory of light. In spite of this, for over one hundred years, Newton's particle model was firmly believed and preferred over the wave model. This was partly because of its simplicity and partly because of Newton's influence on contemporary physics.

Then in 1801, Young performed his double-slit experiment and observed interference fringes. This phenomenon could be explained only by wave theory. It was realized that diffraction was also another phenomenon which could be explained only by wave theory. In fact, it was a natural consequence of Huygens idea of secondary wavelets emanating from every point in the path of light. These experiments could not be explained by assuming that light consists of particles. Another phenomenon of polarisation was discovered around 1810, and this too could be naturally explained by the wave theory. Thus wave theory of Huygens came to the forefront and Newton's particle theory went into the background. This situation again continued for almost a century.

Better experiments were performed in the nineteenth century to determine the speed of light. With more accurate experiments, a value of 3×10^8 m/s for speed of light in vacuum was arrived at. Around 1860, Maxwell proposed his equations of electromagnetism and it was realized that *all* electromagnetic phenomena known at that time could be explained by Maxwell's four equations. Soon Maxwell showed that electric and magnetic fields could propagate through empty space (vacuum) in the form of electromagnetic waves. He calculated the speed of these waves and arrived at a theoretical value of 2.998×10^8 m/s. The close agreement of this value with the experimental value suggested that light consists of electromagnetic waves. In 1887 Hertz demonstrated the generation and detection of such waves. This established the wave theory of light on a firm footing. We might say that while eighteenth century belonged to the particle model, the nineteenth century belonged to the wave model of light.

Vast amounts of experiments were done during the period 1850-1900 on heat and related phenomena, an altogether different area of physics. Theories and models like kinetic theory and thermodynamics were developed which quite successfully explained the various phenomena, except one.

Every body at any temperature emits radiation of all wavelengths. It also absorbs radiation falling on it. A body which absorbs all the radiation falling on it is called a *black body*. It is an ideal concept in physics, like concepts of a point mass or uniform motion. A graph of the intensity of radiation emitted by a body versus wavelength is called the *black body spectrum*. No theory in those days could explain the complete black body spectrum!

In 1900, Planck hit upon a novel idea. If we assume, he said, that radiation is emitted in packets of energy instead of continuously as in a wave, then we can explain the black body spectrum. Planck himself regarded these quanta, or packets, as a property of emission and absorption, rather than that of light. He derived a formula which agreed with the entire spectrum. This was a confusing mixture of wave and particle pictures – radiation is emitted as a particle, it travels as a wave, and is again absorbed as a particle! Moreover, this put physicists in a dilemma. Should we again accept the particle picture of light just to explain one phenomenon? Then what happens to the phenomena of interference and diffraction which cannot be explained by the particle model?

But soon in 1905, Einstein explained the photoelectric effect by assuming the particle picture of light. In 1907, Debye explained the low temperature specific heats of solids by using the particle picture for lattice vibrations in a crystalline solid. Both these phenomena belonging to widely diverse areas of physics could be explained only by the particle model and not by the wave model. In 1923, Compton's x-ray scattering experiments from atoms also went in favour of the particle picture. This increased the dilemma further.

Thus by 1923, physicists faced with the following situation. (a) There were some phenomena like rectilinear propagation, reflection, refraction, which could be explained by either particle model or by wave model. (b) There were some phenomena such as diffraction and interference which could be explained only by the wave model but *not* by the particle model. (c) There were some phenomena such as black body radiation, photoelectric effect, and Compton scattering which could be explained only by the particle model but *not* by the wave model. Somebody in those days aptly remarked that light behaves as a particle on Mondays, Wednesdays and Fridays, and as a wave on Tuesdays, Thursdays and Saturdays, and we don't talk of light on Sundays!

In 1924, de Broglie proposed his theory of wave-particle duality in which he said that not only photons of light but also 'particles' of matter such as electrons and atoms possess a dual character, sometimes behaving like a particle and sometimes as a wave. He gave a formula connecting their mass, velocity, momentum (particle characteristics), with their wavelength and frequency (wave characteristics)! In 1927 Thomson, and Davisson and Germer, in separate experiments, showed that electrons did behave like waves with a wavelength which agreed with that given by de Broglie's formula. Their experiment was on diffraction of electrons through crystalline solids, in which the regular arrangement of atoms acted like a grating. Very soon, diffraction experiments with other 'particles' such as neutrons and protons were performed and these too confirmed with de Broglie's formula. This confirmed wave-particle duality as an established principle of physics. Here was a principle, physicists thought, which explained all the phenomena mentioned above not only for light but also for the so-called particles.

But there was no basic theoretical foundation for wave-particle duality. De Broglie's proposal was merely a qualitative argument based on symmetry of nature. Wave-particle duality was at best a principle, not an outcome of a sound fundamental theory. It is true that all experiments whatever agreed with de Broglie formula. But physics does not work that way. On the one hand, it needs experimental confirmation, while on the other hand, it also needs sound theoretical basis for the models proposed. This was developed over the next two decades. Dirac developed his theory of radiation in about 1928, and Heisenberg and Pauli gave it a firm footing by 1930. Tomonaga, Schwinger, and Feynman, in late 1940s, produced further refinements and cleared the theory of inconsistencies which were noticed. All these theories mainly put wave-particle duality on a theoretical footing.

Although the story continues, it grows more and more complex and beyond the scope of this note. But we have here the essential structure of what happened, and let us be satisfied with it at the moment. Now it is regarded as a natural consequence of present theories of physics that electromagnetic radiation as well as particles of matter exhibit both wave and particle properties in different experiments, and sometimes even in the different parts of the same experiment.

Chapter 11:- Dual Nature of Matter and Radiation

Multiple Choice Questions (MCQs)

1. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

(a) H (b) $H^{1/2}$

(c) H^0 (d) $H^{-1/2}$

2. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nuclear with 1 MeV energy is nearly

(a) 1.2 nm (b) 1.2×10^{-3} nm

(c) 1.2×10^{-6} nm (d) 1.2×10 nm

3. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then,

(a) no electrons will be emitted as only photons can emit electrons

(b) electrons can be emitted but all with an energy, E_0

(c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)

(d) electrons can be emitted with any energy, with a maximum of E_0

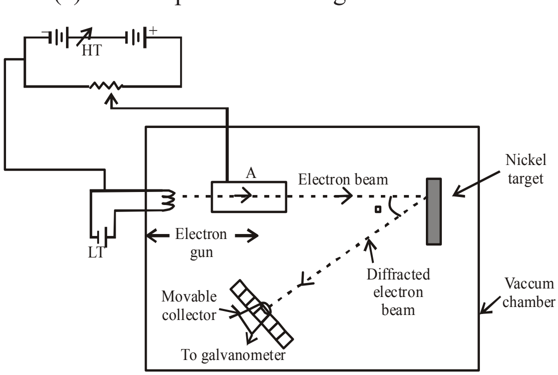
4. Consider figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maximum at value of θ that

(a) will be larger than the earlier value

(b) will be the same as the earlier value

(c) will be less than the earlier value

(d) will depend on the target



5. A proton, a neutron, an electron and an α-particle have same energy. Then, their de-Broglie wavelengths compare as

(a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$

(b) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$

(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$

(d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

6. An electron is moving with an initial velocity $v = v_0 \vec{i}$ and is in a magnetic field $B = B_0 \vec{j}$. Then, it's de-Broglie wavelength

(a) remains constant

(b) increases with time

(c) decreases with me

(d) increases and decreases periodically

7. An electron (mass m) with an initial velocity $v = v_0$ i ($v_0 > 0$) is in an electric field $E = -E_0 \hat{i}$ (E_0 = constant > 0). It's de-Broglie wavelength at time t is given by

(a) $\dfrac{\lambda_0}{\left(1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right)}$ (b) $\left(1+\dfrac{eE_0 t}{mv_0}\right)$

(c) λ_0 (d) $\lambda_0 t$

8. An electron (mass m) with an initial velocity $v = v_0\hat{i}$ is in an electric field $E = E_0\hat{j}$. If $\lambda_0 = h/mv$, it's de-Broglie wavelnegth at time t is given by

(a) λ_0 (b) $\lambda_0\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}$

(c) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ (d) $\dfrac{\lambda_0}{\left(1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}\right)}$

Multiple Choice Questions (MCQs) (More than one option correct)

1. Relativistic corrections become necessary when the expression for the kinetic energy $\frac{1}{2}mv^2$, becomes comparable with mc^2, where m is the mass of the particle. At what de-Broglie wavelength, will relativistic corrections become important for an electron?
(a) $\lambda = 10$ nm (b) $\lambda = 10^{-1}$ nm
(c) $\lambda = 10^{-4}$ nm (d) $\lambda = 10^{-6}$ nm

2. Two particles A_1 and A_2 of masses m_1, m_2 $(m_1 > m_2)$ have the same de-Broglie wavelength. Then,
(a) their momenta are the same
(b) their energies are the same
(c) energy of A_1 is less than the energy of A_2
(d) energy of A_1 is more than the energy of A_2

3. The de-Broglie wavelength of a photon is twice, the de-Brogile wavelength of an electron. The speed of the electron is $v_e = \frac{c}{100}$. Then,
(a) $\frac{E_e}{E_p} = 10^{-4}$ (b) $\frac{E_e}{E_p} = 10^{-2}$
(c) $\frac{p_e}{m_e c} = 10^{-2}$ (d) $\frac{p_e}{m_e c} = 10^{-4}$

4. Photons absorbed in matter are converted to heat. A source emitting n photon/sec of frequency ν is used to convert 1 kg of ice at 0° C to water at 0° C. Then, the time T taken for the conversion
(a) decreases with increasing n, with ν fixed
(b) decreases with n fixed, ν increasing
(c) remains constant with n and ν changing such that nν = constant
(d) increases when the product nν increases

5. A particle moves in a closed orbit around the origin, due to a force which is directed towards the origin. The de-Broglie wavelength of the particle varies cyclically between two values λ_1, λ_2 with $\lambda_1 > \lambda_2$. Which of the following statement are true?
(a) The particle could be moving in a circular orbit with origin as centre
(b) The particle could be moving in an elliptic orbit with origin as its focus
(c) When the de-Broglie wavelength is λ_1, the particle is nearer the origin than when its value is λ_2
(d) When the de-Broglie wavelength is λ_2, the particle is nearer the origin than when its value is λ_1

Very Short Answer Questions

1. A proton and an α – particle are accelerated, using the same potential difference. How are the de-Broglie wavelengths λ_p and λ_α related to each other?

2. (i) In the explanation of photoelectric effect, we assume one photon of frequency ν collides with an electron and transfers its energy. This leads to the equation for the maximum energy E_{max} of the emitted electron as
$E_{max} = h\nu - \phi_0$
where ϕ_0 is the work function of the metal. If an electron absorbs 2 photons (each of frequency ν), what will be the maximum energy for the emitted electron?
(ii) Why is this fact (two photon absorption) not taken into consideration in our discussion of the stopping potential?

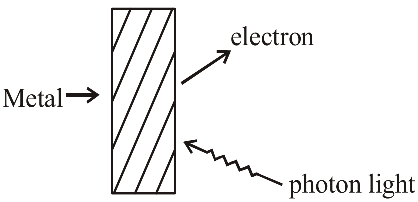
3. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of larger wavelength and emit light of shorter wavelength.

4. Do all the electrons that absorb a photon come out as photoelectron?

5. There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength?

Short Answer Questions

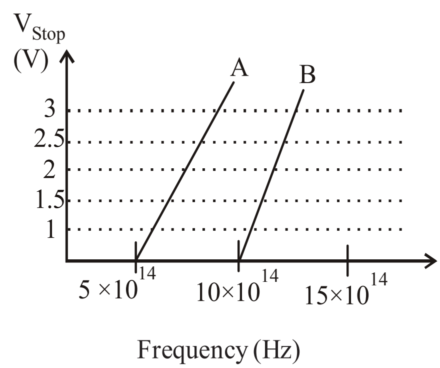
1. Consider figure for photoemission. How would you reconcile with momentum – conservation? Note light (photons) have momentum in a different direction than the emitted electrons.



2. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.
3. Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg uncertainty principle $(\Delta x \times \Delta p \approx h)$. You can assume the uncertainty in position Δx as 1 nm. Assuming $p \approx \Delta p$, find the energy of the electron in electron volts.
4. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies?
5. Two particles A and B of de – Broglie wavelengths λ_1 and λ_2 combine to form a particle C. The process conserves momentum. Find the de – Broglie wavelength of the particle C. (The motion is one – dimensional)
6. A neutron beam of energy E scatters from atoms on a surface with a spacing d = 0.1nm. The first maximum of intensity in the reflected beam occurs at $\theta = 30°$. What is the kinetic energy E of the beam in eV?

Long Answer Questions

1. Consider a thin target (10^{-2} m square, 10^{-3} m thickness) of sodium, which produces a photocurrent of 100 μA when a light of intensity 100 W/m^2 (λ = 660 nm) falls on it. Find the probability that a photoelectron is produced when a photon strikes a sodium atom.
[Take density of Na = 0.97 kg/m^3]
2. Consider an electron in front of metallic surface at a distance d (treated as an infinite plane surface). Assume the force of attraction by the plate is given as $\frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 d^2}$ .Calculate work in taking the charge to an infinite distance from the plate. Taking d = 0.1 nm, find the work done in electrons volts.
[Such a force law is not valid for d > 0.1 nm]
3. A student performs an experiment on photoelectric effect, using two meterials A and B. A plot of V_{stop} versus v is given in figure.



(i) Which material A or B has a higher work function?
(ii) Given the electric charge of an electron = 1.6×10^{-19} C, find the value of h obtained from the experiment for both A and B.
Comment on whether it is consistent with Einstein's theory.
4. A prticle A with a mass m_A is moving with a velocity v and hits a particle B (mass m_B) at rest (one dimensional motion). Find the change in the de – Broglie wavelength of the particle A. Treat the collision as elastic.
5. Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2 eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5 Å.
(i) Estimate number of photons emitted by the bulb per second. [Assume no other losses]
(ii) Will there be photoelectric emission?
(iii) How much time would be required by the atomic disk to receive energy equal to work function (2 eV)?
(iv) How many photons would atomic disk receive within time duration calculated in (iii) above?
(v) Can you explain how photoelectric effect was observed instantaneously?

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(d)** Velocity of a body freely falling from a height H is

$$v = \sqrt{2gH}$$

So, $\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \Rightarrow = \frac{h}{m\sqrt{2g}\sqrt{H}}$

(h, m and g are constant)

Here, $\frac{h}{m\sqrt{2g}}$ is also constant

So, $h \propto \frac{1}{\sqrt{H}} \Rightarrow$ or $\boxed{\lambda \propto H^{-1/2}}$

2. **(b)** Energy of a photon is $E = \frac{hc}{\lambda}$

Where λ is the minimum wavelength of the photon required to eject the proton from nucleus.

Energy of photon must be equal to the binding energy of proton.

So, energy of a photon, $E = 1\text{ MeV} \Rightarrow 10^6 eV$ (given)

Now, $\left(E = \frac{hc}{\lambda}\right)$

So, $\lambda = \frac{hc}{E} = \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^6 eV}\right)$

$\Rightarrow$ So, $\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^6 \times 1.6 \times 10^{-19} J}$

$= 1.24 \times 10^{-9} \times 10^{-3}$

$= 1.24 \times 10^{-3}$ nm

3. **(d)** When a beam of electrons of energy E_0 is incident on a metal surface kept in vacuum or evacuated chamber so electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when part of incident energy of electron is used in liberating the electrons from the surface of metal. So maximum energy of emitted electrons can be E_0.

4. **(c)** We know that,

In Davisson – Germer experiment, the de-Broglie wavelength associated with electron is

$$\lambda_d = \frac{12.27}{\sqrt{V}} Å \quad ...(i)$$

where V is the applied voltage. If there is a maxima of the diffracted electrons at an angle θ, than

$$2d \sin\theta = \lambda \quad ...(ii)$$

From equation (i) if V is inversely proportional to the wavelength λ_d, then the applied voltage V will increase with the decrease in the wavelengh λ_d.

From equation (ii) if wavelength λ_d is directly proportional to sin θ and hence θ.

So, with the decrease in λ_d sin θ, θ will also decrease.

Hence, when the voltage applied to A is increased. The diffracted beam will have the maximum at a value of θ that will be less than the earlier value.

5. **(b)** The relation between λ and K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

So, for the given value of kinetic energy K,

$\frac{h}{\sqrt{2K}}$ is a constant.

Thus, $\lambda \propto \frac{1}{\sqrt{m}}$

$\therefore \Rightarrow \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha$

$\Rightarrow = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$

if $(m_p = m_n)$, then $\lambda_p = \lambda_n$

if $(m_\alpha > m_p)$, then $\lambda_\alpha < \lambda_p$

if $(m_e < m_n)$, then $\lambda_e > \lambda_n$

Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$.

6. **(a)** As given that, $v = v_0 \vec{i}$ and $B = B_0 \vec{j}$

Force on moving electron due to perpendicular magnetic field B is, $F = -e\,(v \times B)$

$$F = -e\left[v_0\hat{i} \times B_0\hat{i}\right] = -e\,v_0\,B_0\,(\hat{i} \times \hat{j})$$

$$\Rightarrow \quad = -ev_0B_0\hat{k} \qquad (\because \hat{k} = \hat{i} \times \hat{j})$$

So, the force is perpendicular to v and B, both as the force is ⊥ to the velocity so the magnitude of v will not change, so momentum is (= mv) will remain same or constant in magnitude. Hence,

de-Broglie wavelength $\lambda = \frac{h}{mv}$ remians constant.

7. **(a)** de-Brogile wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \qquad ...(i)$$

Force on electron

$$\Rightarrow \quad F = -eE = (-e)\left(-E_0\hat{i}\right) = eE_0\hat{i}$$

Acceleration of electron

$$a = \frac{F}{m} = \frac{eE_0\hat{i}}{m} \qquad (\because F = ma)$$

Velocity of electron after time t, is $v = (v_0 + at)$

$$v = v_0\hat{i} + \left(\frac{eE_0\hat{i}}{m}\right)t = \left(v_0 + \frac{eE_0}{m}t\right)\hat{i}$$

$$v = v_0\left(1 + \frac{eE_0}{m}t\right)\hat{i}$$

Now for new de-Broglie wavelength associated with electron at time t is

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad \lambda = \frac{h}{v_0m\left[\left(1 + \frac{eE_0t}{mv_0}\right)\hat{i}\right]} = \frac{\lambda_0}{\left[1 + \frac{eE_0}{mv_0}t\right]}$$

$$\left[\because \lambda_0 = \frac{h}{mv_0}\right]\left[\hat{i} = 1\right]$$

$$\lambda = \frac{\lambda_0}{\left[1 + \left(\frac{eE_0}{mv_0}\right)t\right]}$$

8. **(c)** We know that,

de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0}$$

Force on moving electron due to eletric field,

$$E = F = -eF = -eE_0\hat{j}$$

Acceleration in electron due to force

$$F = ma \text{ or } a = \frac{F}{m} = \frac{-eE_0\hat{j}}{m}$$

It is acting along negative y – axis direction and the initial velocity of electron along x – axis,

$$v_{x_0} = v_0\hat{i}$$

Initial velocity of electron along y – axis,

$$v_{y_0} = 0.$$

So, velocity of electron after time t along y – axis,

$$v = u + at \qquad (\because u = 0)$$

$$v_y = 0 + \left(\frac{eE_0}{m}\hat{j}\right)t = -\frac{eE_0}{m}t\hat{j}$$

Magnitude of velocity of electron after time t is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(\frac{-eE_0}{m}t\right)^2}$$

$$\text{So,} \quad |v| = v_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

de-Broglie wavelength, $\lambda' = \frac{h}{mv}$

$$\text{So,} \quad = \frac{h}{mv_0\sqrt{1 + e^2E_0^2t^2/\left(m^2v_0^2\right)}}$$

$$\left[\because \lambda_0 = \frac{h}{mv_0}\right]$$

$$\lambda' = \frac{\lambda_0}{\sqrt{1+e^2E_0^2t^2/m^2v_0^2}}$$

Hence, option (c) is verified.

Multiple Choice Questions (More Than One Options)

1. **(c, d)** de-Broglie wavelength

As we know that, $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$,

$$\begin{bmatrix} h = 6.6\times10^{-34}\text{Js} \\ m = 9\times10^{-31}\text{kg} \end{bmatrix}$$

Now we consider each option

(a) $(\lambda_1 = 10\text{ nm} = 10\times10^{-9}\text{m} = 10^{-8}\text{ m})$

So, $v_1 = \frac{6.6\times10^{-34}}{(9\times10^{-31})\times10^{-8}}$

$= \frac{2.2}{3}\times10^5$

$\approx 10^5\text{ m/s} < (3\times10^8 \text{ speed of light})$

(b) $\lambda_2 = 10^{-1}\text{nm} = 10^{-1}\times10^{-9}\text{ m} = 10^{-10}\text{ m}$

So, $v_2 = \frac{6.6\times10^{-34}}{(9\times10^{-31})\times10^{-10}}$

$\approx 10^7\text{ m/s}$

$< 3\times10^8$ (speed of ligh)

(c) $\lambda_3 = 10^{-4}\text{nm} = 10^{-4}\times10^{-9}\text{ m} = 10^{-13}\text{ m}$

So, $v_3 = \frac{6.6\times10^{-34}}{(9\times10^{-31})\times10^{-13}}$

$\approx 10^{10}\text{ m/s}$

$> 3\times10^8$ (speed of light)

(d) $\lambda_4 = 10^{-6}\text{nm} = 10^{-6}\times10^{-9}\text{ m} = 10^{-15}\text{ m}$

So, $v_4 = \frac{6.6\times10^{-34}}{9\times10^{-31}\times10^{-15}}$

$\approx 10^{12}\text{ m/s}$

$> 3\times10^8$ (speed of ligh)

So, options (c) and (d) are correct as v_3 and v_4 is greater than 3×10^8 m/s (speed of light).

2. **(a, c)** de-Broglie wavelength $\lambda = \frac{h}{mv}$

where, $p = mv$

So, $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$

Let, h is a constant

then, $p \propto \frac{1}{\lambda} \Rightarrow \frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1}$

$\therefore \quad (\lambda_1 = \lambda_2) = \lambda$ (given)

So, $\frac{p_1}{p_2} = \frac{\lambda}{\lambda} = 1 \Rightarrow p_1 = p_2$

Verified answer (a)

Hence their momenta is same.

Also, $E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2\times m}{m}$

$= \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$

$E \propto \frac{1}{m}$ [as $p_1 = p_2$= constant]

$\therefore \frac{E_1}{E_2} = \frac{m_2}{m_1} < 1 \Rightarrow E_1 < E_2$ [$\because m_1 > m_2$ (given)]

verified answer (c).

3. **(b, c)** We know that,

de-Broglie wavelength for electron

$\lambda_0 = \frac{h}{m_e v_e}, \quad \lambda_p = \frac{h}{m_p v_p}$

where Mass of electron = m_e,

Mass of photon = m_p,
Velocity of electron = v_e and Velocity of photon = v_p

$$= \frac{h}{m_e(c/100)} = \frac{100h}{m_e c}(\text{given}) \quad ....(i)$$

Kinetic energy,

$$E_e = \frac{1}{2} m_e v_e^2,$$

$$\Rightarrow \quad m_e v_e = \sqrt{2E_e m_e}$$

so, $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$, squaring both sides

$$\lambda_e^2 = \frac{h^2}{2m_e E_e} \quad ....(ii)$$

$$\Rightarrow \quad E_e = \frac{h^2}{2\lambda_e^2 m_e}$$

The energy for photon of wavelength λ_p,

$$E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e} \quad (\text{given})\left[\because \lambda_p = 2\lambda_e\right]$$

$$\therefore \quad \frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2\lambda_e^2 m_e}{h^2}$$

$$= \frac{\lambda_e m_e C}{h} = \frac{100h}{m_e c} \times \frac{m_e c}{h} = \frac{100}{1}$$

they, $\frac{E_e}{E_p} = \frac{1}{100} = 10^{-2}$

Now for electron, $p_e = m_e v_e = m_e \times c/100$

$$\left(\because v_e = \frac{c}{100}\right)$$

So, $\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$

4. **(a, b, c)** As given that, the heat energy required to convert 1 kg of ice at 0°C to water at 0° C is Q=ml

So, energy spent to convert ice into water

= mass × latent heat

= (1000 g) × (80 cal/g)

= 80000 cal

Let time T is taken by radiation to melt the ice at 0° C then,

Energy of photons used = nT × E = nT × hν

[$\because$ E = hν]

So, $\quad nTh\nu = mL \Rightarrow T = \frac{mL}{nh\nu}$

$\therefore \quad T \propto \frac{1}{n}$, when ν is constant.

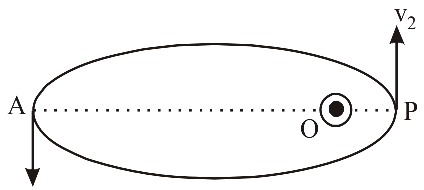
$T \propto \frac{1}{\nu}$, when n is fixed.

$\Rightarrow \quad T \propto \frac{1}{n\nu}$.

So, T is constant, if nν is constant and if nν increases then, T decreases.

5. **(b, d)** The de-Broglie wavelength of the particle varies cyclically between two values λ_1 and λ_2, if is possible when if particle is moving in an elliptical orbit with origin as its one focus.

Consider the figure given below



Let v_1, v_2 be the speed of particle at A and B respectively and origin is at O. If λ_1, λ_2 are the de-Broglie wavelengths associated with particle while moving at A and B respectively. Then,

$$\lambda_1 = \frac{h}{mv_1} \text{ and } \lambda_2 = \frac{h}{mv_2}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1}$$

since($\therefore \lambda_1 > \lambda_2$) given

$\therefore \quad v_2 > v_1$

By law of conservation of angular momentum, the speed of the particle moves faster when it is closer to focus.

From figure, we note that origin O is closed to P than A.

So, the object is close to B than A or the partice is nearer to the origin when wavelength is λ_2 then when wavelength is λ_1.

Very Short Answer Questions

1. As, $\lambda = \frac{h}{\sqrt{2mK}}$ or $\lambda = \frac{h}{\sqrt{2mqv}}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2 m_p q_p \times v_p}} \times \frac{\sqrt{2 m_\alpha q_\alpha v_\alpha}}{h}$$

Given,

$m_\alpha = 4m_p$

$q_\alpha = 2e;$

$q_p = e$

$v_p = v_\alpha = v$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_p q_p}} = \frac{\sqrt{4m_p \times 2e}}{\sqrt{m_p \times e}} = \sqrt{8}$$

$$\lambda_p = \sqrt{8}\lambda_\alpha$$

So, the de-Broglie wavelength of proton is $\sqrt{8}$ times wavelength of α – particle.

15. (i) As given that, an electron absorbs 2 photons each of frequency ν then $\nu' = 2\nu$ where, ν' is the frequency of emitted electron, maximum energy of emitted electron $(E_{max}) = h\nu - \phi_0$ given $(\nu' = 2\nu)$

Now, $E_{max} = h(2\nu) - \phi_0 = 2h\nu - \phi_0$

(ii) The probability of absorbing 2 photons by electron is very low due to their mass difference so possibilities of such emission will be negligible.

16. We know that as the wavelength of photon increase, its frequency decreases or energy increases.

Case first- when the materials which absorb photons of shorter wavelength has the energy of the incident photon on the material is high and the energy of emitted photon is low when it has a longer wavelength.

Case second- the energy of the incident photon is low for the substances which has to absorb photons of larger wavelength and energy of emitted photon is high to emit light of shorter wavelength. So according to this statement material has to supply the energy for the emission of photons.

So, it is not possible for a stable substances material.

17. In photoelectric effect, we can observe that most electrons knocked by photons are scattered into the metal by absorbing a photon.

The number of emitted electrons are very smaller than number of photons absorbed.

So, all the electrons that absorb a photon doesn't come out as from metal surface. Only a few come out of metal whose energy becomes greater than the work function of metal.

18. Let E_1 and E_2 are the energies given by one photon X-rays and visible rays then

$E_1 = h\nu_1$

$E_2 = h\nu_2$

So, Let wavelength of X-rays is λ_1 and the wavelength of visible light is λ_2.

Given,

$P = 100$ W

$\lambda_1 = 1$ nm

and, $\lambda_2 = 500$ nm

Let n_1 and n_2 are number of photons of X-rays and visible light emitted from two sources per sec.

$$\text{So,} \quad \frac{E}{t} = P = n_1 \frac{hc}{\lambda_1} = n_2 \frac{hc}{\lambda_2} \quad [\because E = nh\nu]$$

$$\Rightarrow \quad \frac{n_1}{\lambda_1} = \frac{n_2}{\lambda_2} \qquad [\because \nu = \frac{c}{\lambda}]$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1\,\text{nm}}{500\,\text{nm}} \quad \left[\begin{array}{c}\because \text{Given} \\ \lambda_1 = 1\,\text{nm} \\ \lambda_2 = 500\,\text{nm}\end{array}\right]$$

$$n_1 : n_2 = 1 : 500$$

Short Answer Questions

1. During photoelectric emission, when photons strike to metal surface, photons transfer their momentum to atoms of metal by decreasing its own speed upto zero. At microscopic level, atoms of a metal absorb the photon and its momentum is transferred mainly to the nucleus and electrons.

The excited electron is emitted. So, the conservation of momentum is to be considered as the momentum of incident photon transferred to the nucleus and electrons.

2. We know that

K_{max} (maximum energy) $= h\nu - \phi$

As given that,

For the first case

Wavelength of light $\lambda_1 = 600$ nm

For the second case

Wavelength of light $\lambda_2 = 400$ nm

Let the maximum energies of emitted electron are K_1 and K_2

Also maximum kinetic energy for the second condition is equal to the twice of the kinetic energy in first condition.

i.e., $K_{max2} = 2K_{max1}$

$$K_{max_1} = \frac{hc}{\lambda_1} - \phi \qquad (\because K_{max} = h\nu - \phi)$$

Here, $$K_{max_2} = \frac{hc}{\lambda_2} - \phi$$

$$\Rightarrow \quad 2K_{max_1} = 2\left(\frac{hc}{\lambda_1} - \phi\right)$$

Put (K_{max1}) in above equation

$$\Rightarrow \quad 2\left(\frac{1230}{600} - \phi\right) = \left(\frac{1230}{400} - \phi\right)$$

$[\because hc \simeq 1230 \text{eVnm}]$

$$\Rightarrow \quad \phi = \frac{1230}{1200} = 1.03\text{eV}$$

3. Given, electron revalves in circular path so, Δx

$\Delta x = 1\text{nm} = 10^{-9}$ m, $\Delta p = ?$

As $\Delta x' \Delta p \approx h$

$$\therefore \quad \Delta p = \frac{h}{\Delta x'} = \frac{h}{2\pi\Delta x} \qquad (\because \Delta x' = 2\pi\Delta x)$$

$$\left(\because h = 6.62 \times 10^{-34}\right)$$

$$= \frac{6.62 \times 10^{-34}\text{ JS}}{2 \times (22/7)(10^{-9})\text{m}} = 1.05 \times 10^{-25} \text{ kg m/s}$$

So energy, $$E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \qquad (\because p \approx \Delta p)$$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \text{J}$$

$$\left(\because n = 9.1 \times 10^{-31}\right)$$

$$\Rightarrow \quad = \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{eV}$$

$= 3.8 \times 10^{-2}$ eV.

4. We know that, energy of photons $E = nh\nu$

Let n_A is the number of photons falling per second of beam A and n_B is the number of photons fallings per second of beam B.

So, $n_A = 2n_B$

Energy of falling photon of beam $E_A = h\nu_A n_A$

Energy of falling photon of beam $E_B = h\nu_B n_B$

Now, according to question,

enrgy or intensity of A = energy or intensity of B

$$\therefore \quad n_A h\nu_A = n_B h\nu_B$$

$$\Rightarrow \quad \frac{\nu_A}{\nu_B} = \frac{n_B}{n_A} = \frac{n_B}{2n_B} = \frac{1}{2}$$

$$\Rightarrow \quad \nu_B = 2\nu_A$$

So the frequency of source beam B is twice the source of frequency beam A.

5. By de-Broglie wavelength

$$P = \frac{h\lambda}{\lambda}, \quad P_A = \frac{h\lambda}{\lambda_A}, \quad P_B = \frac{h\lambda}{\lambda_B}, P_C = \frac{h\lambda}{\lambda_C}$$

Given from conservation of momentum,

$$|p_C| = |p_A| + |p_B|$$

$$\Rightarrow \frac{h}{\lambda_C} = \frac{h}{\lambda_A} + \frac{h}{\lambda_B} \quad \left[\because \lambda = \frac{h}{mv} = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}\right]$$

$$\Rightarrow \frac{h}{\lambda_C} = \frac{h\lambda_B + h\lambda_A}{\lambda_A\lambda_B}$$

$$\Rightarrow \frac{\lambda_C}{h} = \frac{\lambda_A\lambda_B}{h\lambda_A + h\lambda_B} \Rightarrow \lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A + \lambda_B}$$

Case I if both p_A and p_B are positive.

then $\lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A + \lambda_B}$

Case II if both p_A and p_B are negative,

So, $\lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A + \lambda_B}$

Case III if $p_A > 0, p_B < 0$ i.e., p_A is positive and p_B is negative,

So, $\frac{h}{\lambda_C} = \frac{h}{\lambda_A} - \frac{h}{\lambda_B} = \frac{(\lambda_B - \lambda_A)h}{\lambda_A\lambda_B}$

$$\Rightarrow \lambda_C = \frac{\lambda_A\lambda_B}{\lambda_B - \lambda_A}$$

Case IV, if $p_A < 0, p_B > 0$, i.e., p_A is negative and p_B is positive,

So, $\frac{h}{\lambda_C} = \frac{-h}{\lambda_A} + \frac{h}{\lambda_B}$

$$\Rightarrow = \frac{(\lambda_A - \lambda_B)h}{\lambda_A\lambda_B} \Rightarrow \lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A - \lambda_B}$$

6. As given that d = 0.1 nm, (spacing distance).

Now, by Bragg's law of diffraction

$\theta = 30° \Rightarrow n = 1$

$2d \sin\theta = n\lambda$ condition for nth maxima is

$2d \sin\theta = n\lambda \Rightarrow 2 \times 0.1 \times \sin 30 = 1\lambda$

$$\Rightarrow \lambda = 0.1\,\text{nm} \Rightarrow 10^{-10}\,\text{m}$$

Now, $\lambda = \frac{h}{mv} = \frac{h}{p} \quad \left(\because p = \frac{hc}{\lambda} \text{ or } \lambda = \frac{hc}{p}\right)$

$$\Rightarrow p = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{10^{-10}} \quad (\because p = mv)$$

$$\Rightarrow = 6.62 \times 10^{-24}\,\text{kg-m/s}$$

Now, $KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$

$$= \frac{1}{2} \times \frac{(6.62 \times 10^{-24})^2}{1.67 \times 10^{-27}}\,\text{J}$$

$$= \frac{1}{2} \times \frac{(6.62 \times 10^{-24})^2}{1.67 \times 10^{-27} \times 1.6 \times 10^{-19}}\,\text{eV}$$

$$\Rightarrow \lambda = 0.082\,\text{eV}$$

Long Answer Questions

1. As given that,

Area of square sheet

$(A) = (10^{-2})m^2 = 10^{-2} \times 10^{-2}\,m^2 = 10^{-4}\,m^2$

Thickness (d) = 10^{-3} m

Current (i) = $100 \times 10^{-6}A = 10^{-4}A$

Intensity (I) = 100 W/m²

$\Rightarrow \lambda = 660\,\text{nm} = 660 \times 10^{-9}\,\text{m}$

$\rho_{Na} = 0.97\,\text{kg/m}^3$

Avogadro's number = 6×10^{26} kg atom

Volume of sodium target = A × d

$= 10^{-4} \times 10^{-3} = 10^{-7}\,m^3$

We know that, 6×10^{26} atoms of Na weights = 23 kg

So, volume of 6×10^{6} Na atoms $= \frac{23}{0.97}\,m^3$

volume occupied by one Na atom

$$= \frac{23}{0.97 \times (6 \times 10^{26})} = 3.95 \times 10^{-26}\,m^3$$

Number of atoms in (Na) target

$$= \frac{10^{-7}}{3.95 \times 10^{-26}} = 2.53 \times 10^{18}\ \text{Atoms,}$$

Let n be the number of photons falling per second on the target,

Energy of each photon = nhν $\left(\because v = \frac{c}{\lambda}\right)$

Total energy falling per second on target

$$= \frac{nhc}{\lambda} = IA \quad (\text{Intensity} \times \text{Area})$$

$$\therefore \quad n = \frac{IA\lambda}{hc}$$

$$\Rightarrow \quad = \frac{100\times10^{-4}\times(660\times10^{-9})}{(6.62\times10^{-34})\times(3\times10^{8})} = 3.3\times10^{16}$$

Let P be the probability of emission of photo electrons per atom per photon.

The number of photoelectrons emitted per second

$N = P.n$ (No. of sodiuma atom)

$N = P \times n \times (Na)$

$= P \times (3.3 \times 10^{16}) \times (2.53 \times 10^{18})$

Now, according to question,

$i = 100\ \mu A = 100 \times 10^{-6} = 10^{-4}$ A.

Current, $i = Ne$

Put i and N value in the equation

$\therefore 10^{-4} = P \times (3.3 \times 10^{16}) \times (2.53 \times 10^{18}) \times (1.6 \times 10^{-19})$

$$\Rightarrow P = \frac{10^{-4}}{(3.3\times10^{16})\times(2.53\times100^{18})\times(1.6\times10^{-19})}$$

$= 7.48 \times 10^{-21} \simeq 7.5 \times 10^{-21}$

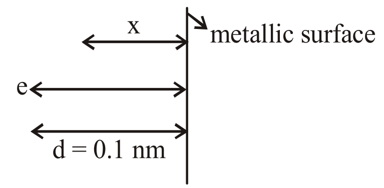
So that, the probability of emission by a single photon on a single atom is very much less than 1. It is due to this reason, the absorption of two photons by an atom is negligible.

2. We know that work done $= \int_0^{\infty} F.dr$

As given question, consider the figure given below

from figure, $d = 0.1$ nm, $= 10^{-10}$ m,

$$F = \frac{q^2}{4\times4\pi\varepsilon_0 d^2}$$



Let at any instance the electron be at distance x from metallic surface. Then, force of attraction between metal surface and electron it is

$$F_x = \frac{q^2}{4\times4\pi\varepsilon_0 x^2}$$

Work done by external agency in taking the electron from distance d to infinity is

$$W = \int_d^{\infty} F_x dx = \int_d^{\infty} \frac{q^2 dx}{4\times4\pi\varepsilon_0} \frac{1}{x^2}$$

$$= \frac{q^2}{4\times4\pi\varepsilon_0} \int_d^{\infty} (x^{-2})dx$$

$$= \frac{q^2}{4\times4\pi\varepsilon_0} \left[\frac{x^{-1}}{-}d\right]^{\infty}$$

$$= \frac{-q^2}{4\times4\pi\varepsilon_0} \left[\frac{1}{\infty} - \frac{1}{d}\right]$$

$$= \frac{q^2}{4\times4\pi\varepsilon_0} \left[\frac{1}{d}\right]$$

$$= \frac{(1.6\times10^{-19})^2\times9\times10^9}{4\times10^{-10}} J$$

$$= \frac{(1.6\times10^{-19})^2\times(9\times10^9)}{(4\times10^{-10})\times(1.6\times10^{-19})} eV = 3.6\ eV$$

3. (i) As given that thresholed frequency of metal A is given by $\nu_{OA} = 5 \times 10^{14}$ Hz and

For metals B, $\nu_{OB} = 10 \times 10^{14}$ Hz

We know that

Work function, $\phi = h\nu_O$

$\Rightarrow \quad \phi_O = \propto \nu_O$

$$\frac{\phi_{OA}}{\phi_{OB}} = \frac{h\nu_{OA}}{h\nu_{OB}}$$

So, $\frac{\phi_{OA}}{\phi_{OB}} = \frac{5\times10^{14}}{10\times10^{14}} = \frac{1}{2}$

$\phi_{OB} = 2\phi_{OA}$

So, $\phi_{OA} < \phi_{OB}$

Hence, work function of B is higher than A.

(ii) For metal A, $h = \frac{2\times e}{(10-5)10^{14}}$

or $h = \frac{2e}{5\times10^{14}} = \frac{2\times1.6\times10^{-19}}{5\times10^{14}}$

$h = 6.4\times10^{-34}$ Js

For metal B, $h = \frac{(2.5-0)e}{(15-10)10^{14}}$

or $h = \frac{2.5\times e}{5\times10^{14}} = \frac{2.5\times1.6\times10^{-19}}{5\times10^{14}}$

$= 8\times10^{-34}$ Js

So, the value of h from experiment for metals A and B is different. Hence, experiment is not consistent with theory.

4. As collision is elastic, so laws of conservation of momentum and kinetic energy are obeyed.

According to law of conservation of momentum,

$m_A v + m_B(0) = m_A v_1 + m_B v_2$

$\Rightarrow m_A(v - v_1) = m_B v_2$ ...(i)

According to law of conservation of kinetic energy

$$\left(\frac{1}{2}m_A v^2 + \frac{1}{2}m_B(0)^2 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_2^2\right)$$

$$\frac{1}{2}m_A v^2 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_2^2$$

$$\Rightarrow m_A\left(v^2 - v_1^2\right) = m_B v_2^2$$

$$\Rightarrow m_A(v - v_1)(v + v_1) = m_B v_2^2 \quad ...(ii)$$

Dividing Eq. (ii) by Eq. (i),

we get, $v + v_1 = v_2$ or $v = v_2 - v_1$ ...(iii)

Solving Eqs. (i) and (iii), we get,

$(m_A v - m_A v_1 = m_B(v + v_1) = m_B v + m_B v_1)$

$(m_A - m_B)v = v_1(m_A + m_B)$

So, $v_1 = \left(\frac{m_A - m_B}{m_A + m_B}\right)v$ and $v_2 = \left(\frac{2m_A}{m_A + m_B}\right)v$

$$\lambda_{initial} = \frac{h}{m_A v}$$

$$\lambda_{final} = \frac{h}{m_A v_1} = \frac{h(m_A + m_B)}{m_A(m_A - m_B)v}$$

Change in de-Broglie wavelength ($\Delta\lambda$)

$= \lambda_{final} - \lambda_{initial}$

$$= \frac{h}{m_A v}\left[\frac{m_A + m_B}{m_A - m_B} - 1\right] = \frac{2m_B h}{m_A(m_A - m_B)v}$$

5. Given, $P = 20$ W, $\lambda = 5000$ Å $= 5000\times10^{-10}$ m

$d = 2$ m, $\phi_0 = 2$ eV, $r = 1.5$ A $= 1.5\times10^{-10}$ m

(i) Let number of photon emitted by bulb per second then

$$P = n'h\left(\frac{c}{\lambda}\right)$$

$$n' = \frac{p}{hv/\lambda} = \frac{p\lambda}{hc}$$

$$= \frac{20\times\left(5000\times10^{-10}\right)}{\left(6.62\times10^{-34}\right)\times\left(3\times10^{8}\right)}$$

$\Rightarrow 5\times10^{19}\,s^{-1}$ (no of photons per sec)

Number of photons emitted by bulb per second $n' = 5\times10^{19}$

(ii) Energy of the incident photon $E = hv$

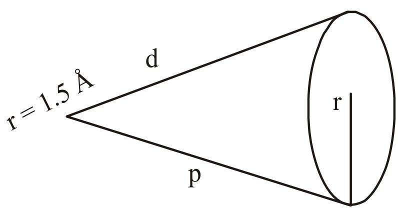
$$= \frac{hc}{\lambda} = \frac{\left(6.62\times10^{-34}\right)\times\left(3\times10^{8}\right)}{5000\times10^{-10}}\,j$$

$$= \frac{\left(6.62\times10^{-34}\right)\left(3\times10^{8}\right)}{5000\times10^{-10}\times1.6\times10^{-19}}\,eV$$

$= 2.48$ eV

As the energy is greater than 2 eV, i.e. the work function of metal surfae hence photoelectric emission takes place.

(iii) Consider the figure



Let Δt be the time spent in getting the energy ϕ (work function of metal).

Energy received by atomic disk in Δt time is

$E = P \times A.\,\Delta t = P \times \pi r^2.\,\Delta t$

energy transfered by bulb in full solid angle $4\pi d^2$ to atoms $= 4\pi d^2\%$

So, $P\pi r^2\,\Delta t = 4\pi d^2\%$

$$\frac{P}{4\pi d^2} \times \pi r^2 \Delta t = \phi_0$$

$$\Rightarrow \quad \Delta t = \frac{4\phi_0 d^2}{Pr^2}$$

$$= \frac{4 \times (2 \times 1.6 \times 10^{-19}) \times 2^2}{20 \times (1.5 \times 10^{-10})^2} \approx 11.4 \text{ sec}$$

(iv) Number of photons received by one atomic disc in time Δt is

$$N = \frac{n' \times \pi r^2}{4\pi d^2} \times \Delta t$$

$$\Rightarrow \quad = \frac{n' r^2 \Delta t}{4d^2}$$

$$= \frac{(5 \times 10^{19}) \times (1.5 \times 10^{-10})^2 \times 11.4}{4 \times (2)^2} \approx 1$$

(1 photon per atom)

(v) As time of emission of electrons is 11.4 sec

So, the photoelectric emission is not instantaneous in the problem. It takes about 11.4 sec in photoelectric emission, there is an collision between incident photon and free electron of the metal surface which lasts for very very short interval of time ($\approx 10^{-9}$ S), so we say photoelectric emission is instantaneous.

ATOMS

SUMMARY

1. Atom, as a whole, is electrically neutral and therefore contains equal amount of positive and negative charges.
2. In *Thomson's model*, an atom is a spherical cloud of positive charges with electrons embedded in it.
3. In *Rutherford's model*, most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus (typically one by ten thousand the size of an atom), and the electrons revolve around it.
4. Rutherford nuclear model has two main difficulties in explaining the structure of atom: (a) It predicts that atoms are unstable because the accelerated electrons revolving around the nucleus must spiral into the nucleus. This contradicts the stability of matter. (b) It cannot explain the characteristic line spectra of atoms of different elements.
5. Atoms of each element are stable and emit characteristic spectrum. The spectrum consists of a set of isolated parallel lines termed as line spectrum. It provides useful information about the atomic structure.
6. The atomic hydrogen emits a line spectrum consisting of various series. The frequency of any line in a series can be expressed as a difference of two terms;

   Lyman series: $\nu = Rc\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$; $n = 2, 3, 4,...$

   Balmer series: $\nu = Rc\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$; $n = 3, 4, 5,...$

   Paschen series: $\nu = Rc\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$; $n = 4, 5, 6,...$

   Brackett series: $\nu = Rc\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$; $n = 5, 6, 7,...$

   Pfund series: $\nu = Rc\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$; $n = 6, 7, 8,...$
7. To explain the line spectra emitted by atoms, as well as the stability of atoms, Niel's Bohr proposed a model for hydrogenic (single elctron) atoms. He introduced three postulates and laid the foundations of quantum mechanics:
   (a) In a hydrogen atom, an electron revolves in certain stable orbits (called stationary orbits) without the emission of radiant energy.
   (b) The stationary orbits are those for which the angular momentum is some integral multiple of $h/2\pi$. (Bohr's quantisation condition.) That is $L = nh/2\pi$, where n is an integer called a quantum number.
   (c) The third postulate states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency (ν) of the emitted photon is then given by

   $h\nu = E_i - E_f$

   An atom absorbs radiation of the same frequency the atom emits, in which case the electron is transferred to an orbit with a higher value of n.

   $E_i + h\nu = E_f$

8. As a result of the quantisation condition of angular momentum, the electron orbits the nucleus at only specific radii. For a hydrogen atom it is given by

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$$

The total energy is also quantised:

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$$

$$= -13.6 \text{ eV}/n^2$$

The $n = 1$ state is called ground state. In hydrogen atom the ground state energy is -13.6 eV. Higher values of n correspond to excited states ($n > 1$). Atoms are excited to these higher states by collisions with other atoms or electrons or by absorption of a photon of right frequency.

9. de Broglie's hypothesis that electrons have a wavelength $\lambda = h/mv$ gave an explanation for Bohr's quantised orbits by bringing in the wave-particle duality. The orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.

10. Bohr's model is applicable only to hydrogenic (single electron) atoms. It cannot be extended to even two electron atoms such as helium. This model is also unable to explain for the relative intensities of the frequencies emitted even by hydrogenic atoms.

POINTS TO PONDER

1. Both the Thomson's as well as the Rutherford's models constitute an unstable system. Thomson's model is unstable electrostatically, while Rutherford's model is unstable because of electromagnetic radiation of orbiting electrons.
2. What made Bohr quantise angular momentum (second postulate) and not some other quantity? Note, h has dimensions of angular momentum, and for circular orbits, angular momentum is a very relevant quantity. The second postulate is then so natural!
3. The orbital picture in Bohr's model of the hydrogen atom was inconsistent with the uncertainty principle. It was replaced by modern quantum mechanics in which Bohr's orbits are regions where the electron may be found with large probability.
4. Unlike the situation in the solar system, where planet-planet gravitational forces are very small as compared to the gravitational force of the sun on each planet (because the mass of the sun is so much greater than the mass of any of the planets), the electron-electron electric force interaction is comparable in magnitude to the electron-nucleus electrical force, because the charges and distances are of the same order of magnitude. This is the reason why the Bohr's model with its planet-like electron is not applicable to many electron atoms.
5. Bohr laid the foundation of the quantum theory by postulating specific orbits in which electrons do not radiate. Bohr's model include only

one quantum number n. The new theory called quantum mechanics supportes Bohr's postulate. However in quantum mechanics (more generally accepted), a given energy level may not correspond to just one quantum state. For example, a state is characterised by four quantum numbers (n, l, m, and s), but for a pure Coulomb potential (as in hydrogen atom) the energy depends only on n.

6. In Bohr model, contrary to ordinary classical expectation, the frequency of revolution of an electron in its orbit is not connected to the frequency of spectral line. The later is the difference between two orbital energies divided by h. For transitions between large quantum numbers (n to $n – 1$, n very large), however, the two coincide as expected.
7. Bohr's semiclassical model based on some aspects of classical physics and some aspects of modern physics also does not provide a true picture of the simplest hydrogenic atoms. The true picture is quantum mechanical affair which differs from Bohr model in a number of fundamental ways. But then if the Bohr model is not strictly correct, why do we bother about it? The reasons which make Bohr's model still useful are:
   (i) The model is based on just three postulates but accounts for almost all the general features of the hydrogen spectrum.
   (ii) The model incorporates many of the concepts we have learnt in classical physics.
   (iii) The model demonstrates how a theoretical physicist occasionally must quite literally ignore certain problems of approach in hopes of being able to make some predictions. If the predictions of the theory or model agree with experiment, a theoretician then must somehow hope to explain away or rationalise the problems that were ignored along the way.

EXERCISES

12.1 Choose the correct alternative from the clues given at the end of the each statement:

(a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

(b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/ Rutherford's model.)

(c) A *classical* atom based on .... is doomed to collapse. (Thomson's model/ Rutherford's model.)

(d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in .... (Thomson's model/ Rutherford's model.)

(e) The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

12.2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

12.3 What is the shortest wavelength present in the Paschen series of spectral lines?

12.4 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

12.5 The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

12.6 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon.

12.7 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1$, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

12.8 The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

12.9 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

12.10 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^{4} m/s. (Mass of earth $= 6.0 \times 10^{24}$ kg.)

ADDITIONAL EXERCISES

12.11 Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

12.12 The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

12.13 Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n-1)$. For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

12.14 Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10}m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, me, and e and confirm that its numerical value has indeed the correct order of magnitude.

12.15 The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

12.16 If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

12.17 Obtain the first Bohr's radius and the ground state energy of a *muonic hydrogen atom* [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton].

Chapter 12:- Atom

Multiple Choice Questions (MCQs)

1. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm (b) 27 pm
(c) 18 pm (d) 13 pm

2. The binding energy of a H – atom, considering an electron moving around a fixed nuclei (proton), is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2}\,(m = \text{electron mass})$$

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2}\,(m = \text{proton mass})$$

This last expression is not correct, because

(a) n would not be integral
(b) Bohr – quantisation applies only two electron
(c) the frame in which the electron is at rest is not inertial
(d) the motion of the proton would not be in circular orbits, even approximately.

As the mass of an electron is negligible as compaterd to Proton. So the centripetal force cannot Provide the electronstatic force $Fi = mv^2/r$

3. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force
(b) of the electrons colliding with each other
(c) of screening effects
(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

4. For the ground state, the electron in the H – atom has an angular momentum = h, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,

(a) because Bohr model gives in correct values of angular momentum
(b) because only one of these would have a minimum energy

(c) angular momentum must be in the direction of spin of electron
(d) because electrons go around only in horizontal orbits

5. O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms
(a) is not important because nuclear forces are short – ranged
(b) is as important as electrostatic force for binding the two atoms
(c) cancels the repulsive electrostatic force between the nuclei
(d) is not important because oxygen nucleus have equal number of neutrons and protons

6. Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced, is
(a) 10.20 eV (b) 20.40 eV
(c) 13.6 eV (d) 27.2 eV

Multiple Choice Questions (MCQs) (More than one option correct)

1. An ionised H – molecule consists of an electron and two protons. The protons are separated by a small distance of the order of angstrom. In the ground state.
(a) the electron would not move in circular orbit.
(b) the energy would be $(2)^4$ times that of a H – atom.
(c) the electrons, orbit would go around the protons
(d) the molecule will soon decay in a proton and a H – atom

2. The Bohr model for the spectra of a H – atom
(a) will not be applicable to hydrogen in the molecular form
(b) will not be applicable as it is for a He – atom
(c) is valid only at room temperature
(d) predicts continuous as well as discrete spectral lines

3. The Balmer series for the H – atom can be observed
(a) if we measure the frequencies of light emitted when an excited atom falls to the ground state
(b) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state
(c) in any transition in a H – atom
(d) as a sequence of frequencies with the higher frequencies getting closely packed

4. Let $E_n = \dfrac{-1me^4}{8\varepsilon_0^2 n^2 h^2}$ be the energy of the nth level of H – atom. If all the H – atoms are in the ground state and radiation of frequency $\dfrac{(E_2 - E_1)}{h}$ falls on it,
(a) it will not be absorbed at all
(b) some of atoms will move to the first excited state
(c) all atoms will be excited to the n = 2 state
(d) no atoms will make a transition to the n = 3 state

5. The simple Bohr model is not applicable to He atom because
(a) He^4 is an inert gas
(b) He^4 has neutrons in the nucleus
(c) He^4 has one more electron
(d) electrons are not subject to central forces

Very Short Answer Questions

1. The mass of a H – atom is less than the sum of the masses of a proton and electron. Why is this?
2. Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr model will be very close. Explain why?
3. When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy?
4. Would the Bohr formula for the H – atom remain unchanged if proton had a charge (+4/3)e and electron a charge (–3/4)e, where $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer.
5. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model?

Short Answer Questions

1. Positronium is just like a H – atom with the proton replaced by the positively charged anti–particle of the electron (called the positron which is as massive as the electron). What would be the ground state energy of positronium?
2. Assume that there is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He–atom.
3. Using Bohr model, calculate the electric current created by the electron when the H – atom is in the ground state.
4. Show that the first few frequencies of light that is emitted when electrons fall to nth level from levels higher than n, are approximate harmonics (i.e., in the ratio 1 : 2 : 3....) when n >>1.
5. What is the minimum energy that must be given to a H – atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

Long Answer Questions

1. The first four spectral in the Lyman series of a H – atom are λ = 1218Å, 1028Å, 974.3Å and 951.4Å. If instead of hydrogen, we consider deuterium, calculate the shift in the wavelength of these lines.
2. Deuterium was discoverd in 1932 by Harold Urey by measuring the small change in wavelength for a particular transition in ^1H and ^2H. This is because, the wavelength of transition depend to a certain extent on the nuclear mass. If nuclear motion is taken into account, then the electrons and nucleus revolve around their common centre of mass.
Such a system is equivalent to a single particle with a reduced mass μ, revolving around the nucleus at a distance equal to the electron – nucleus separation. Here $\mu = m_e M/(m_e + M)$, where M is the nuclear mass and m_e is the electronic mass. Estimate the percentage difference in wavelength for the 1st line of the Lyman series in ^1H and ^2H. (mass of ^1H nucleus is 1.6725×10^{-27} kg, mass of ^2H nucleus is 3.3374×10^{-27} kg, Mass of electron = 9.109×10^{-31} kg.)
3. If a proton had a radius R and the charge was uniformly distributed, calculate using Bohr theory, the ground state energy of a H – atom when (i) R = 0.1Å and (ii) R = 10Å.
4. In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom (This is called an Auger, electron). Assuming the nucleus to be massive, calculate the kinetic energy of an n = 4 Auger electron emitted by Chromium by absorbing the energy from a n = 2 to n = 1 transition.
5. The inverse square law in electrostatic is $|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}$ for the force between an electron and a proton. The $\left(\frac{1}{r}\right)$ dependence of |F| can be understood in quantum theory as being due to the fact that the particle of light (photon) is massless. If photons had a mass m_p, force would be modified to
$$|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right].\exp(-\lambda r) \text{ where } \lambda = \frac{m_p c}{\hbar}$$
and $\hbar = \frac{h}{2\pi}$. Estimate the change in the ground state energy of a H – atom if m_p were 10^{-6} times the mass of an electron.
6. The Bohr model for the H – atom relies on the Coulomb's law of electrostatics. Coulomb's law has not directly been verified for very short distances of the order of angstroms. Supposing Coulomb's law between two opposite charge $+q_1, -q_2$ is modified to
$$|F| = \frac{q_1 q_2}{(4\pi\varepsilon_0)}\frac{1}{r^2}, r \geq R_0$$
$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{R_0^2}\left(\frac{R_0}{r}\right)^{\varepsilon}, r \leq R_0$$
Calculate in such a case, the ground state energy of a H–atom, if E = 0.1, R_0 = 1Å.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. (c) According to Bohr's model of atom rading of an atom in ground stateme is $r = \frac{r_0}{z}$ where r_0 is Bohr's radius and z is a atomic number. Given $r_0 = 53$ pm

The atomic number of lithium is 3, therefore, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about $\frac{1}{3}$ times to that of Bohr radius.

So, the radius of lithim ion is $= \frac{r_0}{z} = \frac{53}{3} \approx 18$ pm.

2. (c) If one decides to work in a frame of reference where the electron is at rest, the given expression is not true as it forms the non – inertial frame of reference.

3. (a) The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. So the nuclear the electrons not being subject to a central force.

4. (a) Accroding Bohr's second postulate states that the electron evolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$ where h is the Planck's constant ($= 6.6 \times 10^{-34}$ J-s). So, the magnitude of angular momentum is kept equal to some integral multiple of $\frac{h}{2\pi}$, where, h is Planck's constant and thus, the Bohr model does not gives correct value of angular momentum.

5. (a) As we know that,

The nuclear force is too much stronger than the Coulomb force acting between charges or the gravitational forces between masses. The nuclear binding force has to dominate over the Coulomb repulsive force between protons inside the nucleus.

The nuclear force between two nucleons falls rapidly to zero as their distance is increase than a few femtometres. So in case of oxygen molecule, the distance between atoms of oxygen is larger as compared to the distance between nuclears in a necleus. So, nuclear force between the nuclei of the two oxygen atoms is not important because nuclear forces are short – ranged.

6. **(a)** We know that,

Electron on the lowest state of the atom, called the ground state have the lowest energy and the electron revolving in the orbit of smallest radius, the Bohr radius, r. The energy of this state (n = 1), E_1 is – 13.6 eV.

Total energy of two H – atoms in the ground state collide in elastically = 2 × (–13.6 eV) = –27.2 eV.

The maximum amount by which their combined kinetic energy is reduced when any one H–atom goes into first excited state after the inelastic collision. So that the total energy of the two H–atoms after the inelastic collision

$$= \left(\frac{13.6}{2^2}\right) + (13.6) = 17.0\text{eV}$$

$[\because \text{for excited state } (n = 2)]$.

So, maximum loss of their combined kinetic energy.

Due to inelastic collision

$= 27.2 - 17.0 = 10.2$ eV

Multiple Choice Questions (More Than One Options)

1. **(a, c)** As we know that,

A hydrogen molecule contain two electrons and two protons whereas ionised H – molecule consists of an electron and two protons.

In H – molecules two protons are separated by a small distance of the order of angstrom. In the ground state the electron would not move in circular orbits, orbit electron around the protons.

2. (a, b) The Bohr model for the spectrum of a H – atom will not be applicable for hydrogen molecular. It will not be applicable as it is for a He – atom. And also it does not depends on minor change in temperature.

3. (b, d) Balmer series for the H – atom can be observed if we measure the frequencies of light emitted due to transitions between higher excited states and the first excited state and as a sequence of frequencies with the higher frequencies getting closely packed. So spectrum lines become closer and electron jumps from higher energy level to ground.

4. (b, d) When all the H – atoms are in the ground state and radiation of photons is $(E_2 - E_1)$ an electron jumpts to next orbit then electron jumps in next energy level (n = 2) after receiving this energy equal to the $(E_2 - E_1)$ energy. The new state is its unstable state – electron jumps from E_2 to E_1 by radiating the energy of same frequency to $(E_2 - E_1)$ So, some of atoms will move to the first excited state and no atoms will make a transition to the n = 3 state.

5. (c, d) Bohr model is valid for H – atom only for one electron but Bohr model is not applicable to He^4 atom because He^4 has one more electron and electrons are not subject to central forces due to longer distance than nuclear size.

Very Short Answer Questions

1. We know that,

The difference in mass of a nucleus and its constituents, ΔM, is called the mass defect and is

$$\Delta M = [Zm_p + (A - Z)m_n] - M$$

and the binding energy is B = mass defect (ΔM) $\times c^2$.

So, the mass of a H – atom is

$m_p + m_e - \frac{B.E}{c^2}$, where B.E = 13.6 eV is the binding energy.

2. We know that,

If we remove one electron from the isotopes of He^4 and He^3, the energy levels, as worked out on the basis of Bohr model will be very close as both the nuclei are very heavy as compared to electron mass. Also after removing one electron from He^4 and He^3 atoms will have one electron as in hydrogen atoms, and the nucleus is much heavier then H atom. So, stability can remain and these atoms are very close to the H atom so the energy levels are as of Hydrogen and these atom will be very close.

3. When electron jumps from a higher energy to a lower energy level can appears in the form of electromagnetic radiation because electrons interact only electromagnetically.

4. As there is no interchange in the position of proton and electron only the magnitude of charge change as coulombian force will be same.

If proton had a charge (+4/3)e and electron a charge (–3/4)e, then the Bohr formula for H – atom remain same, since the Bohr formula involves only the product of the charges which remain constant for given values of charges.

5. We know that,

Bohr's postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$,

where h is planck's constant ($= 6.63 \times 10^{-34}$). Thus, the angular momentum (L) of the orbiting electron is quantised. i.e.,

$$L = \frac{nh}{2\pi}$$

According to Bohr model electrons having different energies belong to different levels having different values of n. So, their angular momentum will be different,

Hence, $$L = \frac{nh}{2\pi} \text{ or } L \propto n$$

Short Answer Questions

1. The total energy of the electron in the stationary states of the hydrogen atom is given by

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2} = (-13.6)\,eV \quad [\text{if } n = 1]$$

where signs are as usual and the m that occurs in the Bohr formula is the reduced mass of electron and proton in hydrogen atom, the total energy of the electron in the ground state of the hydrogen atom is –13.6 eV. For H–atom reduced mass m_e. Whereas for positronium, the reduced mass is

$$m \approx \frac{m_e}{2}$$, by putting m value in En,

$$E_n = \left(\frac{\left(\frac{-me}{2}\right)e^4}{8n^2\varepsilon_0^2h^2}\right)$$

Hence, the total energy of the electron in the ground state of the positronium atom is

$$E'_n = \frac{-13.6eV}{2} = -6.8\,eV\,(\text{at}, n = 1)$$

2. We know that,

The total energy of the electron in the nth stationary states of the hydrogen. Atom of hydrogen like atom of atomic number Z is

$$E_n = \frac{Z\,me^4}{8\varepsilon_0^2n^2h^2}, \; m_e \text{ is mass of electron}$$

For Z = 1 and n = 1; $E_n = -13.6$ eV

$$E_n = Z\frac{-13.6\text{ eV}}{n^2} \quad \left[\because \frac{me^4}{8\varepsilon_0^2h^2} = 13.6\text{ eV}\right]$$

For a He – nucleus with charge 2e and electrons of charge – e, the energy level in ground state is for Z = 4, n = 1

$$E_n = Z\frac{-13.6\text{ eV}}{n^2} = (4)\frac{-13.6\text{ eV}}{1^2} = -54.4\text{ eV}$$

(Total ground state energy).

So, the ground state will have two electrons each of energy E.

3. The electron in Hydrogen atom in ground state revolves on a circular path whose radius is equal to the Bohr radius (a_n). Let the velocity to electron of H atom is v, and radius of orbit is a_0.

So, the number of revolutions per unit time $= \frac{2\pi a_0}{v}$

The electric current is given by $i = \frac{q}{t}$, if q charge flows in time t. Here, q = e So, i = –eV

The electric current is given by $i = \frac{-2\pi a_0}{v}e$.

–ve sign shows that the direction of current is opposite to the direction of motion of electron.

4. The frequency of radiations in the spectrum of H atoms corresponding to the transition of electrons from (n + p) level to nth energy level can be expressed as a difference of two terms,

$$v_{mn} = cRZ^2\left[\frac{1}{(n+p)^2} - \frac{1}{n^2}\right]$$

where, m = n + p, (p = 1, 2, 3, ...) and R is Rydberg constant.

$$v_{mn} = cRZ^2\left[\frac{1}{n^2}\left[\frac{1}{\left(1+\frac{P}{n}\right)^2}\right] - \frac{1}{n^2}\right]$$

For p << n

$$v_{mn} = cRZ^2\left[\frac{1}{n^2}\left(1+\frac{P}{n}\right)^{-2} - \frac{1}{n^2}\right]$$

$$v_{mn} = cRZ^2\left[\frac{1}{n^2} - \frac{2p}{n^3} - \frac{1}{n^2}\right]$$

[By binomial theorem $(1 + x)^n = 1 + nx$ if $|x| < 1$]

$$v_{mn} = cRZ^2 \frac{2p}{n^3} \simeq \left(\frac{2cRZ^2}{n^3}\right) p$$

So, the first few frequencies of light that is emitted when electrons fall from (n + p) to the nth energy level, are approximate harmonic (i.e., in the ratio 1 : 2 : 3....) when (n >>> 1).

5. H_γ in Balmer series corresponds to transition n = 5 to n = 2. So, the electron in ground state i.e., from n = 1 must first be placed in state n = 5, energy of electron in ground state of H atom = –13.6 eV, energy of electron in nth energy level

$$= \left(\frac{-13.6}{n^2}\right).$$

Energy required for the transition from n = 1 ton = 5 is

$$\text{So, } E_1 = -13.6,\ E_5 = \frac{-13.6}{25}$$

$$H_\gamma = E_5 - E_1 = (-0.54 = 13.06 \text{ eV} + 13.6)$$

Since, angular momentum is conserved, so, angular momentum of H_γ photon = change in angular momentum of electron.

$$= L_5 - L_2 = 5\,h - 2h = 3h$$

$$= 3 \times 6.63 \times 10^{-34} = 19.89 \times 10^{-34}\ \text{kg}^{\ n^2/s}$$

Long Answer Questions

1. The total energy of the electron in the stationary states of the hydrogen atom is given by

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2 h^2}$$

where signs are as usual and the m that occurs in the Bohr formula is the reduced mass of electron and proton in hydrogen atom.

By Bohr's model,

$$h\nu_{if} = E_{n_i} - E_{n_f}$$

On simplifying,

$$\nu_{if} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Since, $\lambda \propto \frac{1}{\mu}$ ....(i)

Thus, $\lambda_{if} \propto \frac{1}{\mu}$

where μ is the reduced mass. (here, μ is used in place of m)

Reduced mass for (H atom)

$$H = \mu_H = \frac{m_e}{1 + \frac{m_e}{M}} = m_e\left(1 - \frac{m_e}{M}\right)$$

Reduced mass for (Deuterium)

$$D = \mu_D = m_e\left(1 - \frac{m_e}{2m}\right)$$

∴ (M = 2M)

$$= m_e\left(1 - \frac{m_e}{M}\right)\left(1 + \frac{m_e}{2M}\right)$$

Total energy for nth orbit

$E_n = \frac{-me^4}{8n^2\varepsilon_0^2 h^2}$ where m is the reduced mass of electron and proton in H-atom.

So, $(h\nu) = E_{n_i} - E_{n_f}$

$$h\nu = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

$$\left(\nu = \frac{c}{\lambda}\right) \text{ or } \left(\lambda \propto \frac{1}{\mu}\right)$$

If for hydrogen deuterium, the wavelength is $\frac{\lambda_H}{\lambda_D}$

From eq. (i)

$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} \simeq \left(1 + \frac{m_e}{2M}\right)^{-1} \simeq \left(1 - \frac{1}{2 \times 1840}\right)$$

$$\lambda_D = \lambda_H \times (0.99973)$$

On substituting the values, we have

So, lines are 1217.7Å, 1027.7Å, 974.04Å, 951.143Å.

2. We know that :

The total energy of the electron in the nth states of the H (hydrogen) like atom of atomic number Z is given by

$$E_n = -\frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

$$\Rightarrow \quad E_H = \frac{\mu_H Z^2 e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ and } E = hv$$

where signs are as usual and the μ that occurs in the Bohr formula is the reduced mass of electron and proton.

Let μ_H be the reduced mass of hydrogen and μ_D that of deuterium. Then, the energy for hydrogen is

$$h\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left(1-\frac{1}{4}\right) = \left[\frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\right]\times\frac{3}{4}$$

$(\therefore n_1 = 1, n_2 = 2, z = 1)$

So, frequency of the transition is

$$\nu_H = \frac{3}{4}\left[\frac{\mu_H e^4}{8\varepsilon_0^2 h^3}\right]$$

Similarly, frequency of the transition for Deuterium is

$$\nu_D = \frac{3}{4}\left[\frac{\mu_D e^4}{8\varepsilon_0^2 h^3}\right]$$

$\therefore \quad \Delta\lambda = \lambda_D - \lambda_H$

We know that, the percentage difference is

$$100\times\frac{\Delta\lambda}{\lambda_H} = \frac{\lambda_D - \lambda_H}{\lambda_H}\times 100 = \frac{\mu_D - \mu_H}{\mu_H}\times 100$$

$$\left[\because \mu_D = \frac{m_e M_D}{m_e + M_D}\right]$$

$$= \frac{\dfrac{m_e M_D}{(m_e + M_D)} - \dfrac{m_e M_H}{(m_e - M_H)}}{\dfrac{m_e M_H}{(m_e + M_H)}}\times 100$$

$$\left[\because \mu_H = \frac{m_e M_H}{m_e - M_H}\right]$$

$$= \left[\left(\frac{m_e + M_H}{m_e + M_D}\right)\frac{M_D}{M_H} - 1\right]\times 100$$

Since, $m_e << M_H << M_D$

$$\Rightarrow \quad = \left[\frac{M_H}{M_D}\times\frac{M_D}{M_H}\left(\frac{1+\dfrac{m_e}{M_H}}{1+\dfrac{m_e}{M_D}}\right) - 1\right]\times 100$$

$$= \left[\left(1+\frac{m_e}{M_H}\right)\left(1+\frac{m_e}{m_D}\right)^{-1} - 1\right]\times 100$$

$$\simeq \left[1 + \frac{m_e}{M_H} - \frac{m_e}{M_D} - 1\right]\times 100$$

[By binomial theorem, $(1+x)^n = 1 + nx$ is $|x| < 1$]

$$\simeq m_e\left[\frac{1}{M_H} - \frac{1}{M_D}\right]\times 100 \text{ (given } M_H, M_D)$$

$$= 9.1\times10^{-31}\left[\frac{1}{1.6725\times10^{-27}} - \frac{1}{3.3374\times10^{-27}}\right]\times 100$$

$$= 9.1\times10^{-4}[0.5979 - 0.2996]\times 100$$

$$= 2.714\times10^{-2}\%$$

3. Consider in H atom nucleus as a point charge electron then the electrostatic force of attraction between positively charged nucleus and negatively charged electrons (Coulombian force) provides necessary centripetal force of revolution.

$$\frac{mv^2}{r_B} = -\frac{e^2}{r_B^2}\cdot\frac{1}{4\pi\varepsilon_0} = \frac{+ke^2}{r_B^2} \quad \text{..... (i)}$$

$$\left(\because k = \frac{1}{4\pi\varepsilon_0}\right) \text{ (neglect – ve sign)}$$

By Bohr's postulates in ground state, $\left(\hbar = \frac{nh}{2\pi}\right)$

$$mvr_B = \hbar = \frac{nh}{2\pi}$$

So, $v = \frac{nh}{2\pi m r_B}$ Put v value in above eq. (i)

On solving , $\left(\frac{mn^2h^2}{4\pi^2 m^2 r_B^2 r_B} = \frac{ke^2}{r_B^2}\right)$

$$\Rightarrow \left(\frac{m\,h^2n^2}{4\pi^2 m^2 r_B^2}\cdot\frac{1}{r_B}\right) = +\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{r_B^2}$$

$$\therefore \frac{h^2}{4\pi^2 m}\cdot\left[\frac{4\pi\varepsilon_0}{e^2}\right] = \left[r_B = \frac{h^2}{4\pi^2 m\, ke^2}\right] = 0.51\text{Å}$$

[This is Bohr's radius]

The potential energy is given by

$$-\left(\frac{e^2}{4\pi r_0}\right)\cdot\frac{1}{r_B} = -27.2\text{ eV};$$

$$KE = \frac{mv^2}{2} = \frac{1}{2}m.\frac{h^2n^2}{4\pi^2 m^2 r_B^2} = +13.6\text{ eV}$$

Now, for an spherical nucleus of radius R,

If $R < r_B$, same result.

If $R >> r_B$ the electron moves inside the sphere with radius r'_B (r'_B = new Bohr radius).

Charge inside $r'^4_B = e\left(\frac{r'^3_B}{R^3}\right)$

$$\therefore \quad r'_B = \frac{h^2}{m}\left(\frac{4\pi\varepsilon_0}{e^2}\right)\frac{R^3}{r'^3_B}$$

$r'^4_B = (0.51\text{Å}).R^3$ [R = 10Å]

$= 510\ (\text{Å})^4$

[$\therefore \quad r'_B \simeq (510)^{1/4}$ Å $< R$]

$$KE = \frac{1}{2}mv^2 = \frac{m}{2}\cdot\frac{h^2}{m^2 r_B^2 4\pi^2} = \frac{h}{8m^2\pi}\cdot\frac{1}{r_B^2}$$

$$\left[\because v = \frac{n^2h^2}{4\pi^2 m^2 r'^2_B}\right]$$

$$= \frac{h^2}{8m^2 r_B^2 \pi^2}\cdot\left(\frac{r_B^2}{r'^2_B}\right) = (13.6\text{ eV})\frac{(0.51)^2}{(510)^{1/2}}$$

$$= \frac{3.54}{22.6} = 0.16\text{ eV} \quad \left[\because r'_B = (510)\frac{1}{4}\text{Å}\right]$$

$$PE = +\left(\frac{e^2}{4\pi\varepsilon_0}\right)\cdot\left(\frac{r'^2_B - 3R^2}{2R^3}\right)$$

$$= +\left(\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{r_B}\right)\cdot\left(\frac{r_B\left(r'^2_B - 3R^2\right)}{R^3}\right)$$

$$= +(27.2\text{ eV})\left[\frac{0.51\left(\sqrt{510} - 300\right)}{1000}\right]$$

$[\because R = 10\text{Å}]$

$$P.E = +(27.2\text{ eV}).\frac{-141}{1000} = -3.83\text{ eV}$$

K. E = 0.16 eV

4. The Chromium nucleus is massive, recoil momentum of the atom may be neglected and the entire energy of the transition may be considered transferred to the Auger electron. As there is a single valence electron in Cr, the energy states may be thought of as given by the Bohr model. The energy of the nth state

$$E_n = +Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is the Rydberg constant

for $n_1 = 1$, $n_2 = 2$ and $Z = 24$

The energy released in a transition from 2 to 1 is

$$\Delta E = Z^2R\left(1 - \frac{1}{4}\right) = \frac{3}{4}Z^2R.$$

The energy required to eject a n = 4 electron is

$$E_4 = Z^2R\frac{1}{16}.$$

Thus, the kinetic energy of the Auger electron is

$$= E_n - E_4 = \left(\frac{3}{4}Z^2R - \frac{Z^2R}{16}\right)$$

$$KE = Z^2R\left(\frac{3}{4} - \frac{1}{16}\right) = \frac{1}{16}Z^2R$$

$$= \frac{1}{16}\times 24\times 24\times 13.6\text{ eV} = 5385.6\text{ eV}$$

5. As given that, $m_p = 10^{-6}$ times, the mass of an electron, the energy associated with it is given by

$$E = m_pc^2 = 10^{-6}\times \text{electron mass}\times c^2$$
$$\approx 10^{-6}\times 0.5\text{ MeV}$$
$$\approx 10^{-6}\times 0.5\times 1.6\times 10^{-13}$$
$$E \approx 0.8\times 10^{-19}\text{ J}$$

The wavelength associated with is given by

$$\lambda = \frac{\hbar}{m_pc} = \frac{\hbar c}{m_pc^2} = \frac{10^{-34}\times 3\times 10^8\times 6.62}{0.8\times 10^{-19}}$$

$$\approx 2.4\times 10^{-7}\text{ m} >> \text{Bohr radius } r_3$$

$$|F| = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r^2}+\frac{\lambda}{r}\right]\exp(-\lambda r)$$

where, $\lambda^{-1} = \frac{\hbar}{m_pc} \approx 4\times 10^{-7}\text{ m} >> r_B$

$$\therefore \quad \left(\lambda << \frac{1}{r_B}\text{ i.e., }\lambda r_B << 1\right)$$

$$U(r) = -\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{\exp(-\lambda r)}{r}$$

$$\therefore \quad mvr = \hbar \quad \therefore v = \frac{\hbar}{mr}$$

Also, $\frac{mv^2}{r} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2}+\frac{\lambda}{r}\right]$

$$\therefore \quad \frac{\hbar^2}{mr^3} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2}+\frac{\lambda}{r}\right]$$

$$\therefore \quad \frac{\hbar^2}{m} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[r+\pi r^2\right]$$

If $\lambda = 0$; $r = r_B = \frac{\hbar}{m}\cdot\frac{4\pi\varepsilon_0}{e^2}$

$$\frac{\hbar^2}{m} = \frac{e^2}{4\pi\varepsilon_0}\cdot r_B$$

Since, $\lambda^{-1} >> r_B$, put $r = r_B + \delta$

$r_B = r + \lambda r^2$

$r_B = (r_B + \delta) + \lambda(r_B + \delta)^2$

$\therefore \quad r_B = r_B + \delta + 1\left(r_B^2 + \delta^2 + 2\delta r_B\right)$; neglect δ^2

or $\quad 0 = \lambda r_B^2 + \delta(1 + 2\lambda r_B)$

$$\delta = \frac{-\lambda r_B^2}{1+2\lambda r_B} \approx \lambda r_B^2(1 - 2\lambda r_B)$$

Since, ($\lambda r_B << 1$) is very small

So, $\quad \delta = -\lambda r_B^2$

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{\exp(-\lambda\delta - \lambda r_B)}{(r_B+\delta)}$$

$$= \frac{-e^2}{4\pi\varepsilon_0}\frac{e^{-\lambda(\delta + r_B)}}{r_B}\left[1+\frac{\delta}{r_B}\right]$$

$(\because r = r_B + \delta)$

$$V(r) = -\frac{e^2e^{-\lambda r}}{4\pi\varepsilon_0}\frac{1}{r_B}\left[\left(1-\frac{\delta}{r_B}\right)\right]$$

$V(r) \cong (-27.2\text{ eV})$ remains unchanged

$$KE = -\frac{1}{2}mv^2 = \frac{-1}{2}m.\frac{h^2}{m^2r^2}$$

$$= \frac{-h^2}{2(r_B+\delta)^2 m} = \frac{h^2}{2r_B^2 m}\left(1-\frac{2\delta}{r_B}\right)$$

$$\left(\because v = \frac{h}{mr}\right)$$

$= (13.6\, eV)\,[1+2\lambda r_B]$

Total energy

$$(T.E) = -\frac{e^2}{4\pi\varepsilon_0 r_B} + \frac{h^2}{2r_B^2 m}[1+2\lambda r_B]$$

$$= -27.2 + 13.6[1+2\lambda r_B]\,eV$$

$$= [-27.2+13.6+27.2\lambda r_B]\,eV$$

$$(T.E.) = -13.6 + 27.2\,\lambda r_B eV$$

Change in energy

$$= -13.6 + 27.2\lambda r_B - (-13.6) = 27.2\lambda r_B eV$$

6. Case I when $r \le R_0 = 1Å$

Let $\varepsilon = 2+\delta$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0}.\frac{R_0^\delta}{r^{2+\delta}} = \frac{e.(-e)R_0^\delta}{4\pi\varepsilon_0 r^{2+\delta}}$$

where,

$$\left(\frac{q_1 q_2}{4\pi_0\varepsilon_0} = (1.6\times10^{-19})^2 \times 9\times10^9\right) \left(\begin{matrix}\because q_1 = e \\ \because q_2 = -e\end{matrix}\right)$$

(–ve sign shows force of attraction)

$$|F| = 23.04\times10^{-29}\,N\,m^2 \times\left[\frac{R\delta}{(r^{2+\delta})}\right]$$

The electrostatic force of attraction between positive charged nucleus and negatively charged electrons provides necessary centripetal force.

$$F = \frac{mv^2}{r} = \frac{\Lambda R_0^\delta}{r^{2+\delta}} \Rightarrow v^2 = \frac{\Lambda R_0^\delta}{mr^{1+\delta}} \quad .....(i)$$

Solving this for r, we get

$$r_n = \left[\frac{n^2\hbar^2}{m\Lambda R_0^\delta}\right]^{1-\delta}$$

where, r_n is radius of nth orbit of electron.

For n = 1 and substituting the values of constant, we get

$$r_1 = \left[\frac{h^2}{m\Lambda R_0^\delta}\right]^{\frac{1}{1-\delta}}$$

$$R_0 = 119$$

$$\Lambda = 2.3\times10^{-28}$$

$$\because\ \hbar = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\times3.14}$$

$$= (1.05\times10^{-34})$$

$$\because m = 9.1\times10^{-31}$$

$$\because 1-\delta = 8.9$$

$$r_1 = \left[\frac{(1.05)^2\times10^{-68}}{9.1\times10^{-31}\times2.3\times10^{-28}\times10^{+19}}\right]^{\frac{1}{2.9}}$$

$= 8\times10^{-11}$ m

$= 0.08$ nm (<0.1 nm)

This is radius of orbit of electron in ground state of hydrogen atom.

The speed of electron in ground state

$$v_n = \frac{nh}{mr_n} = nh\left(\frac{m\Lambda R_0^\delta}{n^2\hbar^2}\right)^{\frac{1}{1-\delta}}$$

$$\text{For } n = 1,\ v_1 - \frac{\hbar}{mr_1} = 1.44\times10^6\,m/s$$

The KE of electron in ground state

$$KE = \frac{1}{2}mv_1^2 - 9.43\times10^{-19}\,J = 5.9\,eV$$

PE till $R_0 = -\frac{\Lambda}{R_0}$

The PE of electron in ground state at R_0 to r

$$\text{PE from } R_0 \text{ to } r = +\Lambda R_0^{\delta}\int_{R_0}^{r}\frac{dr}{r^{2+\delta}} = \frac{\Lambda R_0^{\delta}}{1-\delta}\left[\frac{1}{r^{1+\delta}}\right]_{R_0}^{r}$$

$$= \frac{\Lambda R_0^{\delta}}{-(1+\delta)}\left[\frac{1}{r^{1+\delta}}\right]_{R_0}^{r}$$

$$= -\frac{\Lambda R_0^{\delta}}{1+\delta}\left[\frac{1}{r^{1+\delta}} - \frac{1}{R_0^{1+\delta}}\right] = -\frac{\Lambda}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0}\right]$$

$$\text{Put}\begin{pmatrix}\because 1-\delta = 2.9\\ \delta = -1.9\end{pmatrix}$$

$$PE = -\frac{\Lambda}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0} + \frac{1+\delta}{R_0}\right],$$

$$PE = -\frac{\Lambda}{-0.9}\left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1.9}{R_0}\right]$$

$$\left[\because \Lambda = 23\times 10^{-29} \text{ and } R_0 = 10^{19}\right]$$

$$= \frac{2.3}{0.9}\times 10^{-18}\left[(0.8)^{0.9} - 1.9\right]J = -17.3\text{ eV}$$

Total energy = (P.E. + K.E.)

So, energy = (–17.3 + 5.9) = –11.4 eV

This is the required total energy of electron in ground state of H – atom.

Chapter Thirteen

NUCLEI

SUMMARY

1. An atom has a nucleus. The nucleus is positively charged. The radius of the nucleus is smaller than the radius of an atom by a factor of 10^4. More than 99.9% mass of the atom is concentrated in the nucleus.
2. On the atomic scale, mass is measured in atomic mass units (u). By definition, 1 atomic mass unit (1u) is 1/12th mass of one atom of ^{12}C; $1u = 1.660563 \times 10^{-27}$ kg.
3. A nucleus contains a neutral particle called neutron. Its mass is almost the same as that of proton
4. The atomic number Z is the number of protons in the atomic nucleus of an element. The mass number A is the total number of protons and neutrons in the atomic nucleus; $A = Z+N$; Here N denotes the number of neutrons in the nucleus.

   A nuclear species or a nuclide is represented as $^A_Z X$, where X is the chemical symbol of the species.

   Nuclides with the same atomic number Z, but different neutron number N are called *isotopes*. Nuclides with the same A are *isobars* and those with the same N are *isotones*.

   Most elements are mixtures of two or more isotopes. The atomic mass of an element is a weighted average of the masses of its isotopes. The masses are the relative abundances of the isotopes.
5. A nucleus can be considered to be spherical in shape and assigned a radius. Electron scattering experiments allow determination of the nuclear radius; it is found that radii of nuclei fit the formula

   $R = R_0 A^{1/3}$,

   where R_0 = a constant = 1.2 fm. This implies that the nuclear density is independent of A. It is of the order of 10^{17} kg/m^3.
6. Neutrons and protons are bound in a nucleus by the short-range strong nuclear force. The nuclear force does not distinguish between neutron and proton.
7. The nuclear mass M is always less than the total mass, Σm, of its constituents. The difference in mass of a nucleus and its constituents is called the *mass defect*,

   $\Delta M = (Z m_p + (A - Z)m_n) - M$

   Using Einstein's mass energy relation, we express this mass difference in terms of energy as

   $\Delta E_b = \Delta M c^2$

   The energy ΔE_b represents the *binding energy* of the nucleus. In the mass number range A = 30 to 170, the binding energy per nucleon is nearly constant, about 8 MeV/nucleon.
8. Energies associated with nuclear processes are about a million times larger than chemical process.
9. The Q-value of a nuclear process is

   Q = final kinetic energy – initial kinetic energy.

   Due to conservation of mass-energy, this is also,

   Q = (sum of initial masses – sum of final masses)c^2
10. Radioactivity is the phenomenon in which nuclei of a given species transform by giving out α or β or γ rays; α-rays are helium nuclei;

β-rays are electrons. γ-rays are electromagnetic radiation of wavelengths shorter than X-rays;

11. Law of radioactive decay : $N(t) = N(0)\, e^{-\lambda t}$

    where λ is the decay constant or disintegration constant.

    The half-life $T_{1/2}$ of a radionuclide is the time in which N has been reduced to one-half of its initial value. The mean life τ is the time at which N has been reduced to e^{-1} of its initial value

    $$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2$$

12. Energy is released when less tightly bound nuclei are transmuted into more tightly bound nuclei. In fission, a heavy nucleus like $^{235}_{92}\text{U}$ breaks into two smaller fragments, e.g., $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{133}_{51}\text{Sb} + ^{99}_{41}\text{Nb} + 4\,^{1}_{0}\text{n}$

13. The fact that more neutrons are produced in fission than are consumed gives the possibility of a chain reaction with each neutron that is produced triggering another fission. The chain reaction is uncontrolled and rapid in a nuclear bomb explosion. It is controlled and steady in a nuclear reactor. In a reactor, the value of the neutron multiplication factor k is maintained at 1.

14. In fusion, lighter nuclei combine to form a larger nucleus. Fusion of hydrogen nuclei into helium nuclei is the source of energy of all stars including our sun.

Physical Quantity	Symbol	Dimensions	Units	Remarks
Atomic mass unit		[M]	u	Unit of mass for expressing atomic or nuclear masses. One atomic mass unit equals $1/12^{th}$ of the mass of ^{12}C atom.
Disintegration or decay constant	λ	$[T^{-1}]$	s^{-1}	
Half-life	$T_{1/2}$	[T]	s	Time taken for the decay of one-half of the initial number of nuclei present in a radioactive sample.
Mean life	τ	[T]	s	Time at which number of nuclei has been reduced to e^{-1} of its initial value
Activity of a radio-active sample	R	$[T^{-1}]$	Bq	Measure of the activity of a radioactive source.

POINTS TO PONDER

1. The density of nuclear matter is independent of the size of the nucleus. The mass density of the atom does not follow this rule.
2. The radius of a nucleus determined by electron scattering is found to be slightly different from that determined by alpha-particle scattering.

This is because electron scattering senses the charge distribution of the nucleus, whereas alpha and similar particles sense the nuclear matter.

3. After Einstein showed the equivalence of mass and energy, $E = mc^2$, we cannot any longer speak of separate laws of conservation of mass and conservation of energy, but we have to speak of a unified law of conservation of mass and energy. The most convincing evidence that this principle operates in nature comes from nuclear physics. It is central to our understanding of nuclear energy and harnessing it as a source of power. Using the principle, Q of a nuclear process (decay or reaction) can be expressed also in terms of initial and final masses.
4. The nature of the binding energy (per nucleon) curve shows that exothermic nuclear reactions are possible, when two light nuclei fuse or when a heavy nucleus undergoes fission into nuclei with intermediate mass.
5. For fusion, the light nuclei must have sufficient initial energy to overcome the coulomb potential barrier. That is why fusion requires very high temperatures.
6. Although the binding energy (per nucleon) curve is smooth and slowly varying, it shows peaks at nuclides like ^{4}He, ^{16}O etc. This is considered as evidence of atom-like shell structure in nuclei.
7. Electrons and positron are a particle-antiparticle pair. They are identical in mass; their charges are equal in magnitude and opposite. (It is found that when an electron and a positron come together, they annihilate each other giving energy in the form of gamma-ray photons.)
8. In â$^-$-decay (electron emission), the particle emitted along with electron is anti-neutrino ($\overline{\nu}$). On the other hand, the particle emitted in β^+-decay (positron emission) is neutrino (ν). Neutrino and anti-neutrino are a particle-antiparticle pair. There are anti particles associated with every particle. What should be antiproton which is the anti particle of the proton?
9. A free neutron is unstable ($n \rightarrow p + e^- + \overline{\nu}$). But a similar free proton decay is not possible, since a proton is (slightly) lighter than a neutron.
10. Gamma emission usually follows alpha or beta emission. A nucleus in an excited (higher) state goes to a lower state by emitting a gamma photon. A nucleus may be left in an excited state after alpha or beta emission. Successive emission of gamma rays from the same nucleus (as in case of ^{60}Ni, Fig. 13.4) is a clear proof that nuclei also have discrete energy levels as do the atoms.
11. Radioactivity is an indication of the instability of nuclei. Stability requires the ratio of neutron to proton to be around 1:1 for light nuclei. This ratio increases to about 3:2 for heavy nuclei. (More neutrons are required to overcome the effect of repulsion among the protons.) Nuclei which are away from the stability ratio, i.e., nuclei which have an excess of neutrons or protons are unstable. In fact, only about 10% of knon isotopes (of all elements), are stable. Others have been either artificially produced in the laboratory by bombarding α, p, d, n or other particles on targets of stable nuclear species or identified in astronomical observations of matter in the universe.

EXERCISES

You may find the following data useful in solving the exercises:

$e = 1.6\times10^{-19}$C $\quad N = 6.023\times10^{23}$ per mole

$1/(4\pi\varepsilon_0) = 9 \times 10^9$ N m^2/C^2 $\quad k = 1.381\times10^{-23}$J ^{0}K^{-1}

1 MeV = 1.6×10^{-13}J $\quad$ 1 u = 931.5 MeV/c^2

1 year = 3.154×10^7 s

m_H = 1.007825 u $\quad m_n$ = 1.008665 u

$m(^4_2\text{He})$ = 4.002603 u $\quad m_e$ = 0.000548 u

13.1 (a) Two stable isotopes of lithium $^6_3$Li and $^7_3$Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_5$B and $^{11}_5$B. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of $^{10}_5$B and $^{11}_5$B.

13.2 The three stable isotopes of neon: $^{20}_{10}$Ne, $^{21}_{10}$Ne and $^{22}_{10}$Ne have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

13.3 Obtain the binding energy (in MeV) of a nitrogen nucleus $\left(^{14}_7\text{N}\right)$, given $m\left(^{14}_7\text{N}\right)$=14.00307 u

13.4 Obtain the binding energy of the nuclei $^{56}_{26}$Fe and $^{209}_{83}$Bi in units of MeV from the following data:

$m(^{56}_{26}\text{Fe})$ = 55.934939 u $\quad m(^{209}_{83}\text{Bi})$ = 208.980388 u

13.5 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}$Cu atoms (of mass 62.92960 u).

13.6 Write nuclear reaction equations for

(i) α-decay of $^{226}_{88}$Ra (ii) α-decay of $^{242}_{94}$Pu

(iii) β^--decay of $^{32}_{15}$P (iv) β^--decay of $^{210}_{83}$Bi

(v) β^+-decay of $^{11}_6$C (vi) β^+-decay of $^{97}_{43}$Tc

(vii) Electron capture of $^{120}_{54}$Xe

13.7 A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

13.8 The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_6$C present with the stable carbon isotope $^{12}_6$C. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_6$C, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_6$C dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

13.9 Obtain the amount of ${}^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}\text{Co}$ is 5.3 years.

13.10 The half-life of ${}^{90}_{38}\text{Sr}$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

13.11 Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}\text{Au}$ and the silver isotope ${}^{107}_{47}\text{Ag}$.

13.12 Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}\text{Ra}$ and (b) ${}^{220}_{86}\text{Rn}$.

Given $m\ ({}^{226}_{88}\text{Ra}) = 226.02540$ u, $m\ ({}^{222}_{86}\text{Rn}) = 222.01750$ u,

$m\ ({}^{222}_{86}\text{Rn}) = 220.01137$ u, $m\ ({}^{216}_{84}\text{Po}) = 216.00189$ u.

13.13 The radionuclide ${}^{11}\text{C}$ decays according to

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu: \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$m\ ({}^{11}_{6}\text{C}) = 11.011434$ u and $m\ ({}^{11}_{6}\text{B}) = 11.009305$ u,

calculate Q and compare it with the maximum energy of the positron emitted.

13.14 The nucleus ${}^{23}_{10}\text{Ne}$ decays by β^{-} emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\ ({}^{23}_{10}\text{Ne}) = 22.994466$ u

$m\ ({}^{23}_{11}\text{Na}) = 22.089770$ u.

13.15 The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$$Q = [\, m_A + m_b - m_C - m_d]c^2$$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$

(ii) ${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$

Atomic masses are given to be

$m\ ({}^{2}_{1}\text{H}) = 2.014102$ u

$m\ ({}^{3}_{1}\text{H}) = 3.016049$ u

$m\ ({}^{12}_{6}\text{C}) = 12.000000$ u

$m\ ({}^{20}_{10}\text{Ne}) = 19.992439$ u

13.16 Suppose, we think of fission of a ${}^{56}_{26}\text{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $m\ ({}^{56}_{26}\text{Fe}) = 55.93494$ u and $m\ ({}^{28}_{13}\text{Al}) = 27.98191$ u.

13.17 The fission properties of ${}^{239}_{94}\text{Pu}$ are very similar to those of ${}^{235}_{92}\text{U}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?

13.18 A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

13.19 How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + n + 3.27 \text{ MeV}$$

13.20 Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

13.21 From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

13.22 For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + ^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

ADDITIONAL EXERCISES

13.23 In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504u), $^{25}_{12}Mg$ (24.98584u) and $^{26}_{12}Mg$ (25.98259u). The natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

13.24 The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:

$m(^{40}_{20}Ca)$ = 39.962591 u

$m(^{41}_{20}Ca)$ = 40.962278 u

$m(^{26}_{13}Al)$ = 25.986895 u

$m(^{27}_{13}Al)$ = 26.981541 u

13.25 A source contains two phosphorous radio nuclides $^{32}_{15}P$ ($T_{1/2}$ = 14.3d) and $^{33}_{15}P$($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

13.26 Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}Ra \rightarrow ^{209}_{82}Pb + ^{14}_{6}C$$

$$^{223}_{88}Ra \rightarrow ^{219}_{86}Rn + ^{4}_{2}He$$

Calculate the *Q*-values for these decays and determine that both are energetically allowed.

13.27 Consider the fission of $^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $^{140}_{58}Ce$ and $^{99}_{44}Ru$. Calculate *Q* for this fission process. The relevant atomic and particle masses are

$m(^{238}_{92}U)$ =238.05079 u

$m(^{140}_{58}Ce)$ =139.90543 u

$m(^{99}_{44}Ru)$ = 98.90594 u

13.28 Consider the D–T reaction (deuterium–tritium fusion)

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m(^{2}_{1}H)$=2.014102 u

$m(^{3}_{1}H)$ =3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

(Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = 2(3*kT*/2); *k* = Boltzman's constant, *T* = absolute temperature.)

13.29 Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

$m(^{198}Au)$ = 197.968233 u

$m(^{198}Hg)$ =197.966760 u

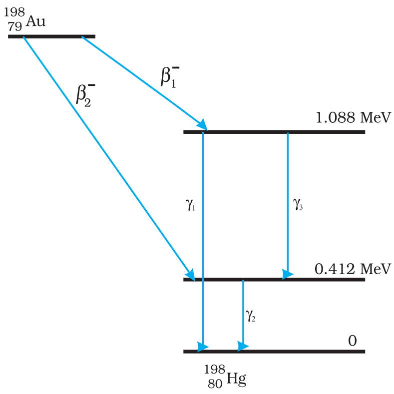


FIGURE13.6

MeV for A = 56. A heavy nucleus say A = 200 has a lower binding energy. Thus, when a heavy nucleus breaks up into two nuclei, (i.e. nuclear fission) the two nuclei produced are more tightly bound. Thus, there is a larger mass defect. Hence, during fission energy will be released.

Similarly, when two lighter nuclei fuse to form a heavier nuclei, (i.e. nuclear fusion), the binding energy increases i.e., the nuclei become more tightly bound. Thus, there will be a large mass defect and energy will be released. Thus, during fusion also energy is released.

(c) The nuclear process of a neutron undergoing a β-decay is given as

$$n \rightarrow p + \overline{e} + \overline{v}$$

where $\overline{v}$ is a neutrino. The neutrinos are difficult to detect because they are chargeless and are of very small non-zero mass. It has very weak interaction with other particles.

4. (a) Aarti has displayed awareness, concern and caring towards the health of her sister.

(b) During the intake of different elements and compounds, the biological organisms absorb them differently. Also, the exact distribution of the elements and their function in the various parts of organisms cannot be known clearly. For this, a radioisotope is made to enter the organism along with the elements and compounds, whose absorption, functioning and distribution to the brain has to be studied. The radioisotope acts as a tag of label for the element or compound under study. the radiation help in detecting the function of the compound by the organisms.

5. (a) Nuclear power plant.

The cause of disaster took place at chernobyl nuclear power plant was fire in the reactor and release of harmful radiations in the atmosphere.

(b) Process of release of energy: Inside a reactor, nuclear energy is first converted to heat energy. This heat energy is then converted to mechanical energy of the turbine which is finally converted to electrical energy.

(c) Values displayed by Asha: Awareness about real-life scenarios, helping nature towards her mother.

Values displayed by Asha's mother: Curiosity towards an worldwide incident.

Chapter 13:- Nuclei

Multiple Choice Questions (MCQs)

1. Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half life of 1 yr. After 1 year,

(a) all the containers will have 5000 atoms of the material

(b) all the contains will conatin the same number of atoms of the material but that number will only be approximately 5000

(c) the containers will in general have different numbers of the atoms of the material but their average will be close to 5000

(d) none of the containers can have more than 5000 atoms.

2. The gravitational force between a H-atom and another particle of mass m will be given by Newton's law

$$F = G\frac{M.m}{r^2}, \text{ where r is in km and}$$

(a) $M = m_{proton} + m_{electron}$

(b) $M = m_{proton} + m_{electron} - \frac{B}{c^2} (B = 13.6 \text{ eV})$

(c) M is not related to the mass of the hydrogen atom

(d) $M = m_{proton} + m_{electron} - \frac{|V|}{c^2}$ (|V| = magnitude of the potential energy of electron in the H-atom.

3. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom
(a) do not change for any type of radioactivity
(b) change for α and β-radioactivity but not for γ-radioactivity
(c) change for α-radioactivity but not for others
(d) change for β-radioactivity but not for others

4. M_x and M_y denote the atomic masses of the parent and the daughter nuclei respectively in radioactive decay. The Q – value for a β^- decay is Q_1 and that for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct?
(a) $Q_1 = (M_x - M_y)\, c^2$ and $Q_2 = [M_x - M_y - 2m_e]\, c^2$
(b) $Q_1 = (M_x - M_y)\, c^2$ and $Q_2 = (M_x - M_y)\, c^2$
(c) $Q_1 = (M_x - M_y - 2m_e)\, c^2$ and $Q_2 = (M_x - M_y + 2c_e)\, c^2$
(d) $Q_1 = (M_x - M_y + 2m_e)\, c^2$ and $Q_2 = (M_x - M_y + 2\, m_e)c^2$

5. Tritium is an isotope of hydrogen whose nucleus triton contains neutrons and 1 proton. Free neutrons decay into $p + \bar{e} + \bar{n}$. If one of the neutrons is Triton decays, it would transform into He^3 nucleus. This does not happen. This is because
(a) Triton energy is less than that of a He^3 nucleus
(b) The electron created in the beta decay process cannot remain in the nucleus
(c) both the neutrons in triton have to decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus.
(d) free neutrons decay due to external perturbations which is absent in triton nucleus

6. Heavy stable nuclei have more neutrons than protons. this is because of the fact that
(a) neutrons are heavier than protons
(b) electrostatic force between protons are repulsive
(c) neutrons decay into protons through beta decay
(d) nuclear force between neutrons are weaker than that between protons

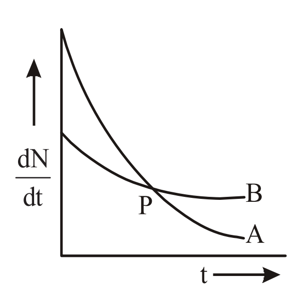
7. In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. the moderator used have light nuclei. Heavy nuclei will not serve the purpose, because
(a) they will break up
(b) elastic collision of neutrons with heavy nuclei will not slow them down
(c) the net weight of the reactor would be unbearably high
(d) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature

Multiple Choice Questions (MCQs) (More than one option correct)

1. Fusion processes, like combining two deuterons to form a He nucleus are impossible at ordinary temperature and pressure. The reasons for this can be traced to the fact
(a) nuclear force have short range
(b) nuclei are positively charged
(c) the origional nuclei must be completely ionized before fusion can take place
(d) the original nuclei must first break up before combining with each other

2. Samples of two radioactive nuclides A and B are taken λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time?
(a) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$
(b) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$
(c) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$
(d) Initial rate of decay of B is same as the rate of decay of A at t = 2h and $\lambda_B < \lambda_A$

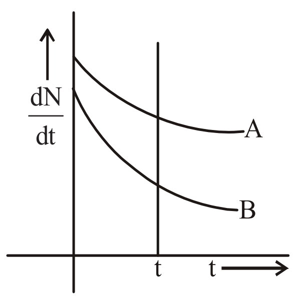
3. The variation of decay rate of two radioactive samples A and B with time is shown in figure. Which of the following statements are true?



(a) Decay constant of A is greater than that of B hence A always decay faster than B
(b) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A
(c) Decay constant of A greater than that of B but it does not always decay faster than B
(d) Decay constant of B is smaller than that of A but still its decay rate becomes equal to that of A at a later instant

Very Short Answer Questions

1. He_2^3 and He_1^3 nuclei have the same mass number. Do they have the same binding energy?
2. Draw a graph showing the variation of decay rate with number of active nuclei.
3. Which sample A or B shown in figure has shorter mean- Life?



4. Which one of the following cannot emit radiation and why? Excited nucleus, excited electron.
5. In pair annihilation, an electron and a positron destroy each other to produce gamma radiations. How is the momentum conserved?

Short Answer Questions

1. Why do stable nuclei never have more protons than neutrons?
2. Consider a radioactive nucleus which decays to a stable nucleus C through the following sequence
$A \to B \to C$
Here B is an intermediate nuclei which is also radioactive. Considering that there are N_0 atoms of A initially, plot the graph showing the variation of number of atoms of A and B versus time.
3. A piece of wood from the ruins of an ancient building was found to have a ^{14}C activity of 12 disintegrations per minute per gram of its carbon content. The ^{14}C activity of the living wood is 16 disintergrations per minute per gram. How long ago did the tree, from which the wooden sample came, die? Given half-life of ^{14}C is 5760 yr.
4. Are the nucleons fundamental particles, or do they consist of still smaller parts? One way to find out is to probe a nucleon just as Rutherford probed an atom. What should be the kinetic energy of an electron for it to be able to probe a nucleon? Assume the diameter of a nucleon to be approximately 10^{-15} m.
5. A nuclide 1 is said to be the mirror isobar of nuclide 2 if $Z_1 = N_2$ and $Z_2 = N_1$. (a) What nuclide is a mirror isobar of $^{23}_{11}Na$? (b) which nuclide out of the two mirror isobars have greater binding energy and why?

Long Answer Questions

1. Sometimes a radioactive nucleus decays into a nucleus which itself is radioactive. An example is

$$^{38}\text{Sulphur} \xrightarrow[=2.48h]{\text{half-life}} {}^{38}\text{Cl} \xrightarrow[=0.62h]{\text{half-life}} {}^{38}\text{Ar(stable)}$$

Assume that we start with 1000 ^{38}S nuclei at time $t = 0$. The number of ^{38}Cl is of count zero at $t = 0$ and will again be zero at $t = \infty$. At what value of t, would the number of counts be a maximum?
2. Deuteron is a bound state of a neutron and a proton with a binding energy $B = 2.2$ MeV. A $\gamma-$ray or energy E is aimed at a deuteron nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. if $E = B$, show that this cannot happen. Hence, calculate how much bigger than B must be E be for such a process to happen.
3. The deuteron is bound by nuclear forces just as H-atom is made up of p and e bound by electrostatic forces. If we consider the force between neutron and proton in deutron as given in the form a coulomb potential but with an effective charge e'

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e'^2}{r}$$

estimate the value of (e' /e) given that the binding energy of a deuteron is 2.2 MeV.

4. Before the neutrino hypothesis, the beta decay process was throught to be the transitions.

$$n \rightarrow p + \bar{e}$$

If this was true, show that if the neutron was at rest, the proton and electron would emerge with fixed energies and calculate them. Experimentally, the electron energy was found to have a large range.

5. The activity R of an unknown radioactive nuclide is measured at hourly intervals. The result found are tabulated as follows

t(h)	0	1	2	3	4
R(MBq)	100	35.36	12.51	4.42	1.56

(i) Plot the graph of R versus t and calculate half-life from the graph.

(ii) Plot the graph of In $\left(\frac{R}{R_0}\right)$ versus t and obtain the value of half-life from the graph.

6. Nuclei with magic number of proton Z = 2, 8, 20, 28, 50, 52 and magic number of neutrons N = 2, 8, 20, 28, 50, 82 and 126 are found to be very stable

(i) Verify this by caculating the proton. Separation energy S_p for ^{120}Sn (Z = 50) and ^{121}Sb(Z = 51).

The proton separation energy for a nuclide is the minimum energy required to separate the least tightly bound proton from a nucleus of that nuclide. It is given by

$S_p = (M_{Z-1'N} + M_H - M_{Z,N})\, c^2$.

Given, ^{119} In = 118.9058 u,

^{120}Sn = 199.902199 u,

^{121}Sb = 120.903824 u, ^{1}H = 1.0078252 u.

(ii) What does the existence of magic number indicate?

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. (c) Radioactivity is a process due to which a radiactive material spontaneously decays. Half-life time for a radioactive remain average half of its or of radioactive atoms will decay. So, the containers will in general have different number of atoms of the material,after one year means one half life i.e., average atom of radioactive subtance remain after 1 year, in each container is equal to $\frac{1}{2}$ of 10000 = 5000 atoms.

2. (b) Due my formation of H-atom some mass of necleons convert into energy by E = mc^2, this energy is used to bind the nucleons along with nucleus, so mass of atom becomes slightly less than sum of actual masses of electron and nucleons.

$$\text{Given, } F = \frac{GMm}{r^2}$$

M = effective mass of hydrogen atom

Actual mass of H-atom

$\left(\because \frac{B}{c^2} \text{ is binding energy}\right)$

$$= \underset{(Me)}{\text{mass of electron}} + \underset{(Mp)}{\text{mass of proton}} - \frac{B}{c^2}$$

where B is BE of hydrogen atom = 13.6 eV.

3. (b) β-particle carries one unit of negative charge and α-particle carries 2 units of positive charge and γ-particle carries no charge, so electronic energy levels of the atom charges in emission of α and β particle, but not in γ-decay.

4. (a) Let the nucleus is $_ZX^A$ radiate β^+ decay is represented as

$$_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + v + Q_2$$

where Z and A have usual meaning : –

$\therefore \quad Q_2 = [m_n(_ZX^A) - m_n(_{Z-1}Y^A) - 2m_e]\, c^2$

$= [m_n(_ZX^A) + Zm_e - m_n(_{Z-1}Y^A) - (Z-1)\, m_e - 2m_e]c^2$

$= [m(_ZX^A) - m(_{Z-1}Y^A) - 2m_e]\, c^2$

$= (M_x - M_y - 2m_e)\, c^2$

Let the parent nuclei $_ZX^A$ is radioactive atom and β^- decay as under is represented as

$\Rightarrow \quad _ZX^A \rightarrow {}_{Z+1}A^Y + {}_{-1}e^0 + \bar{v} + Q_1$

$Q_1 = [m_n(_ZX^A) - m_n(_{Z+1}Y^A) - m_e]\, c^2$

$= [m_n(_ZX^A) + Zm_e - m_n(_{Z+1}Y^A) - (Z+1)\, m_e]\, c^2$

$= [m(_ZX^A) - m(_{Z+1}Y^A)]c^2$

$= (M_x - M_y)c^2$

5. (a) Tritium ($_1H^3$) has proton and 2 neutrons. If a neutron decays as $n \to P + \bar{e} + \bar{v}$, then the nucleus may have 2 protons and neutron, i.e., tritium atom will convert into $_2He^3$ (2 protons and 1 neutron).
Binding energy of $_1H^3$ is less than that of $_2He^3$ nucleus, So, transformation is not possible energetically.

6. (b) Stable heavy nuclei have more neutrons than protons because electrostatic force between protons is repulsive, which causes unstability of nucleus.

7. (b) The moderator used have light nuclei (like proton). When protons undergo perfectly elastic collision with the neutron emitted their velocities are exchanged, i.e., neutrons come to rest and protons move with the velocity of neutrons. To slowdown the speed of neutrons substance should be made up of proton for perfectly elastic i.e., we need light nuclei not heavy nuclei because heavy nuclei will not serve the purpose because elastic collisions of neutrons with heavy nuclei will not slow them down or speed but only direction will change.

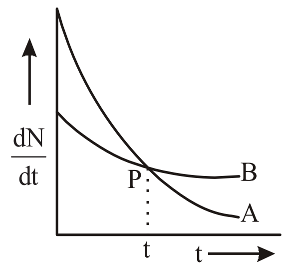
Multiple Choice Questions (More Than One Options)

1. (a, b) Two deuterons can combine to form He atom when their nuclei come close to nuclear range and fusion processes are impossible at ordinary temperature and pressure. So to combine two nuclei, they must reach closer of the range of where nuclear force acts and electrostatic repulsive force does not act the reason is nuclei are positively charged and nuclear forces are short range strongest forces.

2. (b, d) The two radioactive samples of the two nuclides A and B can have the same rate of decay at any time, if the initial rate of decay of A is twice of decay of B and $\lambda_A > \lambda_B$. So, initial rate of decay of B is same as rate of decay of A at t = 2h and decay rate can be same if $\lambda_A > \lambda_B$.

3. (c, d) As the given figure, it is clear that slope of curve A is greater than of B. So rate of decay of A is faster than B.
we know that

$$\frac{dN}{dt} = -\lambda t$$



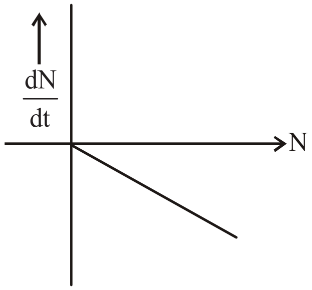
or $\left(\frac{dN}{dt}\right) \propto, \lambda$, at any instant of time (t) hence we can say that $\lambda_A > \lambda_B$. At point P the intersecting point of two graphs at time (t) is same and, rate of decay for both graph A and B is also same.

Very Short Answer Questions

1. Nuclei He_2^3 and He_1^3 have the same mass number. He_2^3 has two proton and one neutron He_1^3 has one proton and two neutron. $_2He^3$ give electrostatic force of repulsion but in, $_1He^3$ between nucleons has only nuclear force So, the repulsive force between neutrons is missing in $_1He^3$ so the binding energy of $_1He^3$is larger than that of $_2He^3$.

2. By law of radioactive decay is, N is the number of radioactive nuclei in the same, i.e., So $\frac{dN}{dt}$ can be negative

$$\frac{-dN}{dt} = \lambda N,$$



where the variation of decay (λ) is constant for a given radioactive material or active nuclei, So graph between N and $\frac{dN}{dt}$ is a straight line as shown in the diagram.

3. As the given figure,
Initially at $t = 0$,

$$\left(\frac{dN_0}{dt}\right)_A = \left(\frac{dN_0}{dt}\right)_B$$

So, $(N_0)_A = (N_0)_B$

Initially both samples has equal number of radioactive atoms Considering any instant t by drawing a line perpendicular to time axis, from figure then,

$$\left(\frac{dN}{dt}\right)_A > \left(\frac{dN}{dt}\right)_B$$

$\Rightarrow \quad \lambda_A N_A > \lambda_B N_B$

$\because \quad N_A > N_B$ (rate of decay of B is slower)

$\therefore \quad \lambda_B > \lambda_A$

$\because \quad -dN/dt = \lambda N$

$$\Rightarrow \quad \frac{1}{\tau_A} < \frac{1}{\tau_B}$$

So, $\tau_A > \tau_B$ $\quad \left[\because \text{Average life } \tau = \frac{1}{\lambda}\right]$

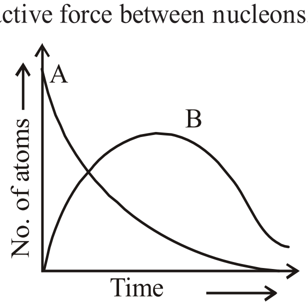
4. Excited electron has energy in the range of eV and not MeV (mega electron volt), γ–radiations have energy of the order of MeV.

5. In pair annihilation when a electron and a positron they destroy each other to produce 2γ photons which move in opposite directions to conserve linear momentum So, the annihilation is ${}_0e^{-1} + {}_0e^{+1} \rightarrow 2\gamma$ (photons).

Short Answer Questions

1. If in a stable nucleus number of protons are larger than neutrons then the replusive force becomes so great in nuclei so, that an excess of neutrons which produce only attractive forces is, required for stability. So, for stability repulsive force between protons must be smaller than nuclear attractive force between nucleons.

2.



At $t = 0$, $N_A = N_0$ (maximum) while $N_B = 0$. As time increases, N_A decreases exponentially and the number of atoms of B increases. They becomes (N_B) maximum and finally the drop to zero exponentially by radioactive decay law.

3. Rate of disintegration in old wood sample of C – 14 radioactive atom is 12 atom per min per gram.
As, $R = 12$ dis/min per g, $R_0 = 16$ dis/min per g,
$T_{1/2} = 5760$ yr
Let t be the span of the tree.
According to radioactive decay law,

$$N = N_0 e^{-\lambda t}$$

or $\quad R = R_0 e^{-\lambda t}$ or $\frac{R}{R_0} = e^{-\lambda t}$ or $e^{\lambda t} = \frac{R_0}{R}$

Taking log on both the sides

$$\lambda t \log_e e = \log_e \frac{R_0}{R} \Rightarrow \lambda t = \left(\log_{10} \frac{16}{12}\right) \times 2.303$$

$$t = \frac{2.303 \times \log_{10}\left[\frac{4}{3}\right]}{\lambda} \text{(half life)}$$

$$t = \frac{2.303(\log 4 - \log 3)}{\lambda}$$

$$= \frac{2.303(0.6020 - 4.771) \times 5760}{0.6931}$$

$$\left(\because \lambda = \frac{0.6931}{T_{1/2}}\right)$$

$= 2391.20$ years.

4. To detect the properties of nucleons inside the nucleus the wavelength of particle which may detect nucleons or each particle (neutron and proton) present inside the nucleus is called nucleon.
Given, wavelength $\lambda = 10^{-15}$ m
To detect separate parts inside a nucleon the electron must have wavelength less than 10^{-15} m.
We know that energy of free electron

$KE = PE$ ...(i)

$$\text{Energy } E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{hc}{\lambda} \quad \text{...(ii)}$$

From Eq. (i) and Eq. (ii),
kinetic energy of electron

$$PE = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{10^{-15}\times1.6\times10^{-19}} eV$$

$= 1.24 \times 10^9$ eV

KE $= 1.24 \times 10^9$ eV

5. (a) As per question,a nuclide 1 is said to be mirror isobar of nuclide 2, if $Z_1 = N_2$ and $Z_2 = N_1$. Where Z is atomic number and N is no. of neutron in ${}_{11}Na^{23}$. So,

$Z_1 = 11, N_1 = 23 - 11 = 12$

So Mg is isobar of ${}_{11}Na^{23}$.

$\therefore$ Mirror isobar of ${}_{11}Na^{23}$ is ${}_{12}Mg^{23}$, for which $Z_2 = 12 = N_1$ and $N_2 = 23 - 12 = 11 = Z_1$

(b) As ${}^{23}_{12}Mg$ contains even number of proton (12) against ${}^{23}_{11}Na$ which had odd number of protons (11), So, ${}^{23}_{11}Mg$ has more binding energy than ${}_{11}Na^{23}$.

Long Answer Questions

1. Consider the given two decays

$$^{38}S \xrightarrow[2.48h]{} {}^{38}Cl \xrightarrow[0.62h]{} {}^{38}Ar$$

Initially at time t = 0. Let the number of radioactive atom of
${}^{38}S$ have N_1 (t) active nuclei and ${}^{38}Cl$ have N_2 (t) active nuclei.

$$\frac{dN_1}{dt} = -\lambda_1 N_1 = \text{rate of formation of } Cl^{38}.$$

Also, $$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1$$

But $N_1 = N_0 e^{-\lambda_1 t}$

$$\frac{dN_2}{dt} = \lambda_1 N_0 e^{-\lambda_1 t - \lambda_2 N_2} \quad ...(i)$$

Multiplying by $e^{\lambda 2^t}$ dt and rearranging

$$e^{\lambda_2 t} dN_2 + \lambda_2 N_2 e^{\lambda_2 t} dt = \lambda_1 N_0 e^{(\lambda_2-\lambda_1)^t} dt$$

Integrating both sides

$$N_2 e^{\lambda_2^t} = \frac{N_0\lambda_1}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)t} + C$$

$$\left[\because e^{\lambda_2 t} dN_2 .dt = 0\right]$$

Since, ${}^{38}Cl$ atoms is formed after d is integration of S^{38}, So initially number of Cl^{38} atom at t = 0, $N_2 = 0$,

So, $$C = -\frac{N_0\lambda_1}{\lambda_2-\lambda_1}$$

$$\therefore \quad N_2 e^{\lambda_2 t} = \frac{N_0\lambda_1}{\lambda_2-\lambda_1} e^{(\lambda_2-\lambda_1)t} - 1) \quad ...(ii)$$

Multiplying by $e^{-\lambda_2 t}$ to both sides then

We get, $$N_2 = \frac{N_0\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_2\lambda_2 - N_2\lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_0 e^{-\lambda_2 t}$$

where N_0 are the numbers of S^{38} atoms and no. of Cl^{38} atoms after time (t) will be $N_2 = N_0 e^{-\lambda_2 t}$.

So, for maximum count,

$$\frac{dN_2}{dt} = 0$$

$$\Rightarrow \quad N_2\lambda_2 - N_2\lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_2$$

$$\lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2 = 0 \quad \text{[From Eq. (i)]}$$

$$\Rightarrow \quad \frac{N_0}{N_2} = \frac{\lambda_2}{\lambda_1} e^{\lambda_1 t} \quad \text{[from Eq. (ii)]}$$

By putting the value of N_2 in (ii)

$$e^{\lambda_2 t} - \frac{\lambda_2}{\lambda_1}.\frac{\lambda_1}{(\lambda_2-\lambda_1)}\left[e^{\lambda_1 t}[e^{(\lambda_2-\lambda_1)t} - 1\right] = 0$$

or $$e^{\lambda_2 t} - \frac{\lambda_2}{(\lambda_2-\lambda_1)} e^{\lambda_2 t} + \frac{\lambda_2}{(\lambda_2-\lambda_1)} e^{\lambda_1 t} = 0$$

$$1 - \frac{\lambda_2}{(\lambda_2-\lambda_1)} + \frac{\lambda_2}{(\lambda_2-\lambda_1)} e^{(\lambda_1-\lambda_2)t} = 0$$

$$\frac{\lambda_2}{(\lambda_2-\lambda_1)} e^{(\lambda_1-\lambda_2)t} = \frac{\lambda_2}{(\lambda_2-\lambda_1)} - 1$$

$$e^{(\lambda_1-\lambda_2)t} = \frac{\lambda_1}{\lambda_2}$$

$$t = \left(\log_e \frac{\lambda_1}{\lambda_2}\right) \Big/ (\lambda_1 - \lambda_2)$$

$$\left(\frac{\lambda_1}{\lambda_2}\right) = \frac{\frac{0.6931}{2.48}}{\frac{0.631}{0.62}} = \frac{0.62}{2.48} = \frac{1}{4}$$

$$-(\lambda_1 - \lambda_2) = -\left(\frac{0.6931}{2.48} - \frac{0.6931}{0.62}\right) = \frac{-(0.693\times1.86)}{0.62\times2.48}$$

$$t = \frac{\log_e\left[\frac{1}{4}\right] \times 2.48 \times 0.62}{-0.6931 \times 1.86}$$

$$= \frac{\not- \log_e(4) \times 2.48 \times 0.62}{\not- 6.931 \times 1.86}$$

$$= \frac{1.06586}{1.2892} = 0.8267 \text{ Hrs} \qquad \because \lambda = \frac{0.693}{T_{1/2}}$$

So number of Cl^{38} radioactive atom will be maximum at $N_2 = 0.8267$ Hrs.

2. As given binding energy of deuteron (B) = 2.2 MeV

From the energy conservation law,

$$\left[\therefore \; KE = \frac{P^2}{2m}\right]$$

$$\text{So,} \quad E - B = K_n + K_p = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \qquad \text{...(i)}$$

From conservation of momentum of γ-ray of energy (E)

$$p_n + p_p = \frac{E}{c} \qquad \text{...(ii)}$$

$$\text{As} \quad \because \lambda = \frac{h}{p} \text{ or } P = \frac{h\nu}{\lambda\nu} = \frac{h\nu}{c} = \frac{E}{c}$$

if $E = B$,

Eq.(i) $p_n^2 + p_p^2 = 0$

It only happen if $p_n = p_p = 0$ because square of non zero number can never zero.

So, the Eq. (ii) cannot satisfied and the process cannot take place.

If $(E > B)$ then

$E = B + \lambda$ where $\lambda << B$ for the process to take place.

Put value of p_n from Eq. (ii) in Eq. (i), we get

$$(B + \lambda) - B = \frac{p_n^2}{2m} + \frac{P_p^2}{2m} \quad \left[\because \; p_n = \frac{E}{c} - p_p\right]$$

$$\lambda = \frac{\left(\frac{E}{c} - p_p\right)^2}{2m} + \frac{p_p^2}{2m}$$

$$\text{or} \quad 2p_p^2 - \frac{2Ep_p}{c} + \left[\frac{E^2}{c^2} - 2m\lambda\right] = 0$$

It is a quadratic equation so its solution by quadratic formula.

$$a = 2, \; b = \frac{-2E}{c}, \; c = \left(\frac{E^2}{C^2} - 2m\lambda\right)$$

$$p_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using the formula of quadratic equation, we get

$$\text{So,} \quad p_p = \frac{\frac{2E}{c} \pm \sqrt{\frac{4E^2}{c^2} - 8\left(\frac{E^2}{c^2} - 2m\lambda\right)}}{4}$$

For the real and equal value p_p, the discriminant must be zero as the value of

$$\frac{4E^2}{c^2} - 8\left[\frac{E^2}{c^2} - 2m\lambda\right] = 0$$

p_p must be one or positive.

$$\frac{4E^2}{c^2} = 8\left[\frac{E^2}{c^2} - 2m\lambda\right]$$

$$16m\lambda = \frac{4E^2}{c^2}$$

$\therefore$ λ is very small $\quad [\therefore \lambda << B \Rightarrow E \cong B]$

$$\text{So,} \quad \lambda = \frac{E^2}{4mc^2} \cong \frac{B^2}{4mc^2}$$

3. The binding energy is H-atom in ground state

$$E = \frac{me^4}{8\pi\varepsilon_0^2 h^2} = 13.6\text{eV} \qquad \text{...(i)}$$

If proton and netron had charge e' each and governed by the same electrostatic force, then in the above equation we would need to replace electronic mass m by the reduced mass m' of proton-neutron and the electronic charge e replaced by e'.

$$\left(\frac{1}{m'} = \frac{1}{M} + \frac{1}{N}\right)$$

$$\text{or} \quad m' = \frac{M \times N}{M + N} = \frac{M}{2} \qquad \because M = N$$

$$= \frac{1836}{2} = 918 \text{ m}$$

Here, M represents mass of a neutron/proton, m is the mass of electron

$$\therefore E'(\text{Binding energy}) = \frac{918(e')^4}{8\varepsilon_0^2 h^2 \pi} = 2.2 \text{ MeV} \quad ...(ii)$$

Diving Eqs, (ii) and (i), we get

$$\left(\frac{E'}{E}\right) = \left[\frac{918m(e')^4 \times 8\pi\varepsilon_0^2 h^2}{8\varepsilon_0^2 h^2 \pi \times me^4}\right]$$

$$= 918\left(\frac{e'}{e}\right)^4 = \frac{2.2 \text{ MeV}}{13.6 \text{ eV}} = \frac{2.2\times10^6}{13.6}$$

$$\left(\frac{e'}{e}\right)^4 = \frac{2.2\times10^6}{13.6\times918} = 176.21$$

Then, Required ratio

$$\left(\frac{e'}{e}\right)^4 = (176.21)^{1/4} = 3.64.$$

4. Before β decay neutron is at rest

Hence $E_n = m_n c^2, p_n = 0$ as the velocity is zero.

By the law of conservation of momentum.

$$p_n = p_p + p_e$$

Let $p_e = p_p$ then

$$p_p + p_e = 0 \Rightarrow |p_p| = |p_e| = p$$

Also, energy of proton

$$E_p = \left(m_p^2 c^4 + p_p^2 c^2\right)^{\frac{1}{2}} = \sqrt{m_p^2 c^4 + p_p^2 c^2}$$

energy of electron

$$E_e = \left(m_e^2 c^4 + p_e^2 c^2\right)^{\frac{1}{2}} = \sqrt{m_e^2 c^4 + p_e^2 c^2}$$

By the law of conservation of energy

$$E_p + E_e = E_n$$

if $|p_e| = |p_p| = p$

$$\left(m_p^2 c^4 + p^2 c^2\right)^{\frac{1}{2}} + \left(m_e^2 c^4 + p^2 c^2\right)^{\frac{1}{2}} = m_n c^2$$

$$m_p c^2 \approx 936 \text{MeV}, m_n c^2 \approx 938 \text{ MeV},$$

$$m_e c^2 = 0.51 \text{ MeV} \approx 0.5 \text{ MeV}$$

As, the energy difference between n and p is very small, pc will be small, $pc << m_p c^2$, while pc may be greater than $m_e c^2$, So by neglecting $(m_e c^2) = (0.5)^2$

$$\Rightarrow \quad m_p c^2 + \frac{p^2 c^2}{2m_p^2 c^4} = m_n c^2 - pc$$

or

$$m_p c^2 + \frac{p^2 c^2}{2m_p^2 c^4} + pc = m_n c^2$$

To first order

$$pc \approx m_n c^2 - m_p c^2$$

$$= 938 \text{ MeV} - 936 \text{ MeV} = 2 \text{ MeV}$$

$E = mc^2, E^2 = m^2c^4$

E is the energy of either proton or neutron, then

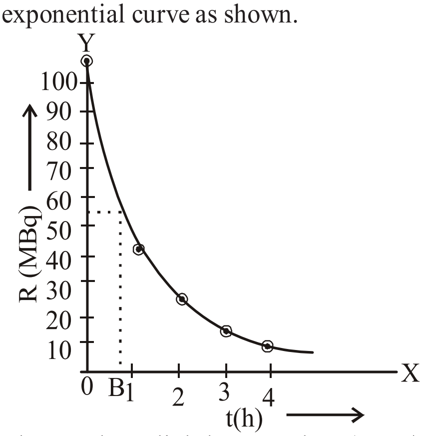
$$E_p = \left(m_p^2 c^4 + p^2 c^2\right)^{\frac{1}{2}} = \sqrt{936^2 + 2^2} = 936 \text{ MeV}$$

$$E_e = \left(m_e^2 c^4 + p^2 c^2\right)^{\frac{1}{2}} = \sqrt{(0.51)^2 + 2^2} = 2.06 \text{ MeV}$$

5. As given table we have listed values of R (MB_q) and t(h), then In $\left(\frac{R}{R_0}\right)$ is, where ($R_0 = 100$)

t(h)	0	1	2	3	4
$R(MB_q)$	100	35.36	12.51	4.42	1.56
$\log_e\left(\frac{R}{R_0}\right)$	–	–1.04	–2.08	–3.11	–4.16

(i) The graph between R versus t, is an exponential curve as shown.

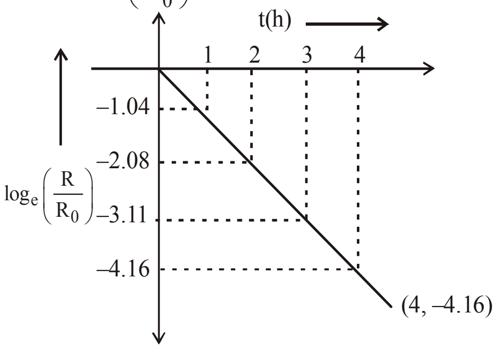


From the graph at slightly more than (t = 1/2) then the R reduces to 50% in t = OB ≈ 42 min at (R = 50%)

So, $t_{1/2} = 0.7 \text{ h} = 0.7 \times 60 \text{ min} = 42 \text{ min}$

(ii) The graph showing the variation of

$\log_e\left(\frac{R}{R_0}\right)$ versus t (h) as shown below :–



Slope of this graph = –λ
We know that the disintegration constant

$$\lambda = \frac{\log_e\left(\frac{R}{R_0}\right)}{\left(t_{\frac{1}{2}}\right)}$$

from the graph,

$$\lambda = -\left(\frac{-4.16 - 3.11}{1}\right) \Rightarrow 1.05 \text{ per hour}$$

for, half-life, $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.05} = 0.66\text{h}$
$= 39.6 \text{ min} \approx 40 \text{ min}$

6. (i) The proton separation energy is

So, S_p for 50

$Sn^{120} = (M_{119.70} + M_H - M_{120,70})\, c^2$

$= (118.9058 + 1.0078252 - 119.902199)\, c^2$

S_p for $Sn^{120} = 0.0114362\, c^2$

at z = 51, z – 1 = 50 for Sn

Similarly for S_p of

$S_n^{/SpSb} = (M_{120,70} + M_H - M_{121,\,70})c^2$

$= (199.902199 + 1.007825^2 - 120.903822)c^2$

$= 0.0059912\, c^2$

Since, $S_{psn} > S_{pSb}$, Sn nucleus is more stable than Sb nucleus.

(ii) The existence of magic numbers indicates that the shell structure of nucleus similar to the shell structure of an atom. This also explains the peaks in binding energy/ nucleon curve.

Chapter Fourteen

SEMICONDUCTOR ELECTRONICS: MATERIALS, DEVICES AND SIMPLE CIRCUITS

SUMMARY

1. Semiconductors are the basic materials used in the present solid state electronic devices like diode, transistor, ICs, etc.
2. Lattice structure and the atomic structure of constituent elements decide whether a particular material will be insulator, metal or semiconductor.
3. Metals have low resistivity (10^{-2} to 10^{-8} Ωm), insulators have very high resistivity ($>10^{8}$ Ω m^{-1}), while semiconductors have intermediate values of resistivity.
4. Semiconductors are elemental (Si, Ge) as well as compound (GaAs, CdS, etc.).
5. Pure semiconductors are called 'intrinsic semiconductors'. The presence of charge carriers (electrons and holes) is an 'intrinsic' property of the material and these are obtained as a result of thermal excitation. The number of electrons (n_e) is equal to the number of holes (n_h) in intrinsic conductors. Holes are essentially electron vacancies with an effective positive charge.
6. The number of charge carriers can be changed by 'doping' of a suitable impurity in pure semiconductors. Such semiconductors are known as extrinsic semiconductors. These are of two types (n-type and p-type).
7. In n-type semiconductors, $n_e >> n_h$ while in p-type semiconductors $n_h >> n_e$.
8. n-type semiconducting Si or Ge is obtained by doping with pentavalent atoms (donors) like As, Sb, P, etc., while p-type Si or Ge can be obtained by doping with trivalent atom (acceptors) like B, Al, In etc.
9. $n_e n_h = n_i^2$ in all cases. Further, the material possesses an *overall charge neutrality*.
10. There are two distinct band of energies (called valence band and conduction band) in which the electrons in a material lie. Valence band energies are low as compared to conduction band energies. All energy levels in the valence band are filled while energy levels in the conduction band may be fully empty or partially filled. The electrons in the conduction band are free to move in a solid and are responsible for the conductivity. The extent of conductivity depends upon the energy gap (E_g) between the top of valence band (E_V) and the bottom of the conduction band E_C. The electrons from valence band can be excited by heat, light or electrical energy to the conduction band and thus, produce a change in the current flowing in a semiconductor.
11. For insulators $E_g > 3$ eV, for semiconductors E_g is 0.2 eV to 3 eV, while for metals $E_g \approx 0$.
12. p-n junction is the 'key' to all semiconductor devices. When such a junction is made, a 'depletion layer' is formed consisting of immobile ion-cores devoid of their electrons or holes. This is responsible for a junction potential barrier.
13. By changing the external applied voltage, junction barriers can be changed. In forward bias (n-side is connected to negative terminal of the battery and p-side is connected to the positive), the barrier is decreased while the barrier increases in reverse bias. Hence, forward bias current is more (mA) while it is very small (μA) in a p-n junction diode.
14. Diodes can be used for rectifying an ac voltage (restricting the ac voltage to one direction). With the help of a capacitor or a suitable filter, a dc voltage can be obtained.
15. There are some special purpose diodes.

16. Zener diode is one such special purpose diode. In reverse bias, after a certain voltage, the current suddenly increases (breakdown voltage) in a Zener diode. This property has been used to obtain *voltage regulation*.
17. p-n junctions have also been used to obtain many photonic or optoelectronic devices where one of the participating entity is 'photon': (a) Photodiodes in which photon excitation results in a change of reverse saturation current which helps us to measure light intensity; (b) Solar cells which convert photon energy into electricity; (c) Light Emitting Diode and Diode Laser in which electron excitation by a bias voltage results in the generation of light.
18. Transistor is an n-p-n or p-n-p junction device. The central block (thin and lightly doped) is called 'Base' while the other electrodes are 'Emitter' and 'Collectors'. The emitter-base junction is forward biased while collector-base junction is reverse biased.
19. The transistors can be connected in such a manner that either C or E or B is common to both the input and output. This gives the three configurations in which a transistor is used: Common Emitter (CE), Common Collector (CC) and Common Base (CB). The plot between I_C and V_{CE} for fixed I_B is called output characteristics while the plot between I_B and V_{BE} with fixed V_{CE} is called input characteristics. The important transistor parameters for CE-configuration are:

    input resistance, $r_i = \left(\dfrac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$

    output resistance, $r_o = \left(\dfrac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$

    current amplification factor, $\beta = \left(\dfrac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$
20. Transistor can be used as an amplifier and oscillator. In fact, an oscillator can also be considered as a self-sustained amplifier in which a part of output is fed-back to the input in the same phase (positive feed back). The voltage gain of a transistor amplifier in common emitter configuration is: $A_v = \left(\dfrac{v_o}{v_i}\right) = \beta\dfrac{R_C}{R_B}$, where R_C and R_B are respectively the resistances in collector and base sides of the circuit.
21. When the transistor is used in the cutoff or saturation state, it acts as a switch.
22. There are some special circuits which handle the digital data consisting of 0 and 1 levels. This forms the subject of Digital Electronics.
23. The important digital circuits performing special logic operations are called logic gates. These are: OR, AND, NOT, NAND, and NOR gates.
24. In modern day circuit, many logical gates or circuits are integrated in one single 'Chip'. These are known as Intgrated circuits (IC).

POINTS TO PONDER

1. The energy bands (E_C or E_V) in the semiconductors are space delocalised which means that these are not located in any specific place inside the solid. The energies are the overall averages. When you see a picture in which E_C or E_V are drawn as straight lines, then they should be respectively taken simply as the *bottom* of conduction band energy levels and *top* of valence band energy levels.

2. In elemental semiconductors (Si or Ge), the n-type or p-type semiconductors are obtained by introducing 'dopants' as defects. In compound semiconductors, the change in relative stoichiometric ratio can also change the type of semiconductor. For example, in ideal GaAs the ratio of Ga:As is 1:1 but in Ga-rich or As-rich GaAs it could respectively be $Ga_{1.1}\ As_{0.9}$ or $Ga_{0.9}\ As_{1.1}$. In general, the presence of defects control the properties of semiconductors in many ways.
3. In transistors, the base region is both narrow and lightly doped, otherwise the electrons or holes coming from the input side (say, emitter in CE-configuration) will not be able to reach the collector.
4. We have described an oscillator as a positive feedback amplifier. For stable oscillations, the voltage feedback (V_{fb}) from the output voltage (V_o) should be such that after amplification (A) it should again become V_o. If a fraction β' is feedback, then $V_{fb} = V_o.\ \beta'$ and after amplification its value $A(v_o.\beta')$ should be equal to V_o. This means that the criteria for stable oscillations to be sustained is $A\ \beta' = 1$. This is known as Barkhausen's Criteria.
5. In an oscillator, the feedback is in the same phase (positive feedback). If the feedback voltage is in opposite phase (negative feedback), the gain is less than 1 and it can never work as oscillator. It will be an amplifier with reduced gain. However, the negative feedback also reduces noise and distortion in an amplifier which is an advantageous feature.

EXERCISES

14.1 In an n-type silicon, which of the following statement is true:
(a) Electrons are majority carriers and trivalent atoms are the dopants.
(b) Electrons are minority carriers and pentavalent atoms are the dopants.
(c) Holes are minority carriers and pentavalent atoms are the dopants.
(d) Holes are majority carriers and trivalent atoms are the dopants.

14.2 Which of the statements given in Exercise 14.1 is true for p-type semiconductos.

14.3 Carbon, silicon and germanium have four valence electrons each. These are characterised by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statements is true?
(a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$
(b) $(E_g)_C < (E_g)_{Ge} > (E_g)_{Si}$
(c) $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$
(d) $(E_g)_C = (E_g)_{Si} = (E_g)_{Ge}$

14.4 In an unbiased p-n junction, holes diffuse from the p-region to n-region because
(a) free electrons in the n-region attract them.
(b) they move across the junction by the potential difference.
(c) hole concentration in p-region is more as compared to n-region.
(d) All the above.

14.5 When a forward bias is applied to a p-n junction, it
(a) raises the potential barrier.
(b) reduces the majority carrier current to zero.
(c) lowers the potential barrier.
(d) None of the above.

14.6 For transistor action, which of the following statements are correct:
(a) Base, emitter and collector regions should have similar size and doping concentrations.
(b) The base region must be very thin and lightly doped.
(c) The emitter junction is forward biased and collector junction is reverse biased.
(d) Both the emitter junction as well as the collector junction are forward biased.

14.7 For a transistor amplifier, the voltage gain
(a) remains constant for all frequencies.
(b) is high at high and low frequencies and constant in the middle frequency range.
(c) is low at high and low frequencies and constant at mid frequencies.
(d) None of the above.

14.8 In half-wave rectification, what is the output frequency if the input frequency is 50 Hz. What is the output frequency of a full-wave rectifier for the same input frequency.

14.9 For a CE-transistor amplifier, the audio signal voltage across the collected resistance of 2 kΩ is 2 V. Suppose the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is 1 kΩ.

14.10 Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 volt, calculate the output ac signal.

14.11 A p-n photodiode is fabricated from a semiconductor with band gap of 2.8 eV. Can it detect a wavelength of 6000 nm?

ADDITIONAL EXERCISES

14.12 The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of Arsenic and 5×10^{20} per m^3 atoms of Indium. Calculate the number of electrons and holes. Given that $n_i = 1.5 \times 10^{16}\ m^{-3}$. Is the material n-type or p-type?

14.13 In an intrinsic semiconductor the energy gap E_g is 1.2eV. Its hole mobility is much smaller than electron mobility and independent of temperature. What is the ratio between conductivity at 600K and that at 300K? Assume that the temperature dependence of intrinsic carrier concentration n_i is given by

$$n_i = n_0 \exp\left(-\frac{E_g}{2k_B T}\right)$$

where n_0 is a constant.

14.14 In a p-n junction diode, the current I can be expressed as

$$I = I_0 \exp\left(\frac{eV}{2k_B T} - 1\right)$$

where I_0 is called the reverse saturation current, V is the voltage across the diode and is positive for forward bias and negative for reverse bias, and I is the current through the diode, k_B is the Boltzmann constant (8.6×10^{-5} eV/K) and T is the absolute temperature. If for a given diode $I_0 = 5 \times 10^{-12}$ A and T = 300 K, then

(a) What will be the forward current at a forward voltage of 0.6 V?

(b) What will be the increase in the current if the voltage across the diode is increased to 0.7 V?

(c) What is the dynamic resistance?

(d) What will be the current if reverse bias voltage changes from 1 V to 2 V?

14.15 You are given the two circuits as shown in Fig. 14.44. Show that circuit (a) acts as OR gate while the circuit (b) acts as AND gate.

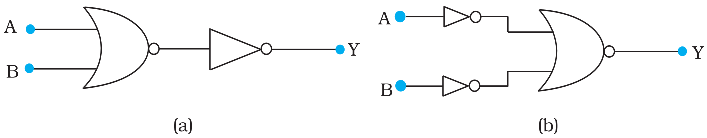


FIGURE 14.44

14.16 Write the truth table for a NAND gate connected as given in Fig. 14.45.

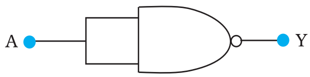


FIGURE 14.45

Hence identify the exact logic operation carried out by this circuit.

14.17 You are given two circuits as shown in Fig. 14.46, which consist of NAND gates. Identify the logic operation carried out by the two circuits.

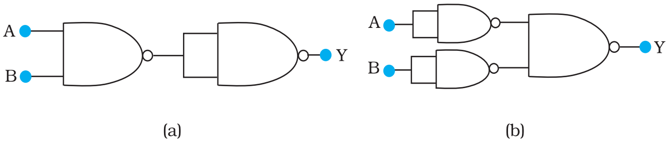


FIGURE 14.46

14.18 Write the truth table for circuit given in Fig. 14.47 below consisting of NOR gates and identify the logic operation (OR, AND, NOT) which this circuit is performing.

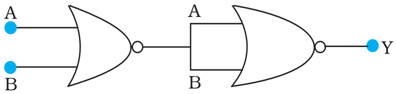


FIGURE 14.47

(Hint: A = 0, B = 1 then A and B inputs of second NOR gate will be 0 and hence Y=1. Similarly work out the values of Y for other combinations of A and B. Compare with the truth table of OR, AND, NOT gates and find the correct one.)

14.19 Write the truth table for the circuits given in Fig. 14.48 consisting of NOR gates only. Identify the logic operations (OR, AND, NOT) performed by the two circuits.

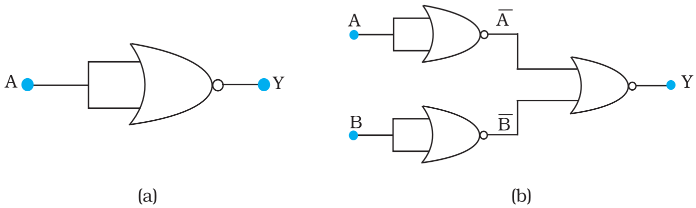


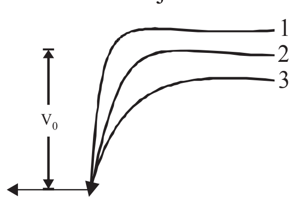
FIGURE 14.48

Chapter 14:- Semiconductor Electronics

Multiple Choice Questions (MCQs)

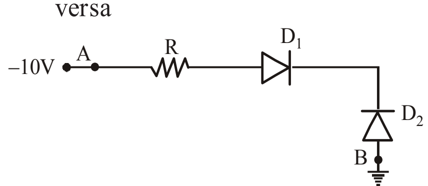
1. The conductivity of a semiconductor increases with increase in temperature because
(a) number density of free current carries increases
(b) relaxation time increases
(c) both number density of carries and relaxation time increase
(d) number density of carries increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

2. In figure given below V_0 is the potential barrier across a p–n junction, when no battary is connected across the junction

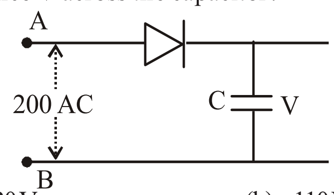


(a) 1 and 3 both correspond to forward bias of junction
(b) 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junctions
(c) 1 corresponds to forward bias and 3 corresponds to reverse bias of junction
(d) 3 and 1 both correspond to reverse bias of junction

3. In figure given below, assuming the diodes to be ideal
(a) D_1 is forward biased and D_2 is reverse biased and hence current flows from A to B
(b) D_2 is forward biased and D_1 is reverse biased and hence no current flows from B to A and vice–versa
(c) D_1 and D_2 are both forward biased and hence current flows from A to B
(d) D_1 and D_2 are both reverse biased and hence no current flows from A to B and vice – versa



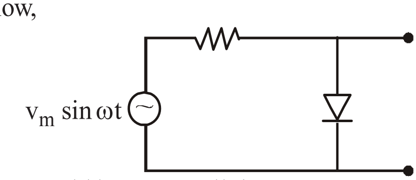
4. A 220 V AC supply is connected between points A and B (figure). What will be the potential difference V across the capacitor?



(a) 220 V (b) 110 V
(c) 0 V (d) $220\sqrt{2}$ V

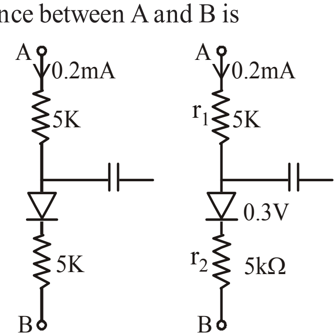
5. Hole in semiconductor is
(a) an anti – particle of electron
(b) a vacancy created when an electron leaves a covalent bond
(c) absence of free electrons
(d) an artificially created particle

6. The output of the given circuit in figure given below,



(a) would be zero at all times
(b) would be like a half wave rectifier with positive cycles in output
(c) would be like a half wave rectifier with negative cycles in output
(d) would be like that of a full wave rectifier

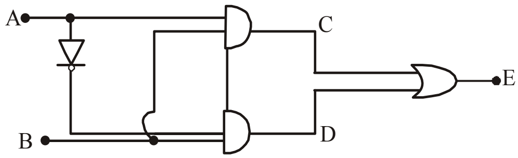
7. In the circuit shown in figure given below, if the diode forward voltage drop is 0.3 V, the voltage difference between A and B is



(a) **(b)**

(a) 1.3 V (b) 2.3 V
(c) 0 (d) 0.5 V

8. Truth table for the given circuit is



(a)

A	B	E
0	0	1
0	1	0
1	0	1
1	1	0

(b)

A	B	E
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	E
0	0	0
0	1	1
1	0	0
1	1	1

(d)

A	B	E
0	0	0
0	1	1
1	0	1
1	1	0

Multiple Choice Questions (MCQs)
(More than one option correct)

1. When an electric field is applied across a semiconductor

(a) electrons move from lower enregy level to higher energy level in the conduction band

(b) electrons move from higher energy level to lower energy level in the conductions band

(c) holes in the valence band move from higher energy level to lower energy level

(d) holes in the valence band move from lower energy level to higher energy level

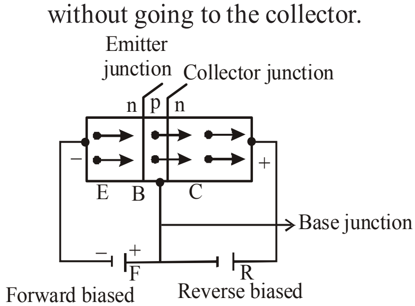
2. Consider an n–p–n transitor with its base emitter junction forward biased and collector juncton reverse biased. Which of the following statements are true?

(a) Electrons crossover from emitter to collector

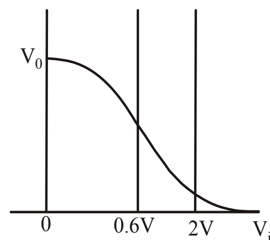
(b) Holes move from base to collector

(c) Electrons move from emitter to base

(d) Electrons from emitter move out of base without going to the collector.



3. Figure given below shows that transfer characteristics of a base biased CE transistor. Which of the following statements are true?



(a) At $V_i = 0.4$ V, transistor is in active state

(b) At $V_i = 1$ V, it can be used as an amplifier

(c) At $V_i = 0.5$ V, it can be used as a switch turned off

(d) At $V_i = 2.5$ V, can be used as a switch tuned on

4. In a n–p–n transistor circuit, the collector current is 10 mA. If 95 per cent of the electrons emitted reach the collector, which of the following statements are true?

(a) The emitter current will be 8 mA

(b) The emitter current will be 10.53 mA

(c) The base current will be 0.53 mA

(d) The base current will be 2mA

5. In the depletion region of a diode

(a) there are no mobile charges

(b) equal number of holes and electrons exist, making the region neutral

(c) recombination of holes and electrons has taken place

(d) immobile charged ions exist

6. What happens during regulation action of a Zener diode?

(a) The current and voltage acorss the Zener remains fixed

(b) The current through the serious Resistance (R_s) changes

(c) The Zener resistance is constant

(d) The resistance offered by the Zener changes

7. To reduce the ripples in rectifier circuit with capacitor filter

(a) R_L should be increased

(b) input frequency should be decreased

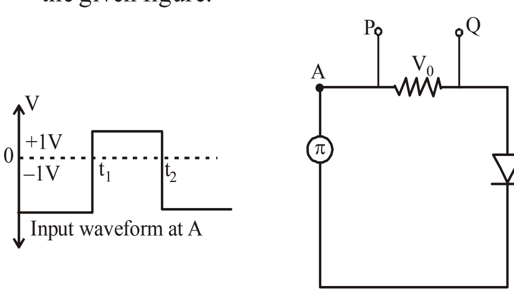
(c) input frequncy should be increased

(d) capacitors with high capacitance should be used

8. The breakdown in a reverse biased p–n junction is more likely to occur due to
   (a) large velocity of the minority charge carriers if the doping concentration is small
   (b) large velocity of the minority charge carries if the doping concentration is large
   (c) strong electric field in a depletion region if the doping concentration is small
   (d) strong electric field in the depletion region if the doping concentration is large

Very Short Answer Questions

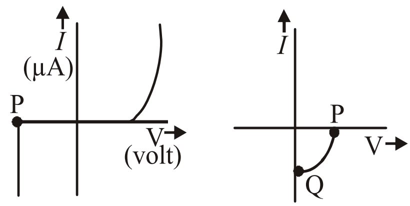
1. Why are elemental dopants for Silicon or Germanium usually chosen from group XIII or group XV?
2. Sn, C and Si, Ge are all group XIV elements. Yet, Sn is a conductor, C is an insulator while Si and Ge are semiconductors. Why?
3. Can the potential barrier across a p–n junction be measured by simply connecting a voltmeter across the junction?
4. Draw the output waveform across the resistor in the given figure.



5. The amplifiers X, Y and Z are connected in series. If the voltage gains of X, Y and Z are 10, 20 and 30, respectively and the input signal is 1 mV peak value, then what is the output signal voltage (peak value)
   (i) if DC supply voltage is 10 V?
   (ii) if DC supply voltage is 5V?
6. In a CE transistor amplifier, there is a current and voltage gain associated with the circuit. In other words there is a power agin. Considering power a measure of energy, does the circuit violate conservation of energy?

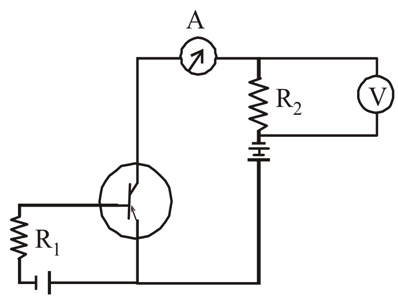
Short Answer Questions

1. (i) Name the type of a diode whose characteristics are shown in figure (a) and (b).
   (ii) What does the point P in fig. (a) represent?
   (iii) What doess the points P and Q in fig. (b) represent?

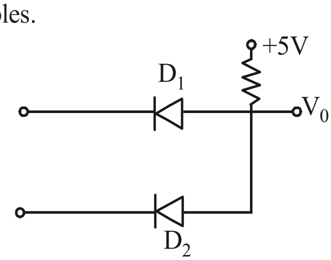


(a) (b)

2. Three photo diodes D_1, D_2 and D_3 are made of semiconductors having band gaps of 2.5eV, 2eV and 3eV, respectively. Which ones will be able to detect light of wavelength 6000 Å?
3. If the resistance R_1 is increased (see figure), how will readings of the ammeter and voltmeter change?

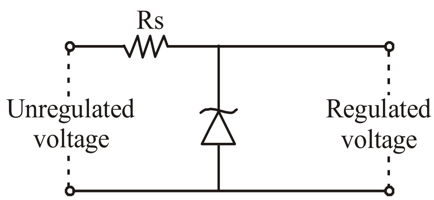


4. Two car garages have a common gate which needs to open automatically when a car enters either of the garages or cars enter both. Devise a circuit that resembles this situation using diodes for this situation.
5. How would you set up a circuit to obtain NOT gate using a trasistor?
6. Explain why elemental semiconductor cannot be used to make visible LEDs.
7. Write the truth table for the circuit shown in figure given below. Name the gate that the circuit resembles.



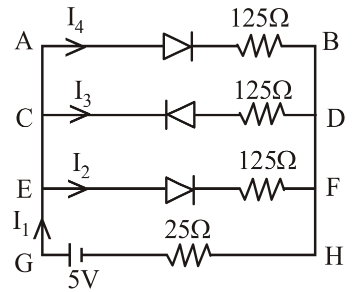
8. A Zener of power rating 1 W is to be used as a voltage regulator. If Zener has a breakdown of 5V and it has to regulated voltage which fluctuated between 3 V and 7V, what should be

the value of R_s for safe operation (see figure)?

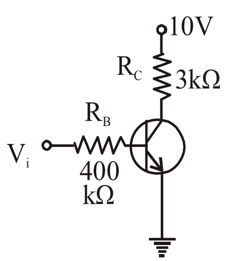


Long Answer Questions

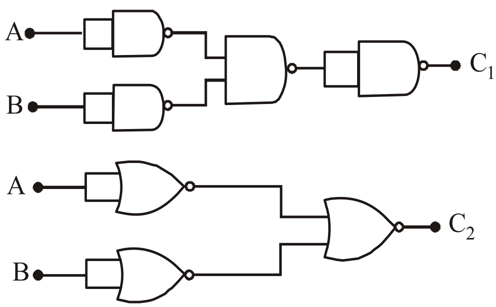
1. If each diode in figure has a foward bias resistance of 25Ω and infinite resistance in reverse bias, what will be the values of the currents I_1, I_2, I_3 and I_4?

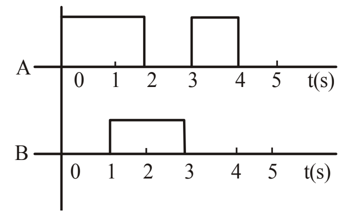


2. In the circuit shown in figure, when the input voltage of the base resistance is 10 V, V_{BE} is zero and V_{CE} is also zero. Find the values of I_B, I_C and β.

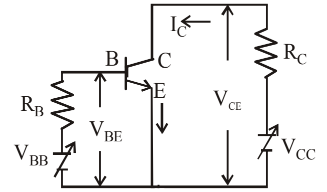


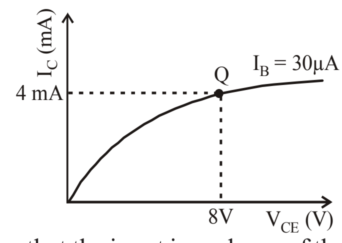
3. Draw the output signals C_1 and C_2 in the given combination of gates.





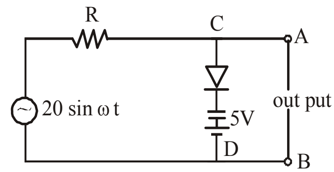
4. Consider the circuit arrangement shown in figure for studying input and output characteristics of n–p–n trasnsistor in CE configuration.
Select the values of R_B and R_C for a transistor whose V_{BE} = 0.7 V, so that the transistor is operating at point Q as shown in the characteristics (see figure).





Given that the input impedance of the transistor is very small and $V_{CC} = V_{BB}$ = 16V, also find the voltage gain and power gain of circuit making appropriate assumptions.

5. Assuming the ideal diode, draw the output waveform for the circuit given in fig. (a), explain the waveform.



6. Suppose a n–type wafer is created by doping Si crystal having 5 × 10^{28} atoms/m^3 with 1 ppm concertration of As. On the surface 200 ppm boron is added to create 'p' region in this wafer. Considering ni = 1.5 × 10^{16} m^{-3}, (i) Calculate the densities of the charge carriers in then n and p regions. (ii) Comment which charge carries would contribute largely for the reverse saturation current when diode is reverse biased.

7. An X OR gate has following truth table.

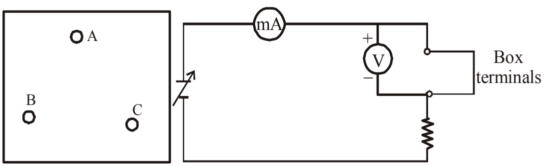
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

It is represented by following logic relation

$$Y = \overline{A}.B + A.\overline{B}$$

Build this gate using AND, OR and NOT gate.

8. Consider a box with three terminals on top of it as shown in figure.



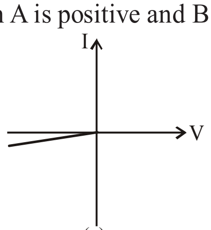
(a) (b)

Three components namely, two germanium diodes and one resistor are connected across these three terminals in some arrangement.

A student performs an experiment in which any two of these three terminals are connected in the circuit shown in figure.

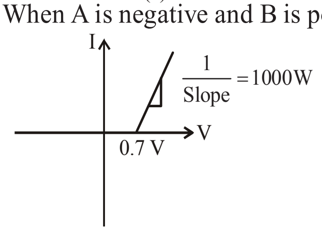
The student obtains graphs of current Voltage characteristics for unknown combination of components between the two terminals connected in the circuit. The graphs are

(i) When A is positive and B is negative



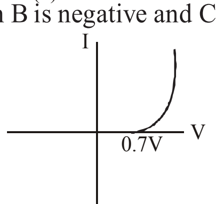
(c)

(ii) When A is negative and B is positive



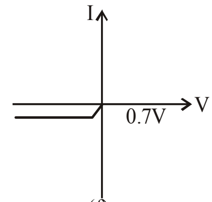
(d)

(iii) When B is negative and C is positive



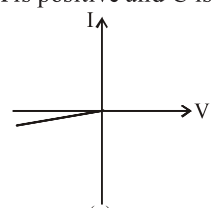
(e)

(iv) When B is positive and C is negative



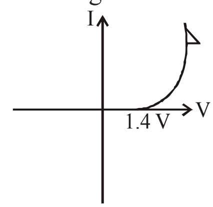
(f)

(v) When A is positive and C is negative



(g)

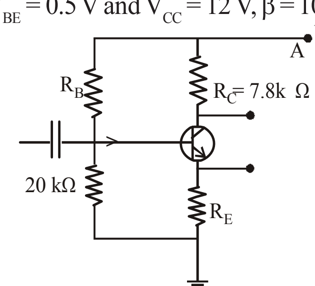
(vi) When A is negative and C is positive



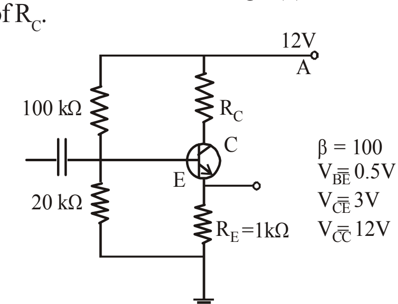
(h)

From these graphs of current – voltage characteristic shown in fig. (c) to (h) determine the arrangement of components between A, B and C.

9. For the transistor circuit shown in figure, evaluate V_E, R_B, R_E, given $I_C = 1$ mA, $V_{CE} = 3$ V, $V_{BE} = 0.5$ V and $V_{CC} = 12$ V, $\beta = 100$.



10. In the circuit shown in fig . (a), find the values of R_C.



NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. (d) In semiconsuctor the density of change carriers (electron hole) are very small, so its resistance is high when the conductivity of a semiconductor increases with increase in temperature, because the number density of current carries increases then the speed of free electron increase and relaxation time decreases but effect of decrease in relaxation is much less than increase in number density.

2. (b) When p–n junction is forward biased then the depleton layer is compresses or decrease so it opposes the potential junction resulting decrease in potential barrier junction when p–n junction is reverese biased, it supports the potential barrier junction, resulting increase in potential across the junction.

3. (b) As the given circuit, p–side of p–n function D_1 is connected to lower voltage and n – side of D_1 of higher voltage.

So, D_1 is reverse biased.

In circuit A is at –10V and B is at 0 (zero) V. So B is positive then A or

The p–side of p–n junction D_2 is at higher potential and n–side of D_2 is at lower potential.

So, D_2 is forward biased.

Hence, No current flows through the junction B to A and vice–versa.

4. (d) As the given figure p–n junction conducts during positive half cycyle only, then diode connected here will work is positive half cycle. Potential difference across C will be peak voltage when diode is in forword bias then the peak voltage of the given AC voltage

$$= V_0 = V_{rms}\sqrt{2} = 220\sqrt{2}\text{V}$$

5. (b) Atom of semiconductor are bounded by covalent bonds between the atoms of same or different type. The concept of hole describes the lack of an electron at a position where one could exist in an atom or atomic lattice. If an electron is excited into a higher state, it leaves a hole in its old state. So, hole can be defined as a vacancy created when an electron leaves a covalent bond.

6. (c) When the diode will be in forward biased during positive half cycle of input AC voltage, the resistance of p–n junction is low. The current in the circuit is maximum. So, a maximum potential difference will appear across resistance connected in series of circuit. So, potential across PN junction will be zero. When the diode will be in reverse biase during negative half cycle of AC voltage, the resistance of p–n juntion becomes high which will be more than resistance in series. So, there will be voltage across p–n junction with negative cycle in output.

7. (b) Let the potential difference between A and B is V, Given here $r_1 = 5\,k\Omega$ and $r_2 = 5\,k\Omega$ are resistance in series connection.

So,

$$V_{AB} - 0.3 = [(r_1 + r_2)\, 10^3] \times (0.2 \times 10^{-3})] \quad [\because V = ir]$$

$$(V_{AB} - 0.3) = 10 \times 10^3 \times 0.2 \times 10^{-3} = 2$$

So, $V_{AB} = 2 + 0.3 = 2.3$ V

8. (c) As the given figure the output of C,D and E are :

$C = A.B$ and $D = \overline{A}.B$

$E = C + D = (A.B) + (\overline{A}.B)$

So, the truth table of given arrangement of gates can be written as :

A	B	$\overline{A}$	$C = A.B$	$D = \overline{A}.B$	$E = (C+D)$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

Multiple Choice Questions (More Than One Options)

1. **(a, c)** When electric field is applied across a semiconductor. The electrons in the conduction band get accelerated and acquire energy then they move from lower energy level to higher energy level. While the holes are in valence band move from higher energy level to lower energy level, where they will be having more energy.

2. **(a, c)** As the Drawing figure, the emitter – base junction is forward biased i.e., the positive pole of emitter base battery is connected to base and its negative pole to emitter. Also, the collector base junction is reverse biased, because the positive pole of the collector base battery is connected to collector and negative pole to base. So, electrons are repelled by forward bias from emitter to collector approx 5% of electron combine with holes of base and rest 95% electrons of emitter are attracted by reverse bias of CB junction. So, electron move from emmiter to base and crossover from emitter to collector.

3. **(b, c, d)**

   Each option the given transfer characteristics of a base biased common emitter transistor, So consider

   (i) At $V_i = 0.4$ V, there is no collector. The transistor circuit is not in active state.

   (ii) At $V_i = 1$V When transistor is in between 0.6 V to 2V, then transistor circuit is nactive state and is used as an amplifier

   (iii) At $V_i = 0.5$ V, when there is no collector current then the transistor is in cut off state and transistor circuit can be used as a switch tuned off.

   (iv) At $V_i = 2.5$ V, the collector current becomes maximum and transistor is in saturation state and there is no effect on changing input voltage. So, it can used as a switch turned on state. So trasnsistor is in beyond active region.

4. **(b, c)** As given that

   $I_c = 10$ mA

   So, $I_c = 95\%$ of I_e

   $$I_c = \frac{95}{100} I_e \Rightarrow I_e = \frac{10 \times 100}{95} = 10.53\text{mA}$$

   We know that $I_E = (I_b + I_c)$

   Also, $I_b = I_e - I_c = 10.53 - 10 = 0.53$ mA

5. **(a, b, d)** During formation of p–n junction electrons from n side and holes from p side move towards each other and form a potential barrier across the junction. And the space–charge regions on both the sides of p–n junction which has immobile ions and entirely lacking of any charge carries will form a region called depletion region of a diode. The number of ionized acceptors on the p–side equals the number of ionized donors on the n–side.

6. **(b, d)** During action of regulation of a Zener diode, the current through the R_s changes and resistance offered by the Zener diode changes the current through the zener charges but the voltage across the Zener remains constant.

7. **(a, c, d)** The Ripple factor (r) of a full wave rectifier using capacitor filter is

   $$r = \frac{1}{4\sqrt{3}R_L C_V}$$

   $$r \propto \frac{1}{R_L} \Rightarrow r \propto \frac{1}{C}, r \propto \frac{1}{\nu}$$

   As we know that

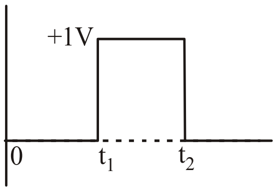
   So, Ripple factor is inversely proportional to R_L, C and ν.

   To reduce r, R_L, should be increased and input frequency (ν) should be increased.

8. **(a, d)** In reverse biasing, minority charge carriers will be accelerated due to high electric field applied or so the minority charge carries in both junction accelerated toward depletion layer which on striking with atoms cause ionization resulting secondary electrons and So more number of charge carries more current in reverse directions. When doping concetration is large, there will be large number of ions in the depletion region, which will give rise to a strong electric field.

Very Short Answer Questions

1. The size of the dopant atom should be equivalent to the size of SIORGE. So that the symmetry of pure SIORGE such that their presence in the pure semiconductor does not disturb or distort the semiconductor but easily contribute the charge carries on forming covalent bonds with Si or Ge atoms. As the silicon and germanium belongs to XIVth group so similar size of atom which are provided by group XIII or group XV elements.
2. A material will be conduct current if in its energy band diagram, there is no energy gap between conduction and valence band. For insulator, the energy gap is large and for semiconductor the energy gap is moderate. So, energy gap decrease from insulator to semiconductor. The energy gap for Sn is 0 eV, for C is 5.4 eV, for Si is 1.1 eV and Ge is 0.7 eV, related to their atomic size. So the Sn is a conductor,C is an insulator while Ge and Si are semiconductors.
3. We cannot measure the potential barrier across a p–n junction by a voltmeter because the resistance across depletion layer is smaller than resistance of μ voltmeter so current to measure the potential will not flow in voltmeter and current passes through the junction resistance.
4. In the given circuit waveform is connected at A and waveform obtained when the diode only works in forward biased, so the output obtained only when positive input is given so the output waveform will be only t_1 to t_2 which shown in figure :–



5. As given that, $Av_x = 10$, $Av_y = 20$, $Av_z = 30$;

$\Delta V_i = 1\text{mV} = 10^{-3}\,\text{V}$

As we know that

Total voltage amplification or voltage gain

$$= \frac{\text{Output Signal Voltage}(\Delta V_0)}{\text{Input Singal Voltage}(\Delta V_i)}$$

So, Toatl voltage gain = $Av_x \times Av_y \times Av_z$

$\Rightarrow \quad \Delta V_0 = Av_x \times Av_y \times Av_z \times \Delta V_i$

$= 10 \times 20 \times 30 \times 10^{-3} = 6\,\text{V}$

(Output signal voltage)

(i) When DC supply voltage is 10 V, then output is 6V, as the theoretical gain is equal to practical gain, i.e., output can never be greater than 6V.

(ii) When DC supply voltage is 5V, or $V_{cc} = 5\text{V}$. Then, output peak will not exceed 5V. So output will be $V_0 = 5\text{V}$.

6. In CE transistor amplifier the DC supply is connected to give energy to signal due to this the power gain is very high. In this circuit, the extra power required for amplified output is obatined from DC source. Hence the circuit used does not violet the law of conservation.

Short Answer Questions

1. (i) The type of diode in the characteristics curve (a) shows the Zener diode and curve (b) shows the solar cell.

(ii) The point P in figure (a) represents Zener break down voltage.

(iii) In figure (b), point Q represents zero voltage and negative current. It means light energy falling on solar cell with atleast minimum threshold frequency gives the current in opposite direction to that due to a battary connected to solar cell. But for the point P, the battary is short circuited. Hence represents the short circuit current.

In figure (b), point P represents some positive voltage on solar cell with zero current through solar cell.

So, there is a battery connected to a solar cell which gives rise to the equal and opposite current to that in solar cell by virtue of light falling on it.

So, current is zero for point P, hence we say P represents open circuit voltage.

2. As given that, wavelength of light

$\lambda = 6000\,\text{Å} = 6000 \times 10^{-10}$ meter

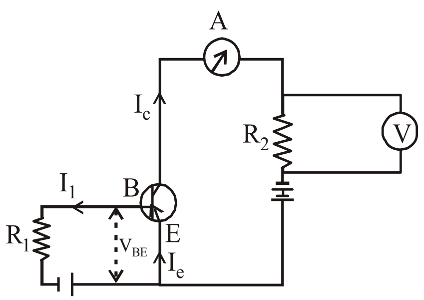
So, Energy of the light photon

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}}\,\text{eV} = 2.06\,\text{eV}$$

The incident radiation is detected by the photodiode D_2 having energy should be greater than the band–gap or knee voltage 2eV. So, it is only valid for diode D_2. Diode D_2 will detect less than incident radiation of 2.06 eV.

3. Consider the figure given below to find the change in reading of output E – C circuit.

So, $I_B = \dfrac{V_{BB} - V_{BE}}{R_i}$

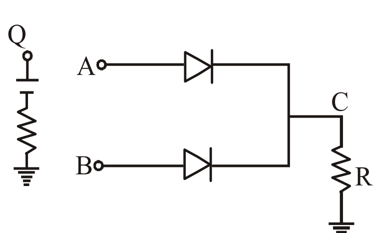


As R_i increased the P.D. across R_i also increase So potential across BE decrease. So I_B is decreased.

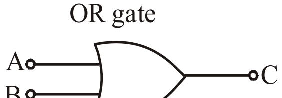
Now, the current in ammeter is collector current I_c.

$I_c = \beta I_B$ as I_B decreased I_c also decreased and the reading of voltmeter and ammeter will also decreased.

4. When a car enters in the gate, any one or both are open. So, the device is shown in figure :–



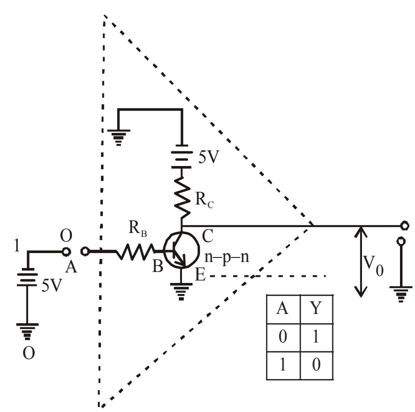
OR gate



So, OR gate gives the desired output shown in truth table :–

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

5. The NOt gate is a device which has only one input and one output i.e., $\overline{A} = Y$ means Y equals NOT A. So, the gate cannot be realised by using diodes. However it can be realised by making use of a transistor. That can be shown in the shown figure



Here, the base B of the transistor is connected to the input A through the input resistance R_B and the emitter E is earthed. The collector is connected to 5 V battery. The output Y is the voltage at C with respect to earth.

The resistor R_B and R_C are so chosen that if emitter–base junction is unbiased, the transistor is in cut off mode and if emitter– base junction is in forward biased by 5V then the transistor is in saturation state.

6. To make the visible LED we cannot used elemental semiconductor because the band gap is such that emissions are in infrared region and not in visible region.

7. As the given circuit resemble AND gate. The boolean expression of this circuit is, V_0 = A.B i.e., V_0 equals A AND B. The truth table of this gate is

A	B	$V_0 = A.B$
0	0	0
0	1	0
1	0	0
1	1	1

8. As given that, power = 1Waft
Voltage for Zener breakdown Vz = 5V
Minimum voltage $V_{min} = 3V$
Maimum voltage $V_{max} = 7V$

$$\text{Current } I_{Z_{max}} = \frac{P}{v_Z} = \frac{1}{5} = 0.2\text{Amp}$$

then the value of R_s for safe operation

$$R_s = \left[\frac{V_{max} - V_{min}}{I_{Z_{max}}}\right] = \left[\frac{7-3}{0.2}\right] = \frac{4}{0.2} = 20\Omega$$

Long Answer Questions

1. As given that,
forward biased resistance = 25Ω
Reverse biased resistance = ∞, V = 5 volt
As the diode in branch CD is in reverse biased which having resistance infinite,
So ($I_3 = 0$)
Diode D_1 and D_3 are in forward bias D_2 is in reverse bias,
So, Resistance in branch AB = 25 + 125 = 150 Ω say R_1
Resistance in branch EF = 25 + 125 = 150 Ω say R_2
AB is parallel to EF.
So,

$$\text{Net resistance } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{150} + \frac{1}{150} = \frac{2}{150}$$

$$\Rightarrow R' = 75\Omega$$

Total resistance R = R' + 25 = 75 + 25 = 100 Ω

$$\text{So, } V = IR, \text{ current } I_1 = \frac{V}{R} = \frac{5}{100} = 0.05\text{Amp}$$

As per figure $I_1 = I_4 + I_2 + I_3 \qquad (\because I_3 = 0)$

So, $I_1 = I_4 + I_2$
Here resistance R_1 and R_2 are same.
i.e., $I_4 = I_2$
So, $I_1 = 2I_2$

$$\Rightarrow I_2 = \frac{I_1}{2} = \frac{0.05}{2} = 0.025 \text{ Amp}$$

and $I_2 = 0.025$ Amp, $I_4 = 0.025$ Amp ($\therefore I_4 = I_2$)
So, $I_1 = 0.05$ Amp $I_2 = 0.025$ Amp, $I_3 = 0$
and $I_4 = 0.025$Amp

2. As given that,
voltage across R_B = 10 V
Resistance R_B = 400 kΩ
$V_{BE} = 0$, $V_{CE} = 0$
$R_C = 3k\Omega$

$$I_B = \frac{\text{voltage across } R_B}{R_B}$$

$$= \frac{10}{400 \times 10^3} = 25 \times 10^{-6} A = 25\mu A$$

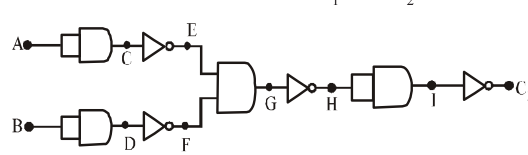
$$I_C = \frac{(\text{voltage across} R_C)}{R_C} = \frac{10}{3 \times 10^3}$$

$= 3.33 \times 10^{-3}$ A = 3.33 mA

$$\text{Current gain } \beta = \frac{I_C}{I_B} = \frac{3.33 \times 10^{-3}}{25 \times 10^{-6}}$$

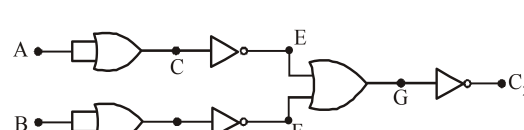
$= 1.33 \times 10^2 = 133$

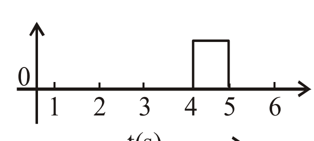
3. As per given figure
to draw the truth table of C_1 and C_2.



A	B	C	D	E	F	G	H	I	C_1
0	0	0	0	1	1	1	0	0	1
1	0	1	0	0	1	0	1	1	0
0	1	0	1	1	0	0	1	1	0
1	1	1	1	0	0	0	1	1	0

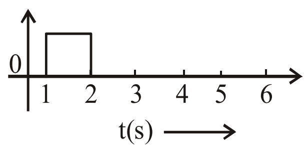
So, the waveform of C_1



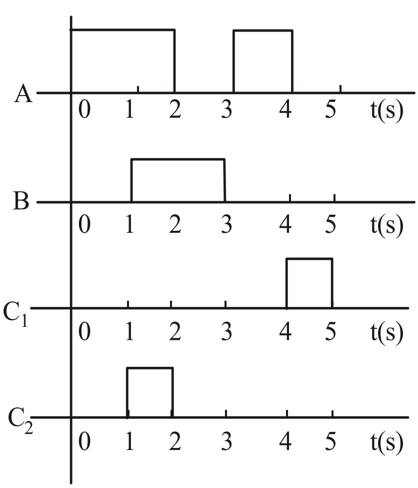


A	B	C	D	E	F	G	C_2
0	0	0	0	1	1	1	0
1	0	1	0	0	1	1	0
0	1	0	1	1	0	1	0
1	1	1	1	0	0	0	1

So, the waveform of C_2



So, the wave form for C_1 and C_2 is –



4. As given that,

$V_{BE} = 0.7\,V, V_{CC} = V_{BB} = 16\,V$

$V_{CE} = 8V$ (from graph)

$I_C = 4mA = 4 \times 10^{-3}\,A$

$I_B = 30\mu A = 30 \times 10^{-6}\,A$

For ouput characteristic at Point Q, Applying Kirchhoff's law in collector emitterloop.

$$V_{CC} = I_C R_C + V_{CE}$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{16-8}{4\times10^{-3}} = \frac{8\times1000}{4} = 2k\Omega$$

$R_C = 2k\Omega$

Now applying Kirchhoff's law in base–emitter loop

$$V_{BB} = I_B R_B + V_{BE}$$

$$R_B = \frac{V_{BB} - V_{BE}}{I_B} = \frac{16-0.7}{30\times10^{-6}} = \frac{15.3\times10^6}{30}$$

$= 510 \times 10^3\Omega = 510\,k\Omega$

For voltage gain = $Av = +\beta\frac{R_C}{R_B}$

$$\beta = \frac{I_C}{I_B} = \frac{4\times10^{-3}}{30\times10^{-6}} = 133$$

For Average voltage gain

$$= \beta\frac{R_C}{R_B} = \frac{133\times2\times10^3}{510\times10^3} = 0.52$$

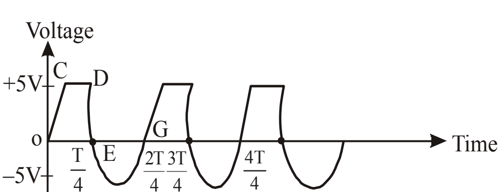
Power gain = $\beta \times$ Voltage gain

$= 133 \times 0.52 = 69.33$

5. When the input voltage (20 sin ωt) is equal to or less than 5V, diode will be reverse biased. It will offer high resistance in comparison to resistance (R) in series. Now, diode appears in open circuit. So input signal will not pass through diode. The input signal is then passed to the output terminals. Then the result with sin wave input is to dip off all positive going portion above 5V.

If input voltage (20 sin ωt) is more than + 5V, diode will be conducting as if forward biased offering low resistance in comparison to R. But there will be no voltage in output beyond 5 V as the voltage beyond + 5V will appears across R. So the current passes through diode and battery and output remian 5V.

When input voltage is negative. there will be opposition to 5V battery in p–n junction input voltage becomes more than – 5V, the diode will be revese biased. It will offer high resistance in comparison to resistance R in series. Now junction diode appears in open circuit. The input wave form is then passed on to output terminals. The output waveform is shown.



6. When As (pentavalent) is implanted in Si crystal, n – type wafer is created. So, the number of majority carries electrons in n type wafer due to doping of As is

$$n_e = (N_D \& Dsi) = \frac{1}{10^6} \times 5 \times 10^{28}$$

$$= 5 \times 10^{22} / m^3$$

For number of minority carriers (holes) in n–type wafer is

$$n_e.n_h = x_2^2 \qquad \text{(given)}$$

$$\text{So,} \quad n_h = \frac{n_i^2}{n_e} = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}}$$

$$(\because n_2 = 1.5 \times 10^{16} / m^3)$$

$$= 0.45 \times 10^{10}/m^3$$

When B Boron (trivalent) is implanted Si crystal, p–type wafer is formed with number of holes, $n_h = (N_D \times n \text{ of Si})$

$$= \frac{200}{10^6} \times (5 \times 10^{28}) = 1 \times 10^{25} / m^3$$

$$nh = 10^{25}/m^3$$

Minority carries (electrons) created in p–type wafer is

$$n_e.n_h = xn_2^2$$

$$\text{So,} \quad n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{1 \times 10^{25}} = 2.25 \times 10^{32-25}$$

$$= 2.25 \times 10^7/m^3.$$

When p–n junction is reverse biased, then the minority charge carrier moves toward depletion layer holes of n–region from n side ($n_h = 0.45 \times 10^{10}/m^3$) would contribute more to the reverse saturation current than minority carrier electrons ($n_e = 2.25 \times 10^7/m^3$) of p region from p side move towards junction and make the depletion layer thicker.

7. In given, the logic relation

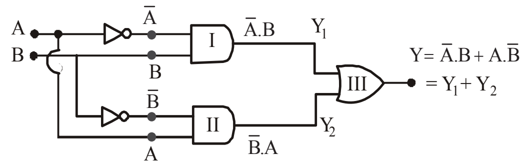
$$Y = \bar{A}.B + A.\bar{B} = Y_1 + Y_2$$

So, $Y_1 = A.B$ and $Y_2 = A.\bar{B}$

Y_1 can be obtained as output of AND gate I for which one Input is A through NOT gate and another input is of B. Y_2 can be obtained as output of AND gate II for which one input is of A and other input is of B through NOT gate.

So, $Y_1 = \bar{A}.B, Y_2 = A.\bar{B}$

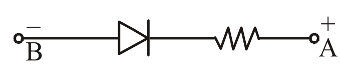
So, the given table can be obtianed form the logic circuit given as : –



Now, Y_1 and Y_2 are feed into the two terminals of OR gate to get (Y).

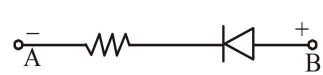
$$Y = Y_1 + Y_2 \text{ or } Y = \bar{A}.B + A.\bar{B}$$

8. (a) For condition (i) of graph shows a reverse characteristics is shown in fig. (c). Here A is connected to n – side of p–n junction I and B is connected to p–n junction I with a resistance in series.

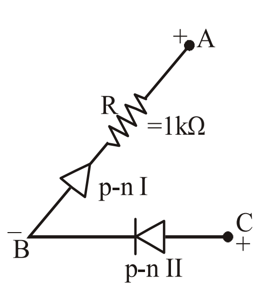


(b) For condition (ii) of graph shows a forward characteristics is shown in fig. (d), where 0.7 V is the knee voltage of p–n junction 1 1/slope = (1/ 1000) Ω.

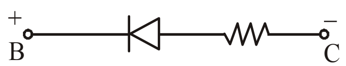
It means A is connected to n–side of p–n junction I and B is connected to p–side of p–n junction I and resistance R is in series of p–n junction I between A and B.



(c) For condition (iii) of graph shows a forward characteristics is shown in figure (e), where 0.7 V is knee voltage. In this case p–side of p–n junction II is connected to C and n–side of p–n junction II to B.

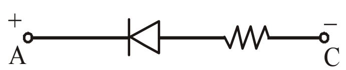


(d) For condition (iv) of graph shows reverse biased characteristic of p–n junction II. n side with B and P side with c along with a resistance in series. So,

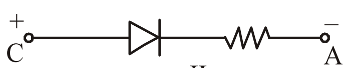


p-n II

(e) Similarly condition (v) of graph shows reverse biased characteristics.



(f) For condition (vi) of graph shows forward biased characteristics of p–n junction II So,



p-n-II

9. As per given figure

$I_C = I_B + I_E$

So, $I_C \approx I_E \; (\because I_B << I_C)$

given, $I_C = 1\text{mA}, R_C = 7.8\,\text{k}\Omega$

From the given figure,

$I_C(R_C + R_E) + V_{CE} = V_{CC}$ (using Kirchoff's rule)

$(R_E + R_C) \times 1 \times 10^{-3} + 3 = 12$

$R_E + R_C = 9 \times 10^3 = 9\text{k}\Omega$

$R_E = 9 - 7.8 = 1.2\,\text{k}\Omega$

$V_E = I_E \times R_E = I_C \times R_E \quad [\because I_E = I_C]$

$= 1 \times 10^{-3} \times 1.2 \times 10^3 = 1.2\text{ V}$

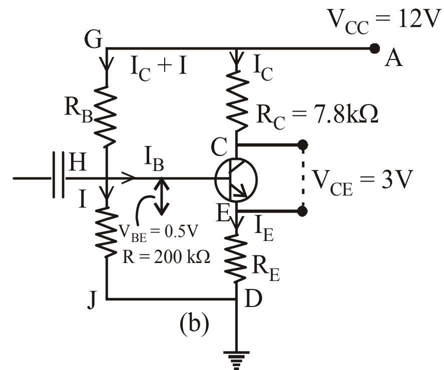
Voltage $V_B = V_E + V_{BE} = 1.2 + 0.5 = 1.7\text{ V}$

Current $I_B = \dfrac{V_B}{20\times10^3} = \dfrac{1.7}{20\times10^3} = 0.085\text{ mA}$

Resistance $R_B = \dfrac{V_E - V_B}{\left(\dfrac{I_c}{\beta} + I_B\right)} = \dfrac{(12-1.7)}{\left[\dfrac{1}{100} + 0.085\right]10^{-3}}$

$= \dfrac{10.3\times10^3}{[0.01+0.085]} \quad [\text{Given } \beta = 100]$

$= 108\text{ k}\Omega$



10. In the given figure,

By applying Kirchoff's rule at transistor

$I_E = I_C + I_B$ and $I_C = \beta I_B$ ...(i)

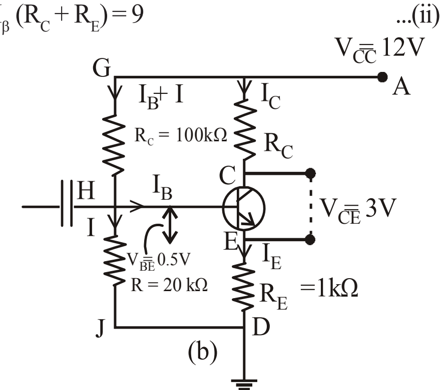
$I_C R_C + V_{CE} + I_E R_E = V_{CC}$ or $I_C R_C + 3 + I_E R_E = 12$

$I_C R_C + I_E R_E = 9 \quad (\because I_B << I_C)$

$I_E \cong I_C = \beta I_B$

So, $I_C R_C + (\beta I_B) R_E = 9$

$\beta I_\beta (R_C + R_E) = 9$ ...(ii)



By Kirchoff's Rule :–

$(RI_B + V_{(BE)} + R_E I_E = V_{CC})$

$\Rightarrow (I_B R + \beta I_B R_E) = V_{CC} - V_{BE}$

So, $(R + \beta R_E) I_B = V_{CC} - V_{BE}$

$\Rightarrow I_B = \dfrac{V_{CC} - V_{BE}}{R + \beta . R_E} = \dfrac{12 - 0.5}{(20 + 100\times1)k} = \dfrac{11.5}{120\times10^3}$

$= 0.096\text{ mA}$

From Eq. (ii), $\beta I_B (R_C + R_E) = 9$

$(R_C + R_E) = \dfrac{9}{\beta I_B} = \dfrac{9}{(100\times0.096)\text{mA}}$

$R_C + 1000\Omega = 0.938\text{ k}\Omega$

$R_C = 62\Omega$

Chapter Fifteen

COMMUNICATION SYSTEMS

SUMMARY

1. Electronic communication refers to the faithful transfer of information or message (available in the form of electrical voltage and current) from one point to another point.
2. Transmitter, transmission channel and receiver are three basic units of a communication system.
3. Two important forms of communication system are: Analog and Digital. The information to be transmitted is generally in continuous waveform for the former while for the latter it has only discrete or quantised levels.
4. Every message signal occupies a range of frequencies. The bandwidth of a message signal refers to the band of frequencies, which are necessary for satisfactory transmission of the information contained in the signal. Similarly, any practical communication system permits transmission of a range of frequencies only, which is referred to as the bandwidth of the system.
5. Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier signal by a process known as modulation.
6. In modulation, some characteristic of the carrier signal like amplitude, frequency or phase varies in accordance with the modulating or message signal. Correspondingly, they are called Amplitude Modulated (AM), Frequency Modulated (FM) or Phase Modulated (PM) waves.
7. Pulse modulation could be classified as: Pulse Amplitude Modulation (PAM), Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM).
8. For transmission over long distances, signals are radiated into space using devices called antennas. The radiated signals propagate as electromagnetic waves and the mode of propagation is influenced by the presence of the earth and its atmosphere. Near the surface of the earth, electromagnetic waves propagate as surface waves. Surface wave propagation is useful up to a few MHz frequencies.
9. Long distance communication between two points on the earth is achieved through reflection of electromagnetic waves by ionosphere. Such waves are called sky waves. Sky wave propagation takes place up to frequency of about 30 MHz. Above this frequency, electromagnetic waves essentially propagate as space waves. Space waves are used for line-of-sight communication and satellite communication.
10. If an antenna radiates electromagnetic waves from a height h_T, then the range d_T is given by $\sqrt{2Rh_T}$ where R is the radius of the earth.
11. Amplitude modulated signal contains frequencies $(\omega_c - \omega_m)$, ω_c and $(\omega_c + \omega_m)$.
12. Amplitude modulated waves can be produced by application of the message signal and the carrier wave to a non-linear device, followed by a band pass filter.
13. AM detection, which is the process of recovering the modulating signal from an AM waveform, is carried out using a rectifier and an envelope detector.

POINTS TO PONDER

1. In the process of transmission of message/ information signal, noise gets added to the signal anywhere between the information source and the receiving end. Can you think of some sources of noise?
2. In the process of modulation, new frequencies called sidebands are generated on either side (higher and lower than the carrier frequency) of the carrier by an amount equal to the highest modulating frequency. Is it possible to retrieve the message by transmitting (a) only the side bands, (b) only one side band?
3. In amplitude modulation, modulation index $\mu \leq 1$ is used. What will happen if $\mu > 1$?

EXERCISES

15.1 Which of the following frequencies will be suitable for beyond-the-horizon communication using sky waves?
(a) 10 kHz
(b) 10 MHz
(c) 1 GHz
(d) 1000 GHz

15.2 Frequencies in the UHF range normally propagate by means of:
(a) Ground waves.
(b) Sky waves.
(c) Surface waves.
(d) Space waves.

15.3 Digital signals
(i) do not provide a continuous set of values,
(ii) represent values as discrete steps,
(iii) can utilize binary system, and
(iv) can utilize decimal as well as binary systems.
Which of the above statements are true?
(a) (i) and (ii) only
(b) (ii) and (iii) only
(c) (i), (ii) and (iii) but not (iv)
(d) All of (i), (ii), (iii) and (iv).

15.4 Is it necessary for a transmitting antenna to be at the same height as that of the receiving antenna for line-of-sight communication? A TV transmitting antenna is 81m tall. How much service area can it cover if the receiving antenna is at the ground level?

15.5 A carrier wave of peak voltage 12V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75%?

15.6 A modulating signal is a square wave, as shown in Fig. 15.14.

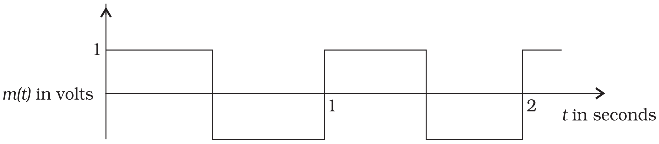


FIGURE 15.14

The carrier wave is given by $c(t) = 2\sin(8\pi t)$ volts.

(i) Sketch the amplitude modulated waveform

(ii) What is the modulation index?

15.7 For an amplitude modulated wave, the maximum amplitude is found to be 10V while the minimum amplitude is found to be 2V. Determine the modulation index, μ.
What would be the value of μ if the minimum amplitude is zero volt?

15.8 Due to economic reasons, only the upper sideband of an AM wave is transmitted, but at the receiving station, there is a facility for generating the carrier. Show that if a device is available which can multiply two signals, then it is possible to recover the modulating signal at the receiver station.

Chapter 15:- Communication Systems

Multiple Choice Questions (MCQs)

1. Three waves A, B and C of frequencies 1600 kHz, 5MHz and 60 MHz, respectively are to be transmitted from one place to another. Which of the following is the most appropriate mode of communication?
   (a) A is transmitted via space wave while B and C are transmitted via sky wave
   (b) A is transmitted via ground wave, B via sky wave and C via space wave
   (c) B and C are transmitted via ground wave while A is transmitted via sky wave
   (d) B is transmitted via ground wave while A and C are transmitted via space wave

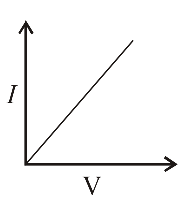
2. A 100 m long antenna is mounted on a 500m tall building. The complex can become a transmission tower for waves with λ
   (a) ~400 m (b) ~25 m
   (c) ~150 m (d) ~2400 m

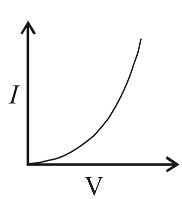
3. A 1 kW signal is transmitted using a communication channel which provides attenuation at the rate of – 2dB per km. If the communication channel has a total length of 5 km, the power of the signals received is [gain in $dB = 10 \log\left(\frac{P_0}{P_i}\right)$]
   (a) 900 W (b) 100 W
   (c) 990 W (d) 1010 W

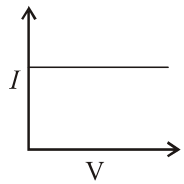
4. A speech signal of 3 kHz is used to modulate a carrier signal of frequency 1 MHz, using amplitude modulation. The freqeuncies of the side bands will be
   (a) 1.003 MHz and 0.997 MHz
   (b) 3001 kHz and 2997 kHz
   (c) 1003 kHz and 1000 kHz
   (d) 1 MHz and 0.997 MHz

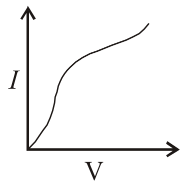
5. A message signal of frequency ω_m is superposed on a carrier wave of frequency w_c to get Amplitude Modulated Wave (AM). The frequency of the AM wave will be
   (a) ω_m (b) ω_c
   (c) $\frac{\omega_c + \omega_m}{2}$ (d) $\frac{\omega_c - \omega_m}{2}$

6. I – V characteristics of four devices are shown in figure.

(i)


(ii)


(iii) 

(iv) 

Identify devices that can be used for modulation
(a) (i) and (iii)
(b) only (iii)
(c) (ii) and some regions of (iv)
(d) All the devices can be used

7. A male voice after modulation–transmission sounds like that of a female to the receiver. The problem is due to
(a) poor selection of modulation index (selected $0 < m < 1$)
(b) poor bandwidth selection of amplifiers
(c) poor selection of carrier frequency
(d) loss of energy in transmission

8. A basic communication system consists of
A. transmitter
B. information source
C. user of information
D. channel
E. receiver
Choose the correct sequence in which these are arranged in a basic communication system.
(a) ABCDE (b) BADEC
(c) BDACE (d) BEADC

9. Identify the mathematical expression for amplitude modulated wave
(a) $A_c \sin [\{\omega_c + k_1 V_m (t)\} t + \phi]$
(b) $A_c \sin \{\omega_c t + \phi + k_2 V_m (t)\}$
(c) $\{A_c + k_2 V_m (t)\} \sin (\omega_c t + \phi)$
(d) $A_c V_m (t) \sin (\omega_c t + \phi)$

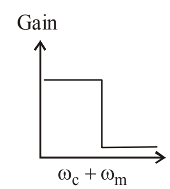
Multiple Choice Questions (MCQs) (More than one option correct)

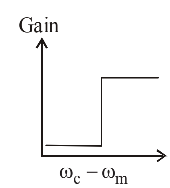
1. An audio signal of 15 kHz frequency cannot be transmitted over long distances without modulation, because
(a) the size of the required antenna would be least 5 km which is not convenient
(b) the audio signal can not be transmitted through sky waves
(c) the size of the required antenna would be at least 20 km which is not convenient
(d) effective power transmitted would be very low, if the size of the antenna is less than 5 km

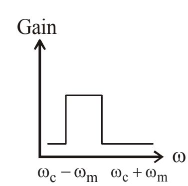
2. Audio sine waves of 3 kHz frequency are used to amplitude modulate a carrier signal of 1.5 MHz. Which of the following statements are true?
(a) The side band frequencies are 1506 kHz and 1494 kHz
(b) The bandwidth required for amplitude modulation is 6kHz
(c) The bandwidth required for amplitude modulation is 3 MHz
(d) The side band frequencies are 1503 kHz and 1479 kHz

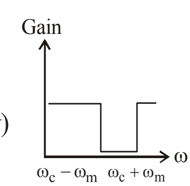
3. A TV transmission tower has a height of 240 m. Signals broadcast from this tower will be received by LOS communication at a distance of (assume the radius of earth to be (6.4×10^6m)
(a) 100 km (b) 24 km
(c) 55 km (d) 50 km

4. The frequency response curve (figure) for the filter circuit used for production of AM wave should be

(i) 

(ii) 

(iii) 

(iv) 

(a) (i) followed by (ii)
(b) (ii) followed by (i)
(c) (iii) (d) (iv)

5. In amplitude modulation, the modulation index m, is kept less than or equal to 1 because
(a) $m > 1$, will result in interference between carrier frequency and message frequency, resulting into distortion
(b) $m > 1$, will result in overlapping of both side bands resulting into loss of information
(c) $m > 1$, will result in change in phase between carrier signal and message signal
(d) $m > 1$, indicates amplitude of message signal greater than amplitude of carrier signal resulting into distortion

Very Short Answer Questions

1. Which of the following would produce analog signals and which would produce digital signals?
(a) A vibrating tuning fork
(b) Musical sound due to a vibrating sitar string
(c) Light pulse
(d) Output of NAND gate

2. Would sky waves be suitable for transmission of TV signals of 60 MHz frequency?

3. Two waves A and B of frequencies 2MHz and 3MHz, respectively are beamed in the same direction for communication via sky wave. Which one of these is likely to travel longer distance in the ionosphere before suffering total internal reflection?

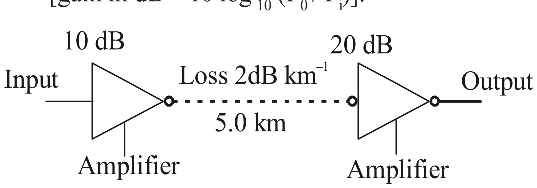
4. The maximum amplitude of an AM wave is found to be 15 V while its minimum amplitude is found to be 3V. What is the modulation index?

5. Compute the LC product of a tuned amplifier circuit required to generate a carrier wave of 1 MHz for amplitude modulation.

6. Why is a AM signal likely to be more noisy than a FM signal upon transmission through a channel?

Short Answer Questions

1. Figure shows a communication system. What is the output power when input signal is 1.01 mW? [gain in dB = 10 log $_{10}$ (P_0 / P_i)].



2. A TV transmission tower antenna is at a height of 20 m. How much service area can it cover if the receiving antenna is (i) at ground level, (ii) at a height of 25 m? Calculate the percentage increase in area covered in case (ii) relative to case (i).

3. If the whole earth is to be connected by LOS communication using space waves (no restriction of antenna size or tower height), what is the minimum number of antennas required? Calculate the tower height of these antennas in terms of earth's radius.

4. The maximum frequency for reflection of sky waves from a certain layer of the ionosphere is found to be $f_{max} = 9 (N_{max})^{1/2}$, where N_{max} is the maximum electron density at that layer of the ionosphere on a certain day it is observed that signals of frequencies higher than 5 MHz are not received by reflection from the F_1 layer of the ionosphere while signals of frequencies higher than 8 MHz are not received by reflection from the F_2 layer of the ionosphere. Estimate the maximum electron densities of the F_1 and F_2 layers on that day.

5. On radiating (sending out) and AM modulated signal, the total radiated power is due to energy carried by ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$. Suggest ways to minimise cost of radiation without compromising on information.

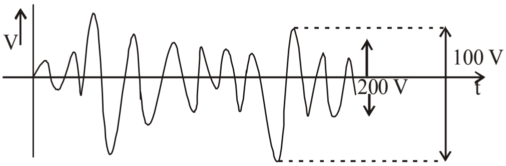
Long Answer Questions

1. The intensity of a light pulse travelling along a communication channel decrease exponentially with distance x according to the relation $I = I_0 e^{-\alpha x}$, where I_0 is the intensity at x = 0 and α is the attenuation constant.
(a) Show that the intensity reduces by 75% after a distance of $\left(\frac{\ln 4}{\alpha}\right)$.
(b) Attenuation of a signal can be expressed in decibel (dB) according to the relation dB = 10 $\log_{10}\left(\frac{I}{I_0}\right)$. What is the attenuation in dB/km for an optical fibre in which the intensity falls by 50% over a distance of 50 km?

2. A 50 MHz sky wave takes 4.04 ms to reach a receiver via re–transmission from a satellite 600 km above Earth's surface. Assuming re–transmission time by satellite negligible, find the distance between source and receiver. If communication between the two was to be done by Line of Sight (LOS) method, what should size and placement of receiving and transmitting antenna be?

3. An amplitude modulated wave is as shown in figure. Calculate
(i) the percentage modulation,
(ii) peak carrier voltage and

(iii) peak value of information voltage



4. (i) Draw the plot of amplitude versus ω for an amplitude modulated were whose carrier wave (ω_c) is carrying two modulating signals, ω_1 and ω_2 ($\omega_2 > \omega_1$).

(ii) Is the plot symmetrical about ω_c? Comment especially about plot in region $\omega < \omega_c$.

(iii) Extrapolate and predict the problems one can expect if more waves are to be modulated.

(iv) Suggest solutions to the above problem. In the process can one understand another advantage of modulation in terms of bandwidth?

5. An audio signal is modulated by a carrier wave of 20 MHz such that the bandwidth required for modulation is 3kHz. Could this wave be demodulated by a diode detector which has the values of R and C as

(i) R = 1kΩ, C = 0.01μF.

(ii) R = 10 kΩ, C = 0.01μF.

(iii) R = 10 kΩ, C = 0.1 μF.

NCERT EXEMPLAR SOLUTIONS

Multiple Choice Questions (MCQs)

1. **(b)** As we know that the range of frequencies for communication

Ground wave propagation – 530 kHz to 1710 kHz

Sky wave propagation – 1710 kHz to 40 MHz

Space wave propagation – 54 MHz to 42 GHz

So given frequency A, B and C are to be transmitted by ground wave, sky wave and space wave.

2. **(d)** Given, length of the building (l) is = 500m, Height or length of antenna from the ground height of building + hight of antenna

= (500 + 100) = 600 m

we know that, wavelength of the wave which can be transmitted by

$\lambda = 4l = 4 \times 600 = 2400$ m

3. **(b)** As given, that power of signal transmitted is given P_i = 1kW = 1000 W

Rate of attenuation of signal = –2dB/km

Length of total path = 5 km

So, gain in dB = 5 × (–2)dB = –10 dB

Also, gain in dB = $10 \log\left(\frac{P_0}{P_i}\right)$ ...(i)

where P_0 is the power of the received signal

Putting the dB values in Eq. (i),

$$-10 = 10\log\left(\frac{P_0}{P_i}\right) \Rightarrow 10\log\left(\frac{P_i}{P_0}\right) = 10$$

$$\log\frac{P_i}{P_0} = 1 \Rightarrow \log_e\left(\frac{P_i}{P_0}\right) = \log_e 10 \quad (\because \log_e 10 = 1)$$

taking antilog

So, $\frac{P_i}{P_0} = 10$

$$\frac{1000\text{W}}{P_0} = 10 \qquad (\because P_i = 1000\text{W})$$

$\Rightarrow P_0 = 100$ W

4. **(a)** Given, frequency of carrier signal is

ω_c = 1MHz

frequency of speech signal W_m = 3kHz

$= 3 \times 10^{-3}$ MHz

ω_m = 0.003 MHz

So frequency of side bands are :

$= (\omega_c \pm \omega_m) = (1 \pm 0.003)$

= 1.003 MHz and 0.997 MHz

5. **(b)** As we know that in amplitude modulation, the frequency of modulated wave is equal to the frequency of carrier wave.

Here, the frequency of carrier wave is ω_c.

So the amplitude modulated wave also has frequency ω_c.

6. **(c)** The device which follows square law can be used for modulation purpose. Characteristics shown by (i) and (iii) corresponds to linear devices as graphs are straight line. Characteristics shown by Graph (ii) corresponds to the square law device. Some part of graph (iv) also shows square law.

Hence, (ii) and (iv) can be used for modulation.

7. **(b)** We know that

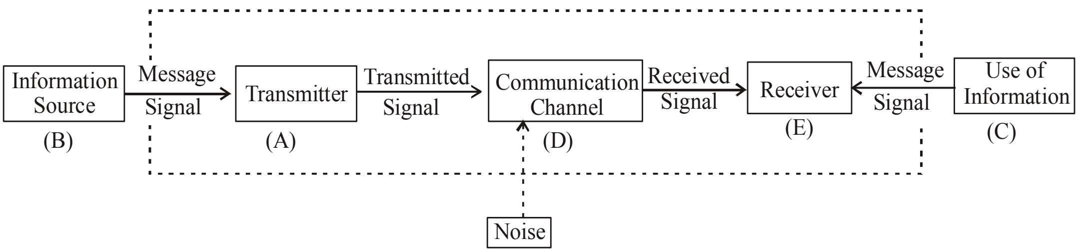
The frequency of male voice smaller than that of a female voice.

So, the frequency of modulated signal is more than input signal which is possible with the poor bandwidth selection of amplifiers.

This happens because bandwidth in amplitude modulation is equal to twice the frequency of modulated signal.

8. **(b)** Communication system is the set–up used in the transmission and reception of information travel from one place to another.

The basic communication system can be represented as the diagram given below



9. **(c)** Let us consider a sinusoidal signal to be modulating is

$m(t) = A_m \sin \omega_m t$

where, A_m = Amplitude of modulating signal

ω_m = (Angular frequency) = ω_m =

$2\pi V_m = \phi V_m$ ...(i)

$A_m = A_c + m(t)$

Now Let us consider a sinusoidal carrier wave is

$C(t) = A_c \sin \omega_c t$ ...(ii)

So, $C_m(t) = A_m \sin \omega_c(t)$

$C_m(t) = (A_c + A_m \sin \omega_m t) \sin\omega_c t$

$$= A_c\left[1 + \frac{A_m}{A_c}\sin \omega_m t\right]\sin \omega_m t$$

$$\because \quad \left(\frac{A_m}{A_c} = \mu\right)$$

$\Rightarrow \quad C_m(t) = (A_c + A_c \times \mu \sin\omega_m t) \sin\omega_m t$ ...(iii)

Now, we know that $A_c \times \mu = K$ [wave constant]

and $\sin \omega_m t = V_m(t)$ [wave velocity]

Thus, Eq. (iii) becomes

$C_m(t) = (A_c + K \times V_m(t)) \sin \omega_c t$

Let us consider a change in phase angle by ϕ then

$\sin \omega_c t \to \sin(\omega_c t + \phi)$

So, $C_m(t) = (A_c + KV_m(t)(\sin \omega_c + \phi)$

Multiple Choice Questions (More Than One Options)

1. **(a, b, d)** Transmission of a signal depends on three factors. These are size of antenna, power of transmission and power of transmitted wave.

As given that frequency of the audio signal to be transmitted is

$v_m = 15 \text{ kHz} = 15 \times 10^3 \text{ Hz}$

$$\text{Wavelength } \lambda_m = \frac{c}{v_m} = \frac{3\times10^8}{15\times10^3} = \frac{1}{5}\times10^5 \text{ m}$$

$= 0.2 \times 10^5$ m

Size of the antenna

$$l = \frac{\lambda}{4} = \frac{0.2\times10^5}{4} = 5\times10^3 \text{ m} = 5\text{km}$$

So, the audio frequency signals are of low frequency waves. So, the audiowave cannot be transmitted by sky waves as they are absorbed by atmosphere.

If the size of the antenna is less than 5 km, the effective power transmission would be very low because of deviation from resonance wavelength of wave and antenna length.

2. **(b, d)** The side band frequency are $(\omega_c \pm \omega_m)$

$=$ Bandwidth $= 2\,\omega_m$.

As given that,

$\omega_m = 3\text{ kHz}$

$\omega_c = 1.5\text{ MHz} = 1500\text{ kHz}$

So, side band frequencies

$= \omega_c \pm \omega_m = (1500 \pm 3)\text{ kHz}$

$= 1503\text{ kHz and } 1479\text{ kHz}$

Also, Bandwidth $= 2\omega_m = 2 \times 3 = 6\text{ kHz}$

3. (b, c, d)

As given that,

height of tower $h_T = 240$ m

Radius of earth $R = 6.4 \times 10^6$ m

For LOS (line of sight) communication range

$$d_T = \sqrt{2Rh_T}$$

So, the maximum distance on earth from the transmitter upto which a signal can be received

is $d_T = \sqrt{2Rh}$ ...(i)

Putting all these values in Eq. (i),

we get $d_T = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 240}$

$= 55.4 \times 10^3\text{ m} = 55.4\text{ km}$

So, the range of 55.4 km covers the distance 24 km, 55 km and 50 km.

4. (a, b, c)

According to the question,

The production of amplitude modulated wave, bandwidth is given by equal to the difference of frequencies of upper side band and lower side band. So,

$\omega = \omega_{USB} - \omega_{LSB} = (\omega_c + \omega_m) - (\omega_c - \omega_m) = 2\omega_m.$

5. (b, d) The modulation index (m) of amplitude modulated wave is

$$(m) = \frac{\text{amplitude of message signal }(A_m)}{\text{amplitude of carrier signal }(A_c)}$$

If $m > 1$, then $A_m > A_c$.

It means, there will be distortion of the resulting signal of amplitude modulated wave.

So, maximum modulation frequency (μ) of A_m wave is

$$m = \frac{\Delta\nu_{max}}{\nu_m(\max)}$$

$$m = \frac{\text{frequency deviation}}{\text{maximum frequency of modulating wave}}$$

If $m_f > 1$, then $\Delta\nu_{max} > \nu_m$ this means, there will be overlapping of both side bands of modulated wave resulting into loss of information.

Very short Answer Questions

1. Analog and digital both signals are used to transmit information, usually through electric signals. In both these technologies, the information such as any audio or video is converted into electrical signals.

In analog technology, information is translated into electric pulses of varying amplitude from zero to maximum. In digital technology, translation of information is into binary formal (zero (0) or one (1)) where each bit is representative of two distinct amplitudes, these amplitudes varying either zero (0) or max value (1).

So, (a) & (b) would produce analog signal and (c) & (d) will produce digital signals.

2. A signal of frequency range of 1710 kHz to 40 MHz, can be transmitted by sky waves.

But, the frequency of TV signals are 60 MHz which is beyond the sky wave of required range. So, sky waves will not be suitable for transmission or TV signals of 60 MHz frequency and given signal can be transmitted by space wave transmission.

3. As the frequency of wave B is more than wave A, it means the refractive index of wave B is more than refractive index of wave A (as refractive index increase with frequency increases). So ($\mu \propto \nu$) and critical angle is inversely proportional to refractive index ($\sin i_c = (\frac{1}{\mu})$). So, critical angle for A will be larger than for B or in other way

$\nu_A < \nu_B$ (given)

So, $\mu_A < \mu_B$ $[\because \mu \propto \nu]$

$$\frac{1}{\sin i_A} < \frac{1}{\sin i_B} \Rightarrow \sin i_A > \sin i_B \Rightarrow i_A > i_B$$

For higher frequency wave (i.e., higher refractive index) the angle of refraction is less i.e., bending is less. So, wave B will travel longer distance in the ionosphere before getting total internal reflection through ionosphere.

4. Consider A_c and A_m be the amplitudes of carrier wave and modulating wave respectively. So,

For AM wave maximum amplitude (A_{max})

$= A_c + A_m = 15\text{ V}$ ...(i)

For AM wave minimum amplitude (A_{min})

$= A_c - A_m = 3\text{V}$ ...(ii)

Adding Eqs. (i) and (ii) them we have

$2A_c = 18$

or $A_c = 9\text{V}$

then $A_m = 15 - 9 = 6\text{V}$ (from (i) and (ii))

So, modulating Index

$$(m) = \frac{A_m}{A_c} = \frac{6}{9} = \frac{2}{3}$$

5. As given that, the frequency of carrier wave is 1, f= 1 MHz

Formula for the frequency of tuned amplifier,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

So equating both of them So,

$$\frac{1}{2\pi\sqrt{LC}} = 1\text{MHz}$$

$$\sqrt{LC} = \frac{1}{2\pi \times 10^6}$$

Squaring both side

$$LC = \frac{1}{\left(2\pi\times 10^6\right)^2} = 2.54\times 10^{-14}\text{ sec}^2$$

So, the product of LC is $2.54 \times 10^{-14}\text{ sec}^2$.

6. In Amplitude Modulated wave the instantaneous voltage of carrier waves is varied by the modulating wave voltage. So, during the transmission, through noise signals can also be added and receiver assumes noise a part of the modulated signal.

In frequency modulated carrier waves, the frequency of carrier waves is changed as the change in the instantaneous voltage and amplitude of modulating waves. That can be done by mixing and not while the signal is transmitting in channel. So, noise does not affect FM signal.

Short Answer Questions

1. As given that

The distance travelled by the signal is 5 km

Loss suffered in path of transmission = 2 dB/km

So, total loss suffered in 5 km = –2 × 5 = – 10 dB

Total gain of a signal in both input and output amplifier.

So, Total amplifier gain = (10dB + 20 dB) = 30 dB

So, Overall gain in output signal (30 – 10) = 20 dB

According to the question, gain in dB = 10

$$\log_{10}\left(\frac{P_0}{P_i}\right)$$

$$\therefore \quad 20 = 10\log_{10}\left(\frac{P_0}{P_i}\right)$$

$$\text{Or} \quad \log_{10}\left(\frac{P_0}{P_i}\right) = 2$$

$$\left(\therefore \log 10^{10} = 1\right)$$

$$\log_{10}\left(\frac{P_0}{P_i}\right) = 2\log_{10}10 = \log_{10}10^2$$

$$\text{So,} \quad \left(\frac{P_0}{P_i}\right) = 100$$

Here, $P_i = 1.01$ mW and P_0 is the output power.

$$\therefore \quad \frac{P_0}{P_i} = 10^2 = 100$$

$\Rightarrow \quad P_0 = P_i \times 100 = 1.01 \times 100$

$P_0 = 101\text{ mW}$

So, the output power is 101 mW.

2. As given that height of antenna $h_T = 20$ m

Radius of earth $= 6.4 \times 10^6$ m

Receiving antenna is at ground level : –

(i) Range of transmitting antenna (r_1)

$$= \sqrt{2hR} = \sqrt{2 \times 20 \times 6.4 \times 10^6}\,m$$

$= 16000m = 16\,km$

Area covered by transmitting antenna = π (range)^2

$= 3.14 \times 16 \times 16 = 803.84\,km^2$

(ii) Receving antenna at a height of $H = 25$ m from ground level. Range due to both antenna (r_2).

$$\text{Range}(r_2) = \sqrt{2hR} + \sqrt{2HR}$$

$$= \sqrt{2 \times 20 \times 6.4 \times 10^6} + \sqrt{2 \times 25 \times 6.4 \times 10^6}$$

$= 16 \times 10^3 + 17.9 \times 10^3 = 33.9 \times 10^3$ m

$= 33.9$ km

Area covered by the transmitted and receiving antenna

$= \pi\,(\text{Range})^2 = 3.14 \times 33.9 \times 33.9$

$= 3608.52\,km^2$

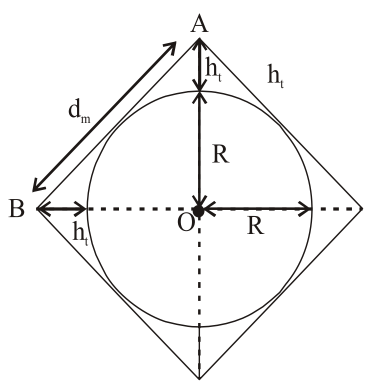
Percentage increase in area in two transmissions

$$= \frac{\text{Difference in area}}{\text{Initial area}} \times 100$$

$$= \frac{(3608.52 - 803.84)}{803.84} \times 100 = 348.9\%$$

Thus, the percentage increase in area covered is 348.9%.

3. Let us consider the figure given below



Let the height of transmitting antenna or receiving antenna in order to cover the entire surface of earth through communication is h_t, and radius of earth is R.

Then, maximum distance between A and B is d_m

By Pythagoras theorem,

$$d_m^2 = (R + h_t)^2 + (R + h_t)^2$$

$= 2\,(R + h_t)^2$ ...(i)

Range of Antenna A and B

$$d_m = \sqrt{2h_tR} + \sqrt{2h_tR} = 2\sqrt{2h_tR}$$

$\Rightarrow d_m^2 = 4\,(2Rh_t)$ ...(ii)

From (i) and (ii),

$\therefore\ 8h_t R = 2\,(R + h_t)^2$ ...(iii)

$\Rightarrow 4h_t R = R^2 + 2Rh_t + h_t^2$

$\Rightarrow R^2 - 2h_t R + h_t^2 = 0$

$\Rightarrow (R - h_t)^2 = 0$

$R = h_t$

So the height of Antenna is equal to the radius of earth.

Therefore, space wave frequency is used $\lambda << h_t$, hence only tower height is to be taken into consideration. In three dimensions of earth, 6 antenna towers of each of height $h_t = R$ would be used to cover the entire surface of earth with communication programme.

4. As given that

The maximum frequency for reflection of sky waves

$f_{max} = 9(N_{max})^{1/2}$ ...(i)

where, N_{max} is a maximum electron density

For F_1 layer,

$f_{max} = 5\,MHz = 5 \times 10^6\,Hz$ ...(ii)

By equating (i) and (ii),

$5 \times 10^6 = 9\,(N_{max})^{1/2}$

Maximum electron density

$$\text{from (i) equation, } N_{max} = \left(\frac{f_{max}}{9}\right)^2$$

$$N_{max} = \left(\frac{5}{9} \times 10^6\right)^2 = 3.086 \times 10^{11}/m^3$$

For F_2 layer,

$f_{max} = 8\ MHz = 8 \times 10^6\ Hz$ ...(iii)

By equating (i) and (iii),

$8 \times 10^6 = 9\ (N_{max})^{1/2}$

Maximum electron density

By equation (i), $N_{max} = \left(\frac{f_{max}}{9}\right)^2$

$$N_{max} = \left(\frac{8 \times 10^6}{9}\right)^2 = 7.9 \times 10^{11}/m^3$$

So maximum electron density for layer F_2 is $N_{max} = 7.9 \times 10^{11}\ e/m^3$.

5. Only side band frequency contain information. So, only $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$ contain information in amplitude modulated signal

Now, as per given question, the total radiated power is due to energy carried by

$\omega_c, (\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$

So to minimise the cost of radiation the information signal without compromising on information ω_c can be send or left either $(\omega_c + \omega_m)$ or $(\omega_c - \omega_m)$ or both $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$.

Long Answer Questions

1. **(a)** As given that intensity of a light pulse $I = I_0 e^{-\alpha x}$

where, I_0 is the intensity at $x = 0$ and α is constant.

According to the question remaining Intensity reduced by 75% so,

remaining

$$I = 25\% \text{ of } I_0 = \frac{25}{100}.I_0 = \frac{I_0}{4} \text{ So, } \left(I = \frac{I_0}{4}\right)$$

Using the formula mentioned in the question,

$I = I_0 e^{-\alpha x}$

$$\frac{I_0}{4} = I_0 e^{-\alpha x}$$

So, $\frac{1}{4} = e^{-\alpha x}$

Taking log on both sides, we get

$(\ln 1 - \ln 4) = \log_e e^{-\alpha x}$

$\ln 1 - \ln 4 = -\alpha x \ln e \qquad (\because \ln_e = 1)$

$0 - \ln 4 = -\alpha x \qquad (\because \ln 1 = 0)$

$$x = \frac{\ln 4}{\alpha}$$

So, at distance $x = \frac{\ln 4}{\alpha}$, the intensity is reduced by 75% of initial intensity.

(b) Let α be the attenuation in dB/km. If x is distance travelled by signal,

Then, $10 \log_{10}\left(\frac{I}{I_0}\right) = -\alpha x$ ...(i)

where, I_0 is the intensity initially.

According to the question Intensity reduced by 50% so remain

$$I = (50\% \text{ of } I_0) = \frac{I_0}{2} \text{ and } x = 50km$$

Putting the value of x in Eq. (i), we get

$$10 \log_{10}\left(\frac{I_0}{2I_0}\right) = -\alpha \times 50$$

$10\ [\log 1 - \log 2] = -50\alpha$

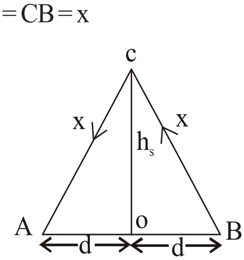
$$\frac{10 \times 0.3010}{50} = \alpha,\ \alpha = 0.0602 \text{ dB per km}$$

So, The attenuation of signal for an optical fibre channel.

$\alpha = 0.0602$ dB/km

2. Let the receiver is at point A and source is at B and the distance between AO = OB = d

CA = CB = x



Velocity of waves = 3×10^8 m/s

Method time to reach a receiver 4.04 ms

$= 4.04 \times 10^{-3}$ sec

So, by echo the height of satellite is given

($h_s = 600$ km).

Radius of earth = 6400 km

Size of transmitting antenna = h_T

We know that,

$$\frac{\text{Distance travelled by wave}}{\text{Time}} = \text{velocity of waves}$$

or $\frac{2x}{t} = V$

$$\frac{2x}{4.04 \times 10^{-3}} = 3 \times 10^8$$

or $x = \frac{3 \times 10^8 \times 4.04 \times 10^{-3}}{2}$

$= 6.06 \times 10^5 = 606$ km

Using Phythagoras theorem,

$$d^2 = x^2 - h_s^2 = (606)^2 - (600)^2 = 7236$$

$d^2 = 7236$ km

$\Rightarrow d = 85.06$ km

The distance between source and receiver = 2d

$= 2 \times 85.06 = 170$ km

For the maximum distance covered by signal from satellite on ground from the transmitter

$$d = \sqrt{2Rh_T}$$

or $\frac{d^2}{2R} = h_T$

or size of antenna $h_T = \frac{7236}{2 \times 6400}$ km

$= 0.565$ km $= 565$ m

We can not obtain signals of ($h_T = 565$ m) at d = 85.06 km, So hight or size of required antenna = 566 m.

3. As the given diagram,

Maximum voltage $V_{max} = \frac{100}{2} = 50V$

Minimum voltage $V_{min} = \frac{20}{2} = 10V$

(i) Percentage of modulation, m

$$= \left(\frac{V_{max} - V_{min}}{V_{max} + V_{min}}\right) \times 100 = \left(\frac{50-10}{50+10}\right) \times 100$$

$$= \frac{40}{60} \times 100 = 66.67\%$$

(ii) Peak carrier voltage,

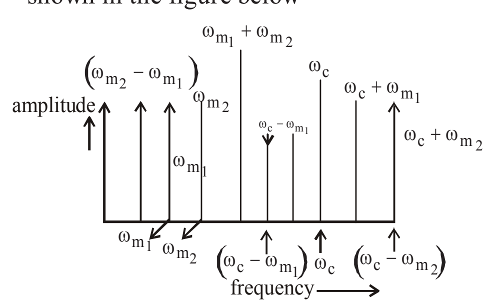
$$V_o = \frac{V_{max} + V_{min}}{2} = \frac{50+10}{2} = 30V$$

(iii) Peak value of information voltage,

$V_{om} = m V_{oc}$

$$= \frac{66.67}{100} \times 30 = 20V$$

4. (i) The plot of amplitude versus ω can be shown in the figure below



$\because (\omega_2 > \omega_1)$

So, increasing order of frequencies is

$(\omega_{m_2} - \omega_{m_1})$

$\omega_{m_1} - \omega_{m_2}$, $(\omega_{m_1} + \omega_{m_2})$ $(\omega_{c_1} - \omega_{m_2})$,

$(\omega_c - \omega_{m_1})$ ω_c,

$(\omega_c + \omega_{m_1})$, $(\omega_c + \omega_{m_2})$. So increasing order amplitude is $(\omega_{m_2} - \omega_{m_1})$, ω_{m_1},

$\omega_{m\frac{2}{2}} (\omega_{m_2} - \omega_{m_1}), \omega_c$,

$(\omega_c + \omega_{m_1})$, $(\omega_c - \omega_{m_2})$, $(\omega_c - \omega_{m_1})$ and $(\omega_c + \omega_{m_2})$.

(ii) From figure we seen that frequency spectrum and graph is not symmetrical about ω_c and more frequency lies below or before ω_c. So, Crowding of spectrum is present for $\omega < \omega_c$.

(iii) If more waves are to be modulated then there will be more frequency lines in the modulating signal in the region $\omega < \omega_c$. Then the more chances of mixing of signals or overlapping of signals.

(iv) To accommodate more signals, we must be increase bandwidth and frequency carrier waves W_C. This shows that large carrier frequency enables to carry more information or modulated signals (i.e., more ω_m) and the same will in turn increase bandwidth.

5. As given, carrier wave frequency

$f_c = 20\text{ MHz} = 20 \times 10^6\text{ Hz}$

$\omega = 2\pi f_c = 2\pi \times 20 \times 10^6 = 40\pi \times 10^6\text{ rad/s}$

Bandwidth required for modulation is $= 2f_m$

(given) $2f_m = 3\text{kHz} = 3 \times 10^3\text{ Hz}$

$$\Rightarrow \quad f_m = \frac{3\times10^3}{2} = 1.5\times10^3\text{ Hz}$$

Demodulation by a diode is possible if the condition $\frac{1}{f_c} << RC < \frac{1}{f_m}$ is satisfied

$$\text{Thus, } \frac{1}{f_c} = \frac{1}{20\times10^6} = \left(5\times10^{-8}\right)\text{s} \qquad ...(i)$$

$$\text{and } \frac{1}{f_m} = \frac{1}{1.5\times10^3\text{ Hz}} = 0.7\times10^{-3}\text{s} \qquad ...(ii)$$

Now, gain through all the options of R and C one by one, we get

(i) $RC = 1\text{ k}\Omega \times 0.01\mu\text{F} = 10^3\,\Omega \times (0.01 \times 10^{-6}\text{ F})$
$= 10^{-5}\text{s}.$

So from condition $\frac{1}{f_c} << RC < \frac{1}{f_m}$ is satisfied because,

$5 \times 10^{-8} << 10^{-5} < 0.7 \times 10^{-3}$ is true.

Hence the wave can be demodulated.

(ii) $RC = 10\text{ k}\Omega \times 0.01\mu\text{F}$
$= 10^4\,\Omega \times 10^{-8}\text{F} = 10^{-4}\text{ s}$

Here condition $\frac{1}{f_c} << RC < \frac{1}{f_m}$ is satisfied because,

$5 \times 10^{-8} << 10^{-4} < 0.7 \times 10^{-3}$ is true.

So, the wave can be demodulated.

(iii) $RC = 10\text{ k}\Omega \times 1\mu\mu\text{F}$
$= 10^4\,\Omega \times 10^{-12}\text{F} = 10^{-8}\text{ s}.$

Here, condition $\frac{1}{f_c} < RC$, Condition,

$\frac{1}{f_c} << RC < \frac{1}{f_m}$ is, not satisfied because

$5 \times 10^{-8} << 10^{-8} < 0.7 \times 10^{-3}$ is not true.

So, the modulated carrier wave cannot be demodulated.